JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 109, 244–257 (1985)

Effect of Rotation in Lubrication Problems: Existence of More Fundamental Solutions

R. S. GUPTA

Department of Mathematics, Manipur University, Imphal-795003, India

AND

MIHIR B. BANERJEE

Department of Mathematics, Himachal Pradesh University, Simla-171005, India

Submitted by E. Stanley Lee

A generalized Reynolds equation is derived in the present paper by taking into account the effect of rotation in lubrication problems. The existence of certain fundamental solutions is shown in this extended framework which is not allowed in the classical Reynolds theory. Results concerning the pressure and the load capacity of the resulting bearing system are obtained and interpreted in the respective cases when the film thickness is a linear or an exponential functions of the coordinate along the bearing length. One of the important results is that while the load capacity decreases with increasing values of α for an exponentially inclined slider in the classical Reynolds theory, it increases with increasing values of α in the present context. © 1985 Academic Press, Inc.

1. INTRODUCTION

By "hydrodynamic lubrication" we mean a process in which two surfaces, moving at some relative velocity with respect to each other, are separated by a fluid film in which forces are generated by virtue of the relative motion only. A two-dimensional theory of lubrication was first developed by Reynolds [1], who showed that the variation of the lubricant pressure in the bearing is described by a partial differential equation known as the "Reynolds equation" and further that if the lubricant layer is to transmit pressure between a shaft and a journal, the layer must have varying thickness otherwise the stresses in the lubricant cannot balance the load of the shaft [2].

It is well known that rotation introduces a number of new elements into

0022-247X/85 $3.00

a hydrodynamic problem, and some of its consequences are, at first sight, unexpected: the role of viscosity is, for example, inverted. The origin of this and other consequences of rotation can be traced to certain general theorems, relating to vorticity, in the dynamics of rotating fluids. Further, there is a fundamental point to remember in the character of the motions which prevail when rotation is present. Rotation induces a component of vorticity in its direction, and the effects arising from it are predominant; for large Taylor numbers it results in the stream lines becoming closely wound spirals with motions principally confined to planes transverse to the direction of rotation. Recently, Banerjee *et al.* [3] have pointed out that almost all the real physical systems are under the effect of rotation though it may be small and they have extended the classical theory of lubrication under the effect of rotation. They have shown that in certain situations, the qualitative features of the bearing system may be different. More importantly, a certain class of fundamental solutions of this problem was omitted in their work [3].

We therefore mathematically analyse the classical lubrication problem [2] under the effect of a uniform rotation about an axis which is transverse to the fluid film. The mathematical equations governing the velocity and the pressure distributions now depend upon another parameter, the rotation number M (the square root of the conventional Taylor number), in addition to density, viscosity, film thickness, and surface and transverse velocities. In order to simplify the mathematical calculations without essentially sacrificing the characteristics of rotation we regard $|M|$ to be small. This leads to a generalized Reynolds equation. The existence of certain fundamental solutions of this equation are shown which are not allowed in the classical Reynolds theory. Results concerning the pressure and the load capacity of the resulting bearing system are obtained and interpreted in the respective cases when the film thickness is a linear or an exponential function of the coordinate along the bearing length.

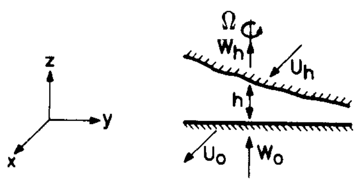

FIG. 1. The fluid film.

2. The Governing Equations of Hydrodynamic Lubrication in a Rotating Frame of Reference

Consider a layer of fluid which is kept rotating at a constant rate. Let Ω denote the angular velocity of rotation about the z-axis (Fig. 1). The hydrodynamical equations of momentum and continuity in the usual tensor notation are [4]:

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = \rho X_i - \frac{\partial}{\partial x_i}\left[p - \frac{1}{2}\rho (E_{ijk}\Omega_i r_k)^2 \right] + \frac{\partial}{\partial x_j}\left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3}\mu \frac{\partial u_k}{\partial x_k} \right] + 2\rho E_{ijk} u_j \Omega_k \tag{1}$$

and

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0. \tag{2}$$

Applying the standard assumptions of lubrication theory [5] Eqs. (1) reduce to

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2} + 2\rho \Omega v \tag{3}$$

$$0 = -\frac{\partial P}{\partial y} + \mu \frac{\partial^2 v}{\partial z^2} - 2\rho \Omega u \tag{4}$$

and

$$0 = -\frac{\partial P}{\partial z} \tag{5}$$

while Eq. (2) for the steady flow gives

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{6}$$

where

$$P = p - \tfrac{1}{2}\rho (E_{ijk}\Omega_i r_k)^2$$

denotes the modified pressure.

Equations (3)–(6) give the governing hydrodynamical equations of momentum and continuity for the problem of steady lubrication.

3. Derivation of Generalized Reynolds Equation

From Eqs. (3), (4) and (5) we obtain the governing equations for u and v as

$$\frac{\partial^4 u}{\partial z^4}+\left(\frac{2\rho\Omega}{\mu}\right)^2 u=-\frac{2\rho\Omega}{\mu^2}\frac{\partial P}{\partial y} \tag{7}$$

and

$$\frac{\partial^4 v}{\partial z^4}+\left(\frac{2\rho\Omega}{\mu}\right)^2 v=\frac{2\rho\Omega}{\mu^2}\frac{\partial P}{\partial x}. \tag{8}$$

The boundary conditions on u and v are given by

$$\begin{aligned} u&=U_0 && \text{at } z=0\\ u&=U_h && \text{at } z=h\\ \mu\frac{\partial^2 u}{\partial z^2}&=\frac{\partial P}{\partial x} && \text{at } z=0 \text{ and } z=h \end{aligned} \tag{9}$$

and

$$\begin{aligned} v&=0 && \text{at } z=0 \text{ and } z=h\\ \mu\frac{\partial^2 v}{\partial z^2}&=\frac{\partial P}{\partial y}+2\rho\Omega U_0 && \text{at } z=0\\ \mu\frac{\partial^2 v}{\partial z^2}&=\frac{\partial P}{\partial y}+2\rho\Omega U_h && \text{at } z=h. \end{aligned} \tag{10}$$

Using the nondimensional quantities defined by

$$\begin{aligned} &\bar{x}=\frac{x}{h_c}, && \bar{u}=\frac{u}{V_c}, && \bar{\mu}=\frac{\mu}{\mu_c}\\ &\bar{y}=\frac{y}{h_c}, && \bar{v}=\frac{v}{V_c}, && P=\frac{h_c P}{\mu_c V_c}\\ &\bar{z}=\frac{z}{h_c}, && \bar{w}=\frac{w}{V_c}, && M=\frac{2\Omega h_c^2\rho_c}{\mu_c}\\ &\bar{h}=\frac{h}{h_c}, && \bar{\rho}=\frac{\rho}{\rho_c}, \end{aligned} \tag{11}$$

and dropping the bars for convenience Eqs. (7)–(10) and (6), respectively, reduce to

$$\frac{\partial^4 u}{\partial z^4}+\frac{M^2\rho^2}{\mu^2}u=-\frac{M\rho}{\mu^2}\frac{\partial P}{\partial y} \tag{12}$$

$$\frac{\partial^4 v}{\partial z^4}+\frac{M^2\rho^2}{\mu^2}v=\frac{M\rho}{\mu^2}\frac{\partial P}{\partial x} \tag{13}$$

$$u=\frac{U_0}{V_c}=U_1 \qquad \text{at } z=0$$

$$u=\frac{U_h}{V_c}=U_2 \qquad \text{at } z=h \tag{14}$$

$$\mu\frac{\partial^2 u}{\partial z^2}=\frac{\partial P}{\partial x} \qquad \text{at } z=0 \text{ and } z=h$$

$$v=0 \qquad \text{at } z=0 \text{ and } z=h$$

$$\mu\frac{\partial^2 v}{\partial z^2}=\frac{\partial P}{\partial y}+M\rho U_1 \qquad \text{at } z=0 \tag{15}$$

$$\mu\frac{\partial^2 v}{\partial z^2}=\frac{\partial P}{\partial y}+M\rho U_2 \qquad \text{at } z=h$$

and

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0. \tag{16}$$

For the case when $|M|$ is small so that terms containing second and higher of M can be neglected as compared to terms containing first and the zeroth of M, the solutions for u and v which satisfy the relevant are given by

$$u=\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)z(z-h)+\left(\frac{h-z}{h}\right)U_1 + \left(\frac{z}{h}\right)U_2-\frac{M\rho}{24\mu^2}\left(\frac{\partial P}{\partial y}\right)z(z^3-2z^2h+h^3) \tag{17}$$

and

$$v=\frac{1}{2\mu}\left(\frac{\partial P}{\partial y}\right)z(z-h) +\frac{M\rho}{24\mu^2}\left(\frac{\partial P}{\partial x}\right)z(z^3-2z^2h+h^3) +\frac{M\rho}{6\mu h}z[(U_2-U_1)z^2+3U_1hz-(2U_1+U_2)h^2]. \tag{18}$$

By replacing u and v by their values from Eqs. (17) and (18), we obtain from the continuity equation (16)

$$\begin{aligned}\frac{\partial}{\partial z}(\rho w) = & -\frac{1}{2}\left[\frac{\partial}{\partial x}\left\{\frac{\rho}{\mu}\left(\frac{\partial P}{\partial x}\right) z(z-h)\right\}\right. \\ & \left. +\frac{\partial}{\partial y}\left\{\frac{\rho}{\mu}\left(\frac{\partial P}{\partial y}\right) z(z-h)\right\}\right] \\ & +\frac{M}{24}\left[\frac{\partial}{\partial x}\left\{\frac{\rho^2}{\mu^2}\left(\frac{\partial P}{\partial y}\right) z(z^3-2z^2h+h^3)\right\}\right. \\ & \left. -\frac{\partial}{\partial y}\left\{\frac{\rho^2}{\mu^2}\left(\frac{\partial P}{\partial x}\right) z(z^3-2z^2h+h^3)\right\}\right] \\ & -\frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-z}{h}\right) U_1+\left(\frac{z}{h}\right) U_2\right\}\right] \\ & -\frac{\partial}{\partial y}\left[\frac{M\rho^2}{6\mu h} z\{(U_2-U_1)z^2+3U_1zh\right. \\ & \left. -(2U_1+U_2)h^2\}\right]. \end{aligned} \tag{19}$$

By integrating with respect to z with the conditions

$$\begin{aligned} w &= \frac{w_0}{V_c} = w_1 \qquad \text{at } z=0 \\ w &= \frac{w_h}{V_c} = w_2 \qquad \text{at } z=h \end{aligned} \tag{20}$$

we have

$$\begin{aligned}\rho(w_2-w_1) = & -\frac{1}{2}\int_0^h \frac{\partial}{\partial x}\left\{\frac{\rho}{\mu}\left(\frac{\partial P}{\partial x}\right) z(z-h)\right\} \\ & \left. +\frac{\partial}{\partial y}\left\{\frac{\rho}{\mu}\left(\frac{\partial P}{\partial y}\right) z(z-h)\right\}\right] dz \\ & +\frac{M}{24}\int_0^h\left[\frac{\partial}{\partial x}\left\{\frac{\rho^2}{\mu^2}\left(\frac{\partial P}{\partial y}\right) z(z^3-2z^2h+h^3)\right\}\right. \\ & \left. -\frac{\partial}{\partial y}\left\{\frac{\rho^2}{\mu^2}\left(\frac{\partial P}{\partial x}\right) z(z^3-2z^2h+h^3)\right\}\right] dz \end{aligned}$$

$$-\int_0^h \frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-z}{h}\right)U_1+\left(\frac{z}{h}\right)U_2\right\}\right]dz$$
$$-\frac{M}{6}\int_0^h \frac{\partial}{\partial y}\left[\frac{\rho^2}{\mu h}z\{(U_2-U_1)z^2+3U_1zh\right.$$
$$\left.-(2U_1+U_2)h^2\}\right]dz. \tag{21}$$

The upper limit h in the last equation is a function of the coordinates x and y and performing the integration before differentiation, which is certainly permissible in the present case, we obtain

$$\rho(w_2-w_1)=\frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\rho h^3}{6\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{6\mu}\frac{\partial P}{\partial y}\right)\right]$$
$$+\frac{M}{24}\left[\frac{\partial}{\partial x}\left(\frac{\rho^2h^5}{5\mu^2}\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\rho^2h^5}{5\mu^2}\frac{\partial P}{\partial x}\right)\right]$$
$$-\frac{1}{2}\left[\rho h\frac{\partial}{\partial x}(U_1+U_2)+(U_1-U_2)\frac{\partial}{\partial x}(\rho h)\right]$$
$$+\frac{M}{24}\left[\frac{\rho^2h^3}{\mu}\frac{\partial}{\partial y}(U_1+U_2)+(U_1+U_2)\frac{\partial}{\partial y}\left(\frac{\rho^2h^3}{\mu}\right)\right] \tag{22}$$

or

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial y}\right)$$
$$+\frac{M}{10}\left[\frac{\partial}{\partial x}\left(\frac{\rho^2h^5}{\mu^2}\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\rho^2h^5}{\mu^2}\frac{\partial P}{\partial x}\right)\right]$$
$$=6(U_1-U_2)\frac{\partial}{\partial x}(\rho h)+6\rho h\frac{\partial}{\partial x}(U_1+U_2)$$
$$-\frac{M}{2}\left[\frac{\rho^2h^3}{\mu}\frac{\partial}{\partial y}(U_1+U_2)+(U_1+U_2)\frac{\partial}{\partial y}\left(\frac{\rho^2h^3}{\mu}\right)\right]$$
$$+12\rho(w_2-w_1). \tag{23}$$

If the bearing surfaces are inelastic in the direction of x and y, U_1 and U_2 will be independent of x and y and this gives

$$\frac{\partial}{\partial x}(U_1+U_2)=0$$
$$\frac{\partial}{\partial y}(U_1+U_2)=0 \tag{24}$$

hence Eq. (23) reduces to

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\mu}\frac{\partial P}{\partial y}\right)$$
$$+\frac{M}{10}\left[\frac{\partial}{\partial x}\left(\frac{\rho^2 h^5}{\mu^2}\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\rho^2 h^5}{\mu^2}\frac{\partial P}{\partial x}\right)\right]$$
$$=6(U_1-U_2)\frac{\partial}{\partial x}(\rho h)-\frac{M}{2}(U_1+U_2)\frac{\partial}{\partial y}\left(\frac{\rho^2 h^3}{\mu}\right)$$
$$+12\rho(w_2-w_1). \tag{25}$$

Equation (25) is the generalized Reynolds equation with which we shall be subsequently concerned in this paper. The above equation can be simplified in some situations. If, for example, it is reasonable to assume that the lubricant is incompressible, that is, the density is constant and further the viscosity of the lubricant does not change, then we have from Eq. (25)

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)$$
$$+\frac{M\rho}{10\mu}\left[\frac{\partial}{\partial x}\left(h^5\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(h^5\frac{\partial P}{\partial x}\right)\right]$$
$$=6\mu(U_1-U_2)\frac{\partial h}{\partial x}-\frac{1}{2}M\rho(U_1+U_2)\frac{\partial}{\partial y}(h^3)$$
$$+12\mu(w_2-w_1). \tag{26}$$

In most practical cases, the bearing is stationary and only the runner in the thrust bearings and the shaft in the journal bearings are moving. In that case Eq. (26) reduces to

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)$$
$$+\frac{M\rho}{10\mu}\left[\frac{\partial}{\partial x}\left(h^5\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(h^5\frac{\partial P}{\partial x}\right)\right]$$
$$=6\mu U\frac{\partial h}{\partial x}-\frac{1}{2}M\rho U\frac{\partial}{\partial y}(h^3)+12\mu(w_2-w_1) \tag{27}$$

which is the same for both thrust and journal bearing with U the sliding velocity of either runner or journal. For the case of pure sliding Eq. (27) becomes

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial P}{\partial y}\right)$$
$$+\frac{M\rho}{10\mu}\left[\frac{\partial}{\partial x}\left(h^5\frac{\partial P}{\partial y}\right)-\frac{\partial}{\partial y}\left(h^5\frac{\partial P}{\partial x}\right)\right]$$
$$=6U\mu\frac{\partial h}{\partial x}-\frac{1}{2}M\rho U\frac{\partial}{\partial y}(h^3). \qquad (28)$$

4. Existence of More Fundamental Solutions of the Generalized Reynolds Equations

In this section we show the existence of more fundamental solutions of the generalized Reynolds equation (28) in one dimension in the sense that such solutions cannot be allowed in the classical Reynolds theory. We do this for the following two situations, namely, when the film thickness is a linear or an exponential function of the coordinate along the bearing length.

4.1. *Plane-Inclined Slider*

By far the most common form of lubricated slider bearing system is the plane-inclined pad illustrated in Fig. 2. As an example of the application of the generalized Reynolds equation to slider bearing systems, we will determine the pressure and load capacity for such a configuration. The bearing system and notation are illustrated in Fig. 2. Let

$$U=+U$$
$$h=h(y) \qquad \text{and} \qquad P=P(y). \qquad (29)$$

Equation (28) then gives

$$\frac{d}{dy}\left(h^3\frac{dP}{dy}\right)=-\frac{1}{2}M\rho U\frac{d}{dy}(h^3). \qquad (30)$$

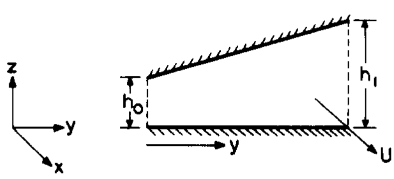

Fig. 2. Plane-inclined slider.

Integrating Eq. (30) w.r.t. y and using the condition

$$\frac{dP}{dy} = 0 \qquad \text{at} \quad h = h^* \tag{31}$$

we have

$$\frac{dP}{dy} = -\frac{M\rho U}{2}\left(\frac{h^3 - h^{*3}}{h^3}\right). \tag{32}$$

This equation must be integrated with respect to y to yield the pressure

$$P = -\frac{M\rho U}{2}\int\left(\frac{h^3 - h^{*3}}{h^3}\right) dy. \tag{33}$$

The film thickness can be expressed at any point as

$$h = h_0\left(1 + \frac{ny}{L}\right) \tag{34}$$

where

$$n = \frac{h_t}{h_0} - 1.$$

This leads to

$$P = -\frac{M\rho U}{2}\int\left[1 - \frac{h^{*3}}{h_0^3\left(1 + \frac{ny}{L}\right)^3}\right] dy \tag{35}$$

$$= -\frac{M\rho U}{2}\left[y + \frac{Lh^{*3}}{2nh_0^3\left(1 + \frac{ny}{L}\right)^2} + A\right] \tag{36}$$

where A is a constant of integration. It will be noted that h^* is the value of h where $dP/dy = 0$, that is, where the pressure has a maximum value. We have two unknowns h^* and A, which must be found by the introduction of two boundary conditions:

$$\begin{aligned} P &= 0 \qquad \text{at} \quad y = 0 \\ P &= 0 \qquad \text{at} \quad y = L. \end{aligned} \tag{37}$$

Note that pressures are expressed as gauge pressures, that is, $P=0$ represents ambient pressure. Substitution of these two conditions gives

$$h^{*3}=\frac{2h_0^3(n+1)^2}{(n+2)} \tag{38}$$

and

$$A=-\frac{L(n+1)^2}{n(n+2)}. \tag{39}$$

These can now be substituted in the pressure equation (36) to give

$$P=-\frac{M\rho U}{2}\left[y+\frac{L(n+1)^2}{n(n+2)\left(1+\frac{ny}{L}\right)^2}-1\right]. \tag{40}$$

It is easily seen that pressure as given by Eq. (40) is non-negative everywhere in the flow domain.

A further integration of the pressure gives the normal load capacity of the bearing system per unit length

$$W=\int_0^L P\,dy. \tag{41}$$

Using Eq. (40) the load capacity for the inclined slider is given by

$$W=\frac{M\rho UL^2}{4}\left(\frac{n}{n+2}\right). \tag{42}$$

4.2. *Exponentially Inclined Slider*

In this case (Fig. 3)

$$\begin{aligned} U &= -U \\ h=h(y) \quad &\text{and} \quad P=P(y). \end{aligned} \tag{43}$$

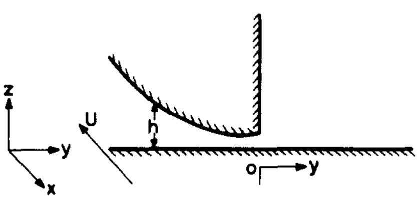

FIG. 3. Exponentially inclined slider.

Equation (26) then gives

$$\frac{d}{dy}\left(h^3\frac{dP}{dy}\right)=\frac{1}{2}M\rho U\frac{d}{dy}(h^3). \tag{44}$$

Integrating Eq. (44) w.r.t. y and using the condition

$$\frac{dP}{dy}=0 \qquad \text{at} \quad h=h^* \tag{45}$$

we have

$$\frac{dP}{dy}=\frac{M\rho U}{2}\left(\frac{h^3-h^{*3}}{h^3}\right). \tag{46}$$

This equation must be integrated with respect to y to yield the pressure

$$P=\frac{M\rho U}{2}\int\left(\frac{h^3-h^{*3}}{h^3}\right)dy. \tag{47}$$

The film thickness can be expressed at any point as

$$h=h_0\exp(-\alpha y). \tag{48}$$

This leads to

$$P=\frac{M\rho U}{2}\int\left[1-\frac{h^{*3}}{h_0^3\exp(-3\alpha y)}\right]dy \tag{49}$$

$$=\frac{M\rho U}{2}\left[y-\frac{h^{*3}\exp(3\alpha y)}{3\alpha h_0^3}+A\right] \tag{50}$$

where A is a constant of integration.

Using the boundary conditions

$$\begin{aligned} P&=0 \qquad \text{at} \quad y=0\\ P&=0 \qquad \text{at} \quad y=-L \end{aligned} \tag{51}$$

we have

$$h^{*3}=-\frac{3\alpha L h_0^3}{\exp(-3\alpha L)-1} \tag{52}$$

and

$$A=-\frac{L}{\exp(-3\alpha L)-1}. \tag{53}$$

These can now be substituted in the pressure equation (50) to give

$$P=\frac{M\rho U}{2}\left[y+L\left\{\frac{\exp(3\alpha y)-1}{\exp(-3\alpha L)-1}\right\}\right]. \tag{54}$$

It is easily seen that pressure as given by Eq. (54) is non-negative everywhere in the flow domain.

A further integration of the pressure gives the normal load capacity of the bearing system per unit length

$$W=\int_{-L}^{0} P\,dy. \tag{55}$$

Using Eq. (54) the load capacity for the exponentially inclined slider is given by

$$W=\frac{M\rho UL}{12\alpha}\left[\frac{6\alpha L+(2+3\alpha L)\{\exp(-3\alpha L)-1\}}{\{1-\exp(-3\alpha L)\}}\right]. \tag{56}$$

5. Discussion and Conclusions

The effect of a uniform rotation on the velocity components u and v is given by Eqs. (7) and (8). These equations show that the governing equations for u and v are of order four, instead of two which is the case when rotation is absent, and as a consequence small extents of rotation induce additional Poiseuille-type flow which is clearly shown by the

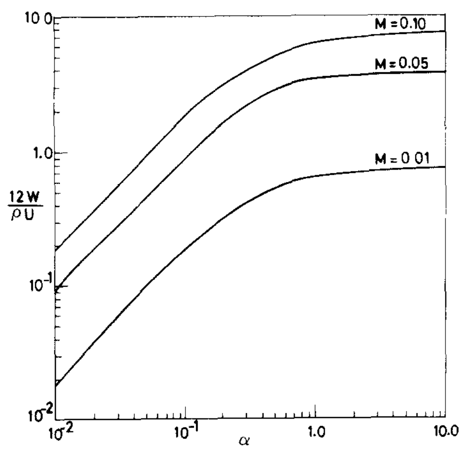

FIG 4. Plot of load capacity versus α for exponentially inclined slider.

solutions for u and v as given by Eqs. (17) and (18). The modified Reynolds equation remains a second-order linear partial differential equation though it contains additional terms. Two of these terms are proportional to M and contain first-order partial derivatives of P while a third term which is also proportional to M is non-homogeneous in character. This non-homogeneous term which contains density and not viscosity is responsible for introducing more fundamental solutions of the generalized Reynolds equation which are not allowed in the classical Reynolds theory. This point is well established in Section 4. The pressures and the load capacities are calculated for the plane-inclined slider and an exponentially inclined slider and some important results are obtained (Fig. 4). Thus, while the load capacity decreases with increasing value of α for an exponentially inclined slider in the classical Reynolds theory [6], it increases with increasing values of α in the present context. Further, while the load capacity has an optimum value with respect to n is the classical case of the plane-inclined slider [5], it is a monotonically increasing function of n in our case.

REFERENCES

1. O. REYNOLDS, On the theory of lubrication and its application to B. Touwers' experiments including an experimental determination of the viscosity of olive oil, *Philos. Trans. Roy. Soc. London* **177** (1886), 157.
2. O. PINKUS AND B. STERNLICHT, "Theory of Hydrodynamic Lubrication," p. 5, McGraw-Hill, New York, 1961.
3. M. B. BANERJEE, R. S. GUPTA, AND A. P. DWIVEDI, The effect of rotation in lubrication problems, *Wear* **69** (1981), 205–218.
4. S. CHANDRASEKHAR, "Hydrodynamic and Hydromagnetic Stability," p. 83, Oxford Univ. Press, London, 1970.
5. J. HALLING, "Principles of Tribology," p. 369, Macmillan & Co., London, 1975.
6. A. CAMERON, "Basic Lubrication Theory," p. 42, Ellis Horwood, England, 1976.