



Charged condensation

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Abstract

We consider Bose–Einstein condensation of massive electrically charged scalars in a uniform background of charged fermions. We focus on the case when the scalar condensate screens the background charge, while the net charge of the system resides on its boundary surface. A distinctive signature of this substance is that the photon acquires a Lorentz-violating mass in the bulk of the condensate. Due to this mass, the transverse and longitudinal gauge modes propagate with different group velocities. We give qualitative arguments that at high enough densities and low temperatures a charged system of electrons and helium-4 nuclei, if held together by laboratory devices or by force of gravity, can form such a substance. We briefly discuss possible manifestations of the charged condensate in compact astrophysical objects.

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1. Introduction and summary

Consider a sphere enclosing massive stable charged spin-1/2 particles with number density \bar{J}_0 , and stable massive spin-0 particles of an equal but opposite charge. At some high temperature the substance in the sphere could form hot plasma. With the decreasing temperature the opposite charges would ordinarily form neutral atoms of half-integer spins. These atoms would not be able to Bose–Einstein condense because of their spin-statistics.

We will discuss in this work a different sequence of events that could take place in the above system. In particular, we will show that under certain conditions, instead of forming neutral atoms, the charged scalars could themselves condense, neutralizing by this condensate the background charge of the fermions.

Especially interesting we find the case when the system has a net overall charge to begin with. In this case, although the resulting substance is charge neutral in the interior of the sphere, the net charge will reside on its surface. The substance in the bulk has distinctive properties. We will show in Section 2 that propagation of a photon in this substance is rather special. Even at zero temperature, the photon acquires a Lorentz non-invariant

mass term. The transverse and longitudinal components of the photon have equal masses; the mass squares are proportional to \bar{J}_0 and inversely proportional to the charged scalar mass. However, the group velocities of the transverse and longitudinal modes are different. The longitudinal mode is similar to a plasmon excitation of cold plasma. The transverse modes of the photon propagate as massive states. We will refer to this phase as the charged condensate, emphasizing that the charged scalars have undergone Bose–Einstein condensation, while the background fermions merely play the role of charge neutralizers in the bulk of the substance, and the net charge of the system is residing on the boundary.

The above mechanism is universal: the gauge field could be a photon or any other $U(1)$ field, while the charged scalar could be a fundamental field, or a composite state made of other particles, in the regime where its compositeness does not matter. This may have applications in particle physics and condensed matter systems.

As a concrete example we imagine a reservoir, or a trap, in which negatively charged electrons and positively charged helium-4 nuclei, with a nonzero net charge, could be put together at densities high enough for an average inter-particle separation to be smaller than the size of a helium atom. In this case, the helium atoms would not form. The results of Section 2 cannot immediately be applied to this case, since electrons are lighter than the helium nuclei. However, we will argue in Sec-

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tion 3 that if temperature of the system is low enough for the helium de Broglie wavelength to be greater than both the average inter-particle separation and the Compton wavelength of the massive photon, then the charged helium-4 nuclei would fall into the condensate. Photons, in the bulk of this substance, would propagate with a delay caused by the acquired mass. Such a system would also have a net surface charge. Quantitative features of this example are discussed in Section 3. Our estimate for the temperature is within the range of the low temperatures that have already been achieved in experiments on Bose–Einstein condensation of atoms, see, e.g., [1].

In the above example the charged condensate containing droplet was assumed to be held together by a rigid boundary or external fields in a laboratory. In Section 4 we point out that gravity could play the role of the stabilizing force, and briefly discuss possible manifestations of the charged condensation in compact astrophysical objects.

A few comments on the literature. The pion condensation due to strong interactions is well known [2]. In this work we discuss condensations due to electromagnetic interactions instead (or in more general case, due to some $U(1)$ Abelian interactions). It was shown in Ref. [3] that the constant charge density strengthens spontaneous symmetry breaking when the symmetry is already broken by the usual Higgs-like nonlinear potential for the scalar. In our work the scalar has a conventional positive-sign mass term. The fact that the conventional-mass scalar could condense in the charged background was first shown in [4]. However, the system considered in [4] is neutral, and thus, is physically different from the one studied in this work (see, brief comments after Eq. (4.6) in [4]). An expanded discussions of the topics covered in the present work, with other possible applications will be presented elsewhere [5].

2. Basic mechanism

We consider a simplest model that exhibits the main phenomenon. Let us start with a system in an infinite volume and at zero-temperature. The classical Lagrangian contains a gauge field A_μ , a charged scalar field ϕ with a right-sign mass term $m_H^2 > 0$, and fermions Ψ^+ , Ψ with mass m_J

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m_H^2\phi^*\phi + \bar{\Psi}i\gamma^\mu D_\mu\Psi - m_J\bar{\Psi}\Psi + \mu\Psi^+\Psi. \quad (1)$$

The chemical potential μ is introduced for the global fermion number carried by Ψ 's (e.g., lepton, baryon or other number). The covariant derivatives in (1) are defined as $\partial_\mu + ig_\phi A_\mu$ for the scalars, and $\partial_\mu + ig_\psi A_\mu$ for the fermions. Their respective charges, g_ϕ and g_ψ , are different in general. For simplicity we assume that $g_\phi = -g_\psi \equiv -g$.

To study the ground state it is convenient to introduce the following notations for the scalar, gauge field and fermions: $\phi = \frac{1}{\sqrt{2}}\sigma e^{i\alpha}$, $B_\mu \equiv A_\mu + \frac{1}{g}\partial_\mu\alpha$, and $\psi = \Psi e^{-i\alpha}$. In terms of the gauge invariant variables σ , B_μ and ψ the Lagrangian, takes the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}g^2 B_\mu^2\sigma^2 - \frac{1}{2}m_H^2\sigma^2 + \bar{\psi}i\gamma^\mu D_\mu\psi - m_J\bar{\psi}\psi + \mu\psi^+\psi, \quad (2)$$

where now $F_{\mu\nu}$ and D are a field-strength and covariant derivative for B_μ , respectively.

Fermions in (2) obey the conventional Dirac equation with a nonzero chemical potential. This implies a net fermion number in the system, \bar{J}_0 . Since the fermions are also electrically charged, they set a background electric charge density. Such charged fermions would repel each other. In our case, however, the charge will be screened by the charged scalar condensate. One way to see this is to assume that such a self-consistent solution exists, and then check explicitly that it satisfied equations of motion, as we will do it below. We consider distance scales that are greater than an average separation between the fermions, so that their spatial distribution could be assumed to be uniform. Then, the background charge density due to the fermions could be approximated as $\bar{J}_\mu = \bar{J}_0\delta_{\mu 0}$, where \bar{J}_0 is a constant. The magnitude of the latter is related to the value of the chemical potential μ . In particular, a self-consistent solution of the equations of motion implies that $\mu - \langle gB_0 \rangle = E_F$, where E_F denotes the Fermi energy of the background fermion sea, and is related to \bar{J}_0 as follows, $E_F = \sqrt{(3\pi\bar{J}_0/4)^{2/3} + m_J^2}$.

The rest of the equations of motion derived from (2) are:

$$\partial^\mu F_{\mu\nu} + g^2 B_\nu\sigma^2 = g\bar{J}_\nu, \quad \square\sigma = g^2 B_\nu^2\sigma - m_H^2\sigma. \quad (3)$$

The Bianchi identity for the first equation in (3), $\partial^\nu(B_\nu\sigma^2) = 0$, can also be obtained by varying the action w.r.t. α . For a constant charge density, $\bar{J}_\mu = \bar{J}_0\delta_{\mu 0}$, the theory with the scalar field (1) admits a static solution with constant B_0 and σ :

$$\langle B_0 \rangle = B_{0c} \equiv \frac{m_H}{g}, \quad \langle \sigma \rangle = \sigma_c \equiv \sqrt{\frac{\bar{J}_0}{m_H}}. \quad (4)$$

The charge density stored in the condensate, $J_0^{\text{scalar}} = -i[\phi^*D_0\phi - (D_0\phi)^*\phi] = -g\sigma^2 B_0$, equals to $-\bar{J}_0$, by virtue of (4). Hence, the total charge density $J_{\text{total}} = \bar{J}_0 + J_0^{\text{scalar}} = 0$, vanishes. The ground state is charge-neutral in its bulk. On the other hand, a nonzero $\langle B_0 \rangle$ in (4) suggests that there must be an uncompensated charge on a surface at infinity, as it will be the case (see below).

Before we continue with studies of small perturbations about the solution (4), we would like to make four essential comments:

(i) The expression for the gauge field in (4) scales as $1/g$, and is non-perturbative in its nature. Moreover, it diverges in the limit $m_H \rightarrow \infty$. This seeming non-decoupling of the charged scalar field results from the fact that we are dealing with a constant background *charge density* in an infinite volume, i.e., with an infinite background charge. It is not surprising then, that such a background is capable of affecting a charged state of an arbitrary mass. Moreover, when m_H exceeds the fermion mass, our averaging procedure over the background charges should not be applicable in general.

(ii) In regard with the above discussions, it is instructive to regularize the problem by considering a finite volume ball of a

radius R . A nonzero $\langle B_0 \rangle$ in (4) suggests that there must be an uncompensated charge on the surface of the ball, which tends to the value, $Q = m_H R/g$, as $R \rightarrow \infty$. Indeed, such a charge Q could give rise to a constant $\langle B_0 \rangle = m_H/g$ in the interior of the ball, where $\langle B_0 \rangle = Q/R$, in analogy with a static potential inside a conducting ball with surface charge Q . This is indeed what happens in the present case. These and other finite volume effects are discussed in detail in Section 3.

(iii) Unlike for the fermions, we have not introduced chemical potential for the scalars. However, nonzero $\langle gB_0 \rangle$ acts as dynamically induced chemical potential for the perturbations of the scalar. Its value in the ground state, $\langle gB_0 \rangle = m_H$, is consistent with the expectation that the chemical potential be equal to the mass of the scalar in Bose–Einstein condensate.

In general, we could have introduced chemical potential for the charged scalar, μ_s . The above described condensation mechanism would still take place with the result, $\langle gB_0 \rangle = m_H + \mu_s$, and $\sigma_c^2 = \bar{J}_0/m_H$, instead of (4). The charge density in the condensate in this case would read, $-(\mu_s - gB_0)\sigma^2 = -\bar{J}_0$, ensuring charge neutrality of the substance in its bulk, but in general there would be a nonzero surface charge, unless $\mu_s = -m_H$ and $\langle gB_0 \rangle = 0$.

(iv) So far our discussions have been classical. Upon quantization the charged condensate can be thought of a zero-momentum state with a nonzero occupation number of the charged scalar field quanta. It is useful to consider small temperature T in the system, in which case the de Broglie wavelength of the condensed scalars, $\lambda_T \sim (1/m_H T)^{1/2}$, will exceeds the average inter-particle separation $\sim \bar{J}_0^{-1/3}$. Thus, it makes sense to think of the charged condensate, as of any other Bose–Einstein condensate, to be a macroscopically occupied mode. The specifics of our case is that this macroscopic state of electrically charged scalars can exist even when the Compton wavelength of the corresponding massive photon is greater than the average interparticle separation between the scalars. In the bulk of the condensate the charge is balanced by the background charge density of fermions.

The uniform fermion background sets a preferred Lorentz frame. We study the spectrum and propagation of perturbations in this background frame. For this we introduce small perturbations of gauge and scalar fields, b_μ and τ , as follows:

$$B_\mu = B_{0c}\delta_{\mu 0} + b_\mu(x), \quad \sigma = \sigma_c + \tau(x). \quad (5)$$

The Lagrangian density for the perturbations reads

$$\mathcal{L}_2 = -\frac{1}{4}f_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\tau)^2 + \frac{1}{2}g^2\sigma_c^2 b_\mu^2 + 2gm_H\sigma_c b_0\tau + \dots \quad (6)$$

Here $f_{\mu\nu}$ denotes the field strength for b_μ , and we dropped all the fermionic terms as well as the cubic and quartic interaction terms of b 's and τ . The last term in (6) is Lorentz violating. Calculations of the spectrum of the theory is non-trivial but straightforward. We briefly summarize the results. First, b_0 is not a dynamical field, as it has no time derivatives in (6). Therefore, it can be integrated out through its equation of motion,

leaving us with the equations for three polarizations of a massive vector b_j , $j = 1, 2, 3$, and one scalar τ . These constitute four physical degrees of freedom of the theory. The transverse part of the vector b_j obeys the free equation

$$(\square + g^2\sigma_c^2)b_j^T = 0, \quad \text{where } b_j^T \equiv b_j - \frac{\partial_j}{\Delta}(\partial_k b_k). \quad (7)$$

Therefore, the two states of the gauge field carried by b_j^T have the following mass

$$m_g^2 = g^2\sigma_c^2 = g^2 \frac{\bar{J}_0}{m_H}. \quad (8)$$

Moreover, the frequency ω and the three-momentum vector \mathbf{p} of these two states obey the conventional dispersion relation, $\omega^2 = \mathbf{p}^2 + m_g^2$.

The longitudinal mode of the gauge field b_j^L , and the scalar τ , on the other hand, give rise to the following Lorentz-violating dispersion relations (valid for $m_g \neq 0$)

$$\omega_\pm^2 = \mathbf{p}^2 + 2m_H^2 + \frac{1}{2}m_g^2 \pm \sqrt{4\mathbf{p}^2 m_H^2 + (2m_H^2 - \frac{1}{2}m_g^2)^2}. \quad (9)$$

The r.h.s. of (9) is positive. Both of these modes have masses which can be obtained by putting $\mathbf{p} = 0$ in (9). One of them coincides with (8), and the other one, has the mass squared equal to $m_s^2 = 4m_H^2$. Interestingly, the group velocities of the transverse and longitudinal modes of the massive vector boson are different. For $m_H \gg m_g$, and for an arbitrary \mathbf{p} , the fastest ones are the transverse modes, they are followed by the scalar, and the longitudinal mode is the slowest.

In the limit $m_H \rightarrow 0$, (9) describes a massive longitudinal component of a vector bosons of mass m_g , and a massless scalar, in agreement with (6). The limit $m_g \rightarrow 0$, however, is discontinuous, since for any nonzero m_g in (6) one has to satisfy the Bianchi identity which would not appear as a constraint if m_g had been set to zero in (6) from the very beginning.

It is important to specify the limits of applicability of the above condensation mechanism. (I) The Lagrangian (1) could contain a quartic interaction term for the scalar $\lambda(\phi^*\phi)^2 = \lambda\sigma^4/4$. It is straightforward to check that our results will hold as long as $\lambda m_g^2 \ll g^2 m_H^2$. (II) The scalar could have an additional Yukawa term, $q(\phi^*\psi_1\Gamma\psi_2 + \text{h.c.})$, where q is a coupling, Γ denotes either the $\mathbf{1}$ or $i\gamma_5$ matrix depending on the spatial parity of ϕ , and $\psi_{1,2}$ denote fermions with different $U(1)$ charges that render the Yukawa term gauge invariant. One, or both of these fermions could be setting the background charge density \bar{J}_0 . The fermion condensate, $\langle \bar{\psi}_1\psi_2 + \text{h.c.} \rangle$, if nonzero, could act as a source for the scalar. In order for this not to change significantly our results, the condition $q\langle \bar{\psi}_1\psi_2 + \text{h.c.} \rangle \ll m_H^2\sigma_c$ should be met.¹ (III) Due to the above Yukawa couplings the scalar ϕ can decay. In order for the condensate phase to form

¹ The Yukawa coupling would also lead to the new terms in the fermion mass matrix. Depending on a concrete context, this may or may not impose additional constraints.

in the first place, the “condensation time” σ_c^{-1} has to be shorter than the lifetime of the ϕ . Through the work we will be checking the conditions (I–III) when appropriate.

If the number density of the background fermions is such that it allows for the average inter-particle separation between them to be greater than the Bohr radius of a fermion-scalar bound state, then, the fermions would likely form a crystalline structure at low temperatures. If the resulting crystal is due to the metallic bonding, that is it supports quantum gas of almost free scalars, then the condensation of the scalars described above would be similar to the condensation of Cooper pairs in superconductors. This case could be realized if $J_0 \lesssim g^6 m_H^3$.

On the other hand, if the average inter-particle separation between the background fermions is much smaller than the would-be Bohr radius of the fermion-scalar bound state, then the conventional quantum-mechanical considerations of the van der Waals, ionic, covalent or metallic bonding would not be applicable. This would correspond to the choice $J_0 \gtrsim g^6 m_H^3$. In this case, the background fermions do not have to form an ordered structure, and yet, we would expect the condensation of scalars. Moreover, the argument that the crystalline structure should be lost at some high density is supported by the discussions in a paragraph below.

A special sub-case of the discussion in the above paragraph is when $J_0 \gg m_H^3/g^6$: It is straightforward to deduce from the results obtained above that the average inter-particle separation in the system, although is smaller than the would-be Bohr radius, is greater than the Compton wavelength of the massive photon. If so, then, the electric charges of the fermions and bosons are screened for all our purposes. The above described condensation mechanism, with a good approximation, would reduce to the standard Bose–Einstein condensation of (almost) free scalars. This system would behave as a two-component substance of free fermions and condensed scalars.

3. Finite-volume regularization

Here we would like to regularize the infinite-volume theory of the previous section. Consider a material ball of a fixed radius R which has a built in constant charge density $g\bar{J}_0$ uniformly distributed over its volume. We will assume that such a ball is prepared “by hands” with appropriate charges, and address the question: How does the electric potential of this ball look like when the charged condensate described in the previous section compensates the fermion charge in its interior? This question is similar in spirit to the one we ordinarily study for, e.g., a uniformly charged insulating ball in electrodynamics.

We shall be looking for static solutions of Eq. (3), which we parametrize as follows:

$$B_0(r) = B_{0c} + \delta B_0(r), \quad \sigma(r) = \sigma_c + \delta\sigma(r). \quad (10)$$

We focus on the solutions that in the interior of the ball satisfy $\delta\sigma/\sigma_c \ll 1$ and $\delta B_0/B_{0c} \ll 1$. Then the equations for δB_0 and $\delta\sigma$ become:

$$-\nabla^2 \delta B_0 + m_g^2 \delta B_0 = -2m_g m_H \delta\sigma, \quad (11)$$

$$-\nabla^2 \delta\sigma = 2m_g m_H \delta B_0, \quad (12)$$

where, as before, $m_g \equiv g\sigma_c$. Explicit solutions of the above equations can be readily found. For simplicity, we will present them for $m_H \gg m_g$, i.e., when the $m_g^2 \delta B_0$ term in the first equation can be neglected.

The solutions in the interior of the ball are

$$\delta B_0(r) = \frac{1}{r} [c_1 \sinh(Mr) \cos(Mr) + c_2 \cosh(Mr) \sin(Mr)], \quad (13)$$

$$\delta\sigma(r) = \frac{1}{r} [-c_1 \cosh(Mr) \sin(Mr) + c_2 \sinh(Mr) \cos(Mr)], \quad (14)$$

where $M \equiv \sqrt{m_g m_H}$, and c_1 and c_2 are constants to be determined from matching these solutions to the exterior ones.

Outside of the ball we approximate the solutions to be

$$B_0 = \frac{Q}{r}, \quad \sigma = k \frac{e^{-m_H(r-R)}}{r}, \quad (15)$$

where Q is a yet-unknown effective charge of the ball, which should be determined from the matching conditions, and which we expect to be mostly concentrated near the surface. By matching the solutions and their first derivatives at $r = R$, we find

$$c_1 = \frac{2}{gD} \left[m_g(m_H R + 1) (\sinh(MR) \sin(MR) + \cosh(MR) \cos(MR)) + m_H \left(\sinh(MR) \sin(MR) - \cosh(MR) \cos(MR) - \frac{m_H}{M} \sinh(MR) \cos(MR) \right) \right], \quad (16)$$

$$c_2 = \frac{2}{gD} \left[m_g(m_H R + 1) (\sinh(MR) \sin(MR) - \cosh(MR) \cos(MR)) - m_H \left(\sinh(MR) \sin(MR) + \cosh(MR) \cos(MR) + \frac{m_H}{M} \cosh(MR) \sin(MR) \right) \right]. \quad (17)$$

While, for the charge Q we obtain the following expressions:

$$Q = \frac{1}{gD} \left[(m_g(m_H R + 1) + m_H(m_H R - 1)) \sinh(2MR) - (m_g(m_H R + 1) - m_H(m_H R - 1)) \sin(2MR) + (2m_H MR - m_H^2/M) \cosh(2MR) + (2m_H MR + m_H^2/M) \cos(2MR) \right], \quad (18)$$

where $D \equiv m_H \sinh(2MR) + m_H \sin(2MR) + 2M \times \cosh(2MR) + 2M \cos(2MR)$. Finally, the constant k is determined as

$$k = \frac{1}{gD} \left[-(m_g + m_H) \sinh(2MR) - (m_g - m_H) \sin(2MR) + 2m_g MR \cosh(2MR) + 2m_g MR \cos(2MR) \right]. \quad (19)$$

In the case of physical interest, $MR \gg 1$, the above solutions have a number of interesting properties. The net charge density in the ball, $g\bar{J}_0^{\text{eff}} = g\bar{J}_0 - g^2\sigma(r)^2B_0(r)$, is exponentially small in the interior, except in a narrow spherical shell near the surface of width M^{-1} . Thus, the charge is screened in the bulk of the ball, but there remains an unscreened surface charge. In this limit the effective charge of the ball is $Q = m_H R/g = g\bar{J}_0 R^3/(m_g R)^2$. This system is characterized by the conserved electric charge Q , and conserved fermion number $N = \bar{J}_0 R^3/3$.

If we increase $R \rightarrow \infty$, with all the other parameters held fixed, the effective charge should also grow linearly with R in order for the condensate phase to be possible inside the ball. Put in other words, in order to prepare a ball of a given radius with the charged condensate phase inside, one has to retain a specific amount of charge Q defined in (18), on its surface. Hence, in the infinite volume limit considered in the previous section, there is “a surface at infinity” that carries charge. This charge is responsible for the constant B_0 in (4).

In the bulk of the ball the electric field and the electromagnetic energy are negligible. Closer to the boundary, however, the surface energy becomes nonzero due to the varying electric field. The resulting expression scales as

$$\text{Energy}_E \propto \frac{Q^2}{R} \propto \frac{m_H^2 R}{g^2}. \quad (20)$$

From our solutions it is also straightforward to get the scaling of the volume energy well within the ball; it reads as $\sim m_H \bar{J}_0 R^3$.

Let us consider an example of a physical system in which the charged condensate could potentially be obtained. Suppose in a laboratory one could prepare a reservoir, or a trap, in which negatively charged electrons and positively charged helium-4 nuclei, with a net negative charge, could be put together. Consider densities of these particles high enough so that the average separation between the particles, $\sim \bar{J}_0^{-1/3}$, is smaller than the size of a helium atom, which we estimate for simplicity to be the Bohr radius $\sim 1/(\alpha_{\text{em}} m_e)$ (α_{em} denotes the fine-structure constant, and m_e is the electron mass; we still stay somewhat lower than nuclear densities). As long as $\bar{J}_0^{1/3} \gtrsim \alpha_{\text{em}} m_e$ the helium atoms in the substance would not form. According to the discussion at the end of Section 2, at high-enough densities (but still somewhat below the nuclear ones) we would not expect the crystalline structure to form either. Can the charged condensate be formed in this system? Strictly speaking, the calculations of the previous section are not directly applicable to this case, because electrons are lighter than the helium-4 nuclei and averaging over the electron positions to calculate the photon mass may not be a good approximation. In this case we would expect the photon mass squared to be determined by $g^2 \bar{J}_0/m_e$, instead of $g^2 \bar{J}_0/m_H$, which should be applicable when the fermions are heavier than the scalars. We can introduce small temperature in the above system to see under what conditions the condensation would take place. Once the thermal de Broglie wavelengths of the helium-4 nuclei have overlaps with each other, and as long as the photon Compton wavelength is shorter than the thermal de Broglie wavelength, the system

can be treated as a macroscopic mode, or the condensate. The former condition, $\lambda_T \sim (1/m_H T)^{1/2} \gtrsim \bar{J}_0^{-1/3}$, would suggest that $T \lesssim 10^{-1} \text{ eV} \sim 10^{-5} \text{ K}$, while the latter, $1/m_g \lesssim \lambda_T$, would give a stronger bound $T \lesssim 10^{-5} \text{ eV} \sim 10^{-9} \text{ K}$ (we use $g^2 \bar{J}_0/m_e$ as the photon mass squared). Temperatures reached in experiments on Bose–Einstein condensation of atoms are within this range, see, e.g., [1].

Let us look at other characteristics of this system in the condensate phase. Suppose the size of the sphere, or the trap we are dealing with, was $\sim 1m$. Then, the number of electrons and helium-4 nuclei would have to be $N \gtrsim (\alpha_{\text{em}} m_e)^3 (1m)^3 \sim 10^{33}$ for helium atoms not to form. The total mass of these particles would be $\gtrsim 10^6 \text{ kg}$. Moreover, the photon in this substance would acquire the mass $m_g \gtrsim 10^4 \text{ eV}$, while the unbalanced charge of $\gtrsim 10^{16}$ units would be residing near the surface, in a narrow spherical shell of size $\sim 1/\sqrt{m_H m_g} \sim 10 \text{ fm}$. (The electric field strength near the surface of such a sphere would be enough to ionize the air, so we assume that it is placed in a vacuum chamber.)

Propagation of light in the bulk of this substance would proceed with a delay caused by the induced photon mass m_g . For simplicity, we have considered above the system of a macroscopic size, but nothing prevents one to look at much smaller systems, e.g., for a 1 mm size system the required number of electrons and helium-4 nuclei would have to be $N \sim 10^{22}$, and the mass of the system $\gtrsim 10^{-5} \text{ kg}$.

Suppose a ball of a fixed radius and charge determined by (18) with the charged condensate had been prepared. What happens if we gradually bring to the ball’s surface additional charges that would decrease or increase Q ? In terms of the theory considered above, this would imply that we are adding a nonzero scalar chemical potential term μ_s , as discussed in the comment (iii) in Section 2. In this case, the value of $\langle gB_0 \rangle$ inside the ball would change to maintain the value of the effective chemical potential, $-\mu_s + \langle gB_0 \rangle$, to be equal to m_H . In this case, one should expect the relation (18) to be modified.

Before turning to the next section, let us comment on certain limiting cases. If $m_H \rightarrow \infty$, for fixed and finite R , we would expect the scalar field to decouple and the solution to turn into the one for the potential of an insulating ball populated by a constant charge density, for which the potential equals to $g\bar{J}_0(\frac{R^2}{2} - \frac{r^2}{6})$ inside, and to $g\bar{J}_0(\frac{R^3}{3r})$ outside. On the other hand, this would imply that $\delta\sigma = -\sigma_c$. However, our expansion breaks down in this regime, and the solutions (13) and (14) are no longer applicable. In the full perturbative expansion, the l.h.s. of Eqs. (11) and (12) include the non-linear terms

$$+gm_H\delta\sigma^2 + 2gm_g\delta\sigma\delta B_0 + g^2\delta\sigma^2\delta B_0, \quad (21)$$

$$-gm_g\delta B_0^2 - 2gm_H\delta\sigma\delta B_0 - g^2\delta\sigma\delta B_0^2, \quad (22)$$

respectively. When $\delta\sigma = -\sigma_c$ these terms become relevant, and in fact recover the standard electrodynamics result: $-\nabla^2 B_0 = g\bar{J}_0$. Moreover, at some point when m_H exceeds the background fermion mass, mobility of the fermions will play a role and, in general, our results should not be immediately applicable.

Alternatively, we could look at the limit in which $m_g \rightarrow 0$ for a fixed m_H , i.e., $\bar{J}_0 \rightarrow 0$. In this case we have a massless photon and a massive scalar, with σ scaling as m_g . Since this implies that $\delta\sigma \rightarrow -\sigma_c$, the same argument as above applies and the solutions (13) and (14) are not applicable.

Finally, in the limit $m_H \rightarrow 0$ we return back to Eqs. (11) and (12) and now take $m_H \ll m_g$ so that we neglect the r.h.s. of the first equations. Then, it would seem that as $m_H \rightarrow 0$ the solutions approach the trivial ones, $B_0 = 0$ and $\sigma = 0$. To see how we arrived at this erroneous result we again return to the non-linear terms (21) and (22) which become significant in this limit. Retaining these terms in our equation for B_0 , we set $\sigma = 0$ and recover the expected electrostatic result.

In the present work we left out a question of existence of a soliton with the charged condensate phase inside, that would be stable due to surface effects. Such an object would be somewhat similar to a droplet in a liquid drop model of the nucleus (see, e.g., [6]). The related issues will be discussed in [5].

4. Comments on compact objects

In this section we will use the power of gravity as a stabilizer to suggest a possible manifestation of the charged condensation in astrophysics. We consider compact objects. In a general setup, due to energy considerations, the condensing scalar would be a lightest charged scalar available in the spectrum [5], that could condense before decaying. If no new light charged scalars exist, then a first candidate would be a charged pion. However, in order for pions not to decay, one should consider high densities, e.g., the conditions similar to the ones for pion condensation in neutron stars [2].

Charged condensate in compact objects with electrons and helium-4 nuclei could also exist. These object could be held together by gravity which is competing against the degeneracy pressure of the fermions.² Since this mechanism is generic, and since we would expect any such object to contain a mixture of various species, we will discuss it in general terms of background fermions and charged scalars.

Consider a distribution of N charged fermions and N_s charged scalars with the net electric charge Q . Such a distribution could collapse under the influence of gravity into a compact object, a droplet. Below we consider a regime in which gravitational force is dominating over the electrostatic forces at the surface of the droplet. Moreover, we will assume that the temperature in the interior is low enough for all particles to be treated non-relativistically. Then, at a certain temperature, there should be a phase transition in the interior into the charged condensate state. At that point the relation $\langle g B_0 \rangle - \mu_s(T_c) = m_H(T_c)$ will be satisfied.

To get qualitative estimates of the size of such a droplet we will ignore the difference between the values of N and N_s , and minimize energy as a function of the radius R at a fixed value

of the charged particle number N . Since these discussions are qualitative, we shall be omitting the factors of order 10 or less. The total energy of a droplet reads:

$$E(R) = m_H N + N \sqrt{p_J^2 + m_J^2} - \frac{GM^2}{R}, \quad (23)$$

where the first term is the energy of the condensate; the second term is the energy of a non-interacting gas of charged particles that give rise to the background density J_0 (hence, the subscripts in p_J, m_J); and the last term is due to gravity, where G denotes the Newton's constant (we shall be using the Planck mass $M_{\text{Pl}} \equiv G^{-1/2}$), and M is the total mass of the droplet which depends on N . We have ignored in (23) the surface terms which are negligible in the regime where gravity is dominant.

The critical radius reads: $R_c \sim B^2/m_J N^{1/3}$ where $B \equiv M_{\text{Pl}}/(m_H + m_J)$. This leads to the expression for the critical energy

$$E_c = (m_H + m_J)N \left[1 - \left(\frac{m_J}{m_H} \right) \left(\frac{N^{1/3}}{B} \right)^4 \right]. \quad (24)$$

The critical radius decreases with increasing N , the bounds on which are:

$$\frac{1}{e^{1/2}} \left(\frac{m_H}{m_J} \right)^{3/4} B^{9/4} \lesssim N \lesssim \min \left\{ \left(\frac{m_H}{m_J} \right)^{3/4} B^3; B^3 \right\}. \quad (25)$$

Here the lower bound is due to the requirement that gravity be dominant in stabilizing this object, and the upper bound is for the relativistic gravitational and fermionic effects to be negligible. These objects are stable as long as the gravitational binding energy in (24) exceeds the electrostatic energy of uncompensated charges on its surface. This constraint is taken into account by the bounds (25).

In a simple case when the droplet is assumed to be made of electrons and the charged condensate of helium-4 nuclei, N has to be close to the upper bound in (25), $N \sim 10^{57}$. The mass of this object is within an order of magnitude of the mass of the Sun, and its size is $\sim 10^6$ m. This object has characteristics that are similar to those of neutron stars (except that it will have some surface charge, that was negligible in our considerations). However, propagation of light through such a cold and dense object will have specific characteristics described in Sections 2 and 3.

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² This is similar to the stabilization mechanism in white dwarfs and neutron stars.

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