Superconductivity proximity effect is numerically evaluated based on McMillan’s tunneling proximity model for a sandwich of a normal metal layer on top of the surface superconducting layer of intrinsic Josephson junctions in a Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) crystal. Due to the very thin thickness of 0.3 nm of the superconducting layer in IJJs, the surface layer is subject to influence of the proximity effect when the top layer is contacted with a normal metal layer. The effect manifests itself as a significant change in the characteristics of the IJJ surface Josephson junction. It is found that when the superconducting layer thickness is smaller than 0.6 nm, the pair potential reduces significantly, leading to an almost complete suppression of the critical Josephson current density for the surface junction. This result can partly explain the experimental results on the IJJ characteristics of a mesa type structure.
Fig. 1. (a) Oscilloscope images of the current-voltage characteristics for a small mesa structure of Bi-2212 IJJs fabricated on top of a crystal surface with the top Ag electrode thin film evaporated immediately after cleavage in vacuum. The mesa lateral size is 2 μm and the thickness is 6 nm. (b) An enlargement of the origin of the current-voltage characteristics.

is needed to attain close contact with the top superconducting layer in order to obtain a very low contact resistance. It was found by Zhao and his coworkers that evaporating an electrode metal layer in vacuum leads to a very low contact resistance [8, 9]. In such a case, a suppression of the maximum Josephson current is often observed [8, 9, 10]. In the present study, this suppression of the Josephson critical current is intended to be explained in terms of the proximity effect.

The superconductivity proximity effect is usually known as a phenomenon in which a normal metal thin film in close contact with a superconductor exhibits superconductivity despite the fact that it alone shows no trace of superconductivity down to a very low temperature. The proximity effect to be dealt with in the present study is concerned with an opposite case. Namely, we are concerned with the case in which a very thin superconducting layer is contacted with a rather thick normal metal layer. In such a case, it occurs that the superconductivity of the thin film is weakened in the presence of a normal metal layer. In the present study, we try to numerically evaluate the Josephson critical current \( I_c \) based on McMillan’s tunneling proximity model [11] for a Josephson junction in which one electrode is composed of a very thin superconducting layer, i.e., 0.3 nm CuO\(_2\) double layer, backed by a normal metal layer. We have found that under some conditions \( I_c \) is significantly reduced, which is almost comparable with the experimental results.

2. Experimental background

Figure 1(a) shows an oscilloscope image of current-voltage (I-V) characteristics for a Bi-2212 IJJ mesa structure. It is clearly seen that the first zero-voltage branch current is almost completely suppressed compared with the other three branches. Figure 1(b) shows an enlarged image of the origin in Fig. 1(a). A suppressed \( I_c \) of a extremely small magnitude is observable. We presume that the suppression of \( I_c \) is caused by the proximity effect in the presence of a normal metal layer contacted with the top layer junction electrode. Such a suppressed zero-voltage branch was observed by Zhao et al. [8] and Zhu et al. [9] in their submicrometer mesa samples cleaved and electrode-evaporated in vacuum. There is no definite statement about the relationship between the suppressed \( I_c \) and the contact resistance, and they argue that the reduction in \( I_c \) is caused by the deficiency in the oxygen content in Bi-2212 which occurred during the cleavage in vacuum. As for the possibility of this different scenario, a relationship between both causes is discussed in Sec. 5.
for \( \alpha \) \( \Re \Delta \), the BCS pair potential which is equivalent to respectively. In McMillan’s tunneling model, the following equations hold. 

\[
\Delta_N(E) = \left( \Delta_N^\text{ph} + \frac{i\Gamma_N\Delta_S(E)}{[E^2 - \Delta_S^2(E)]^{1/2}} \right) \left( 1 + \frac{i\Gamma_N}{[E^2 - \Delta_S^2(E)]^{1/2}} \right)
\]

\[
\Delta_N^\text{ph} = \lambda_N \int_\Delta \Delta^{+\Delta N} \text{Re} \left( \frac{\Delta_N(E)}{[E^2 - \Delta_N^2(E)]^{1/2}} \right) \tanh \frac{E}{2k_B T} dE,
\]

where \( \Gamma_S \) and \( \Gamma_N \) are expressed as follows.

\[
\Gamma_N = \frac{\hbar v_{FS}}{4B_N d_N}, \quad \Gamma_S = \frac{\hbar v_{FN}}{4B_S d_S},
\]

The expressions for \( \Delta_S(E) \) and \( \Delta_N^\text{ph} \) are obtained by exchanging suffixes S and N in the above equations. In the expressions, \( v_{FN} \) and \( v_{FS} \) are the Fermi velocities in N and S layers. \( \alpha \) is the tunneling probability through the proximity interface between the S and N layers. \( \lambda_{S,N} \) are the values self-consistently related to \( T_c \) through \( T_c = 1.13\hbar \omega_{S} \exp(-1/\lambda_S) \) and Eq. (2) in the case of \( d_S = 0 \). In the present case, \( \lambda_S = 0.446 \) is chosen to represent a strong-coupling nature and in the absence of the normal layer \( 2\Delta(0)/k_BT_c \) is approximately 4.3. \( B_N \) and \( B_S \) are functions of scattering time and nearly constant. The Josephson critical current is expressed by Nam’s formula [12] by the following.

\[
I_c(T) = \frac{\hbar}{\pi e R_N} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \frac{p_L(E)p_R(E')}{E - E'} [f(E) - f(E')]
\]

where, \( f(E) = 1/[1 + \exp(E/k_B)] \) is the Fermi function, \( p_L(E) \) are the pair density of states and \( p_L(E) = \text{Re} \Delta_{S,N}(E)/\sqrt{E^2 - \Delta_{S,N}^2(E)^2} \) for the S side of the proximity sandwich while \( p_R(E) \) should be expressed by the BCS pair potential which is equivalent to \( \Delta_N^\text{ph} \) with \( \Gamma_S = 0 \). In the numerical evaluation of \( I_c \), Eqs. (1)-(4) are numerically solved self-consistently.

In the present study, we chose s-wave symmetry for the superconductor since it is sufficient for the present purpose to check the validity of the model in which we need to expect a much larger change in \( I_c \) than that caused by the difference between d-wave and s-wave symmetries.

4. Results

Figure 3 shows the pair density function \( p_{S,N}(E) \) for the S side and N side of the proximity sandwich for \( \alpha = 0.1, d_S = 6 \text{ nm}, d_N = 6 \text{ nm} \), and the other parameters listed in the figure caption. The values for the N layer are nearly compared with those for Ag. As a characteristics feature of a proximity sandwich represented by McMillan’s model, the pair density becomes finite at the N side of the sandwich. The value of \( \alpha = 0.1 \) for the tunneling parameter is not a small value but nearly comparable to a value at the interface.
between elemental metals [13]. Nevertheless, the sharp peak at nearly the original position indicates that the pair diffusion from the superconductor into the normal layer is not affluent. This arises from the fact that the value chosen for the Fermi velocity in the superconductor is small in the calculation, reflecting a large anisotropy in the Bi-2212 system and a low carrier density. The ratio of $v_{FS}/v_{FN}$ implies approximately a value of 0.01 for the carrier density ratio when the effective masses are assumed to be the same. Since the anisotropy is ~100 for the Bi-2212 system, the above value for $v_{FS}$ may be larger than the actual value. It is therefore more than expectation that the diffusion of pairs into the normal layer is pretty slim.

Figure 4 shows the pair density function $p_S(E)$ as a function of energy $E$ at various values for the thickness of the superconducting layer $d_S$. It is seen that the pair density shifts toward lower energies with its peak lowered and broadened as $d_S$ decreases, the magnitude of the pair density itself decreases as a result of diffusion of pair density into the N side. It is seen that $p_S(E)$ is pretty suppressed when $d_S$ is less than 0.6 nm.

Figure 5 shows the temperature dependence of the pair density of states $p_S(E)$ at the S side of the proximity sandwich. The temperature dependence is determined self-consistently from Eq. (2) and its counterpart of the S layer depending on the values for $\lambda_S$ and $\lambda_N$. In the present case, both values were preliminarily determined in the absence of proximity effect to give a $T_c$ of 90 K and a superconducting order parameter magnitude of $4.3k_BT_c$. The curves of the pair density of states in Fig. 4 are the results of such
self-consistent results.

Figure 6 shows the temperature dependence of $I_c$ for various values of $d_S$ from 0.3 to 120 nm. The parameters in this case is $\alpha = 0.09$ and the other parameters are listed in the figure caption, which are almost the same as those in the previous calculations. It is clearly seen that $I_c$ is totally suppressed when $d_S$ is less than 0.3 nm. When $d_S$ is thicker than 12 nm, the magnitude of $I_c$ shows almost no change by the proximity effect. Similarly, when the normal metal layer is thicker than 60 nm, the numerical calculation results show no appreciable change relating to the change in $d_N$. Thus it becomes clear that the proximity effect can cause a significant reduction in the maximum Josephson current of the surface IJJ due to the proximity effect when $\alpha = 0.09$ or larger and $v_{FS} = 3 \times 10^7$ cm/s or larger.

5. Discussion

By the numerical calculation based on McMillan’s tunneling model, it has become known that the Josephson critical current can be reduced to a considerable degree due to the proximity effect. By this point, we have assumed several values for the proximity parameters, of which important are the tunneling probability $\alpha$ and the Fermi velocity $v_{FS}$ for the S layer. Since the degree of the proximity effect in McMillan’s model is determined by the values for $\Gamma_S$ and $\Gamma_N$, the degree of the influence is dominated by the value of $\alpha$ and $v_{FS}/d_S$. Namely, an increase of $\alpha$ exhibits the same effect as an increase in $v_{FS}$ when the magnitude for $\Gamma_S$ is the same. In the present study, we have adopted rather large values for $\alpha$ and $v_{FS}$ as described in the preceding section. Actually, $\alpha$ can be much smaller than 0.1 and in this case, the present result is thought to be as an overestimation of the proximity effect.

In McMillan’s tunneling proximity model, the diffusion of the pair density is mainly taken into account. This concept is effective in the case of a sandwich in which the Fermi velocity is nearly the same for both layers.

Another important point that must be reflected on is the d-wave symmetry of the order parameter in the BSCCO system. Since there is a node in the d-wave superconductor, diffusion of normal electrons into the superconducting layer is probable. The diffusion of normal electrons is not taken into account in the McMillan’s model and therefore it is not reflected on the present result. The interdiffusion of normal electrons is particularly important in high-$T_c$ superconductors, in which $T_c$ and the critical Josephson current density $J_c$ depends on the carrier density or doping level. Since the carrier type of high-$T_c$ superconductors is the p-type, the diffusion of electron causes the superconducting layer to shift toward the underdoped side. This can be a cause of an extended proximity effect which results in a significant decrease in $I_c$. In turn, this implies that the proximity effect is largely mitigated when the doping level of the superconducting layer is in the overdoped region. Therefore, it also implies that the proximity effect becomes profound when the superconducting layer is in the underdoped range.
We cannot rule out the possibility of oxygen reduction from the top layer which causes underdoping in the superconductor. Zhao and his coworkers [8, 9] argue that evaporation of upper electrode metal in vacuum immediately after cleavage of a Bi-2212 crystal inevitably causes oxygen desorption to a greater or lesser degree depending on the evaporation rate. Since it is sufficiently likely that the excess oxygen atoms at the surface BiO layer can be desorbed during the evaporation process in vacuum, this causes the top CuO$_2$ layer to become more or less underdoped. When the top layer becomes underdoped, then the placement of a metal layer in close contact with the surface superconducting layer is sufficiently thought to bring about significant reduction in the Josephson critical current.

6. Conclusion

In order to explain the experimental result that $I_c$ is significantly reduced for the surface junction in a stack of IJJs, we propose a model in which the proximity effect of the top normal layer in close contact with the junction is responsible for the phenomena. Numerical calculation based on McMillan’s tunneling proximity model has revealed that the Josephson critical current $I_c$ between the proximity sandwich and a single IJJ can be reduced to such a small level as observed experimentally under a limited condition. The condition is attained when the tunneling through a single BiO-SrO layer of 0.6 nm is sufficiently flourish or when the Fermi velocity of the superconductor in the normal direction is at least as large as 1/10 of a usual metal like Ag. The result also implies that oxygen reduction in vacuum should be taken into account and the reduced doping level enhances the proximity effect.

References