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ORIGINAL ARTICLE

Natural convection heat transfer from an isothermal horizontal square cylinder

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Abstract Laminar natural convection from a horizontal isothermal square cylinder is numerically investigated. The study covered a range of Rayleigh number, *Ra* from 10^3 to 10^6. A computer program is developed to solve the continuity, momentum and thermal energy equations together with their boundary conditions by using a finite volume method. Streamlines and isotherms were generated to describe the flow around the square cylinder. The local and average Nusselt numbers are calculated and plotted over the four sides of the square cylinder. The numerical results were correlated and compared with previous work.
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1. Introduction

The convective heat transfer is an important problem, because it affects the performance of thermal equipment in various engineering applications such as heat exchangers, natural circulation boilers, nuclear reactors, solar heating systems, dry cooling towers, and cooling of electronic equipment.

Churchill and Usagi [1] proposed a method for correlating convective heat transfer using two asymptotes. Oosthuizen and Bishop [2,3] investigated numerically and experimentally the mixed convection heat transfer over a square cylinder. Hassani [4] introduced an expression for predicting free convection from isothermal two-dimensional bodies of arbitrary cross section. Clemes et al. [5] investigated experimentally natural convection heat transfer in air for horizontal, relatively long isothermal cylinders of different cross sections (circular, square, semi-circular, etc) for Rayleigh numbers ranging from 10^3 to 10^9. Yovanovich [6] studied the laminar natural convection from isothermal convex bodies of complex shapes. Mahmud et al. [7] performed numerical investigation to predict the fluid flow and heat transfer characteristics around a square cylinder at different orientations from 0° to 90° for a range of Grashof number from 10 to 10^5. Radziemska and Lewandowski [8] presented the results of theoretical and experimental studies of the natural convection heat transfer from isothermal cuboids. Popiel and Wojtkowiak [9] presented the results of experimental investigations on natural convection heat transfer from the isothermal vertical surfaces of a vertical short and slender square cylinder to air. These results were obtained with a lumped capacitance method. Zeitoun and Ali [10] investigated numerically the natural convection heat transfer from square and rectangular cross-sectional ducts for wide ranges of Rayleigh number from 700 to 10^8. Kumar and Dalal [11] studied numerically the natural convection heat transfer around a tilted heated square cylinder kept in an enclosure in the range of *Ra* from 10^3 to 10^6. Khodary and Bhattacharyya [12] studied numerically and experimentally laminar

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Nomenclature

a	side length of the square test cylinder, m
B	duct aspect ratio, $B = b/a$
b	duct horizontal width, m
C_p	specific heat at constant pressure, J/kg K
g	gravitational acceleration, m/s^2
Gr	Grashof number, $Gr = a^3\beta g(T_w - T_\infty)/v^2$
H	dimensionless duct height, $H = L_d/a$
h	average heat transfer coefficient on all cylinder sides, W/m^2 K
h_l	local heat transfer coefficient on cylinder side, W/m^2 K
k	fluid thermal conductivity, W/m K
L_d	duct height, m
Nu_l	local Nusselt number on cylinder sides
Nu_n	average Nusselt number for natural convection
p_d	dynamic pressure, N/m^2
P_d	dimensionless dynamic pressure
Pr	Prandtl number, $Pr = C_p\ \mu/k$
Ra	Rayleigh number, $Ra = Gr{\cdot}Pr$
T	local fluid temperature, K
u	velocity in x-direction, m/s
U	dimensionless velocity in x-direction
v	velocity in y-direction, m/s
V	dimensionless velocity in y-direction
x, y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity, $\alpha = k/\rho\ C_{p,}$ m^2/s
β	coefficient of volumetric thermal expansion, K^{-1}
μ	dynamic viscosity, kg/m s
θ	dimensionless temperature
ρ	local fluid density, kg/m^3
υ	kinematic viscosity, $\upsilon = \mu/\rho$, m^2/s

Subscripts

∞	at free stream conditions and outside thermal boundary layer
n	natural
w	at cylinder wall

natural convection from a horizontal isothermal square cylinder inside a vertical adiabatic parallel plate channel for duct aspect ratio, B from 1.25 to 10 and different lateral locations for values of Grashof number, Gr between 1 and 3×10^6. Davis et al. [13] investigated numerically and experimentally the confined flow around a rectangular cylinder in a horizontal channel. Igarashi [14,15] performed a series of experiments investigating fluid flow and heat transfer around a square/rectangular cylinder. He measured local and average heat transfer rates on a square cylinder at various angles of attack and on a rectangular cylinder at various width–to–height ratios. He then correlated heat transfer coefficients with flow characteristics. Goldstein et al. [16] reported the effects of multiple vortices formed around a square cylinder mounted vertically on a base plate on the mass transfer. Yoo et al. [17] investigated the convective mass transfer process from a square cylinder and its base in a flow of air. Breuer et al. [18] carried out a 2-D study for the confined flow around a square cylinder in a channel with Finite–Volume Method (FVM). They used non-equidistant staggered grids for a fixed blockage ratio, $1/B$ of 1/8. Rosales et al. [19] investigated numerically the unsteady flow-field and heat-transfer characteristics for a tandem pair of square cylinders in a laminar channel flow. Turki et al. [20] studied numerically the effect of three blockage ratios, $1/B$ = 1/8, 1/6 and 1/4 on the 2-D unsteady and laminar flow past a square cylinder inside a horizontal channel. Sharma and Eswaran [21] investigated numerically the flow and heat transfer of an isolated square cylinder in cross flow for both steady and unsteady periodic laminar flows in the two-dimensional regime. Dhiman et al. [22] investigated numerically the effect of blockage ratio on the flow characteristics of power-law fluids across a square cylinder confined in a channel for B = 4, 6, and 8.

Dhiman et al. [23] investigated numerically the effects of cross-buoyancy and Prandtl number on the flow and heat transfer characteristics of an isothermal square cylinder in a channel for B = 8.

The objectives of the present study were to investigate numerically the heat transfer and flow characteristics for natural convection from a horizontal isothermal square cylinder over a wide range of Ra. Also, local and average values of Nu are to be investigated and a heat transfer correlation is to be presented.

2. Mathematical formulation

The steady–state dimensionless governing equations for the two-dimensional laminar heat transfer in Cartesian Coordinates system (X-Y) including the Boussinesq approximation are given by (see Fig. 1):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P_d}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P_d}{\partial Y} + Gr\theta + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \quad (3)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

The following dimensionless variables are used:

$$X = \frac{x}{a}, \quad Y = \frac{y}{a}, \quad U = \frac{u}{(v/a)}, \quad V = \frac{v}{(v/a)}, \quad P_d = \frac{p_d}{\rho(v/a)^2}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Pr = \frac{C_p\mu}{k} = \frac{v}{\alpha}, \quad \text{and} \quad Ra = \frac{g\beta(T_w - T_\infty)a^3}{v\alpha}$$

$$B = \frac{b}{a} \quad \text{and} \quad H = \frac{L_d}{a} \quad (5)$$

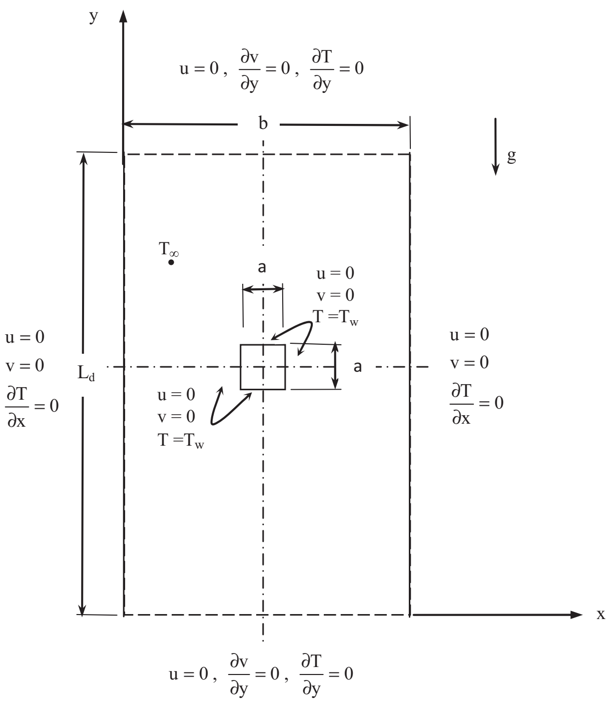

Figure 1 Boundary conditions of the natural convection problem.

The above equations are subjected to the following boundary conditions:

At $\frac{B-1}{2} \leqslant X \leqslant \frac{B+1}{2}$ and $Y = \frac{H-1}{2}$ or $Y = \frac{H+1}{2}$

also at $\frac{H-1}{2} \leqslant Y \leqslant \frac{H+1}{2}$ and $X = \frac{B-1}{2}$ or $X = \frac{B+1}{2}$

$$U = 0, \quad V = 0, \quad \theta = 1 \tag{6a}$$

at $0 \leqslant X \leqslant B$ and $Y = 0$

$$U = 0, \quad \frac{\partial V}{\partial Y} = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \tag{6b}$$

at $0 \leqslant X \leqslant B$ and $Y = H$

$$U = 0, \quad \frac{\partial V}{\partial Y} = 0, \quad \frac{\partial \theta}{\partial Y} = 0 \tag{6c}$$

at $0 \leqslant Y \leqslant H$ and $X = 0$ or $X = B$

$$U = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial X} = 0 \tag{6d}$$

The calculation domain for the natural convection problem is shown in Fig. 1.

2.1. The Nusselt number

The local Nusselt number, Nu_l at any location of the cylinder sides is defined as follows:

$$Nu_l = \frac{h_1 a}{k} = \frac{\partial \theta}{\partial n} \tag{7}$$

where, "n" is the normal to cylinder surface.

The average Nusselt number for natural convection, Nu_n over the inner hot square cylinder sides is defined as follows:

$$Nu_n = ha/k \tag{8}$$

where, 'h' is the average heat transfer coefficient over the cylinder surface.

2.2. The calculation grid

The grid used in the numerical solution is square but non-uniform. The grid spacing in the vicinity of the cylinder in both X- and Y-directions is smaller than the rest of the grid to pick up fine changes in the boundary layers. Different values of grid size were examined to determine the suitable grid size to get an accurate solution. Table 1 shows the effect of grid size on the average Nusselt number for one run. Increasing the number of nodes in X- and Y-directions more than (85×263) did not increase the accuracy of Nu more than 1.62%. It was found that it is good enough to use the mesh size (85×263). The solution domain is taken with duct aspect ratio, $B = 10$ and $H = L_d/a = 10$. Therefore, the centered square cylinder occupies the space $4.5 \leqslant (X \text{ or } Y) \leqslant 5.5$.

2.3. The numerical solution

The governing equations along with the boundary conditions cannot be solved analytically. So, the numerical solution remains the only possible solution one could take. The computer program used to solve the governing equations is based on the finite volume method (FVM) developed by Patankar [24]. This was based on the discretization of the governing equations using the central differencing in space. The discretization equations were solved by the Gauss-Seidel elimination method. The iteration method used in this program is a line by line procedure, which is a combination of the direct method and the resulting Tri-Diagonal Matrix Algorithm (TDMA). The procedure used by Patankar [24] is to solve simultaneously the continuity and momentum equations and then the thermal energy equation.

3. Results

3.1. Streamlines and isotherms

The streamlines and isotherms for the flow over an isothermal horizontal square cylinder due to natural convection are plotted with the aid of the Surfer software version 6.01. The results are obtained at different values of Rayleigh numbers from 10^3 to 10^6, Prandtl number, $Pr = 0.7$ for duct aspect ratio of 10 as shown in Fig. 2(a)–(d).

Table 1 Effect of grid size on *Nu*.

Grid	*Nu*	% Dev.
61×187	14.342	3.151
73×225	14.298	2.853
85×263	14.119	1.621
97×299	13.890	0

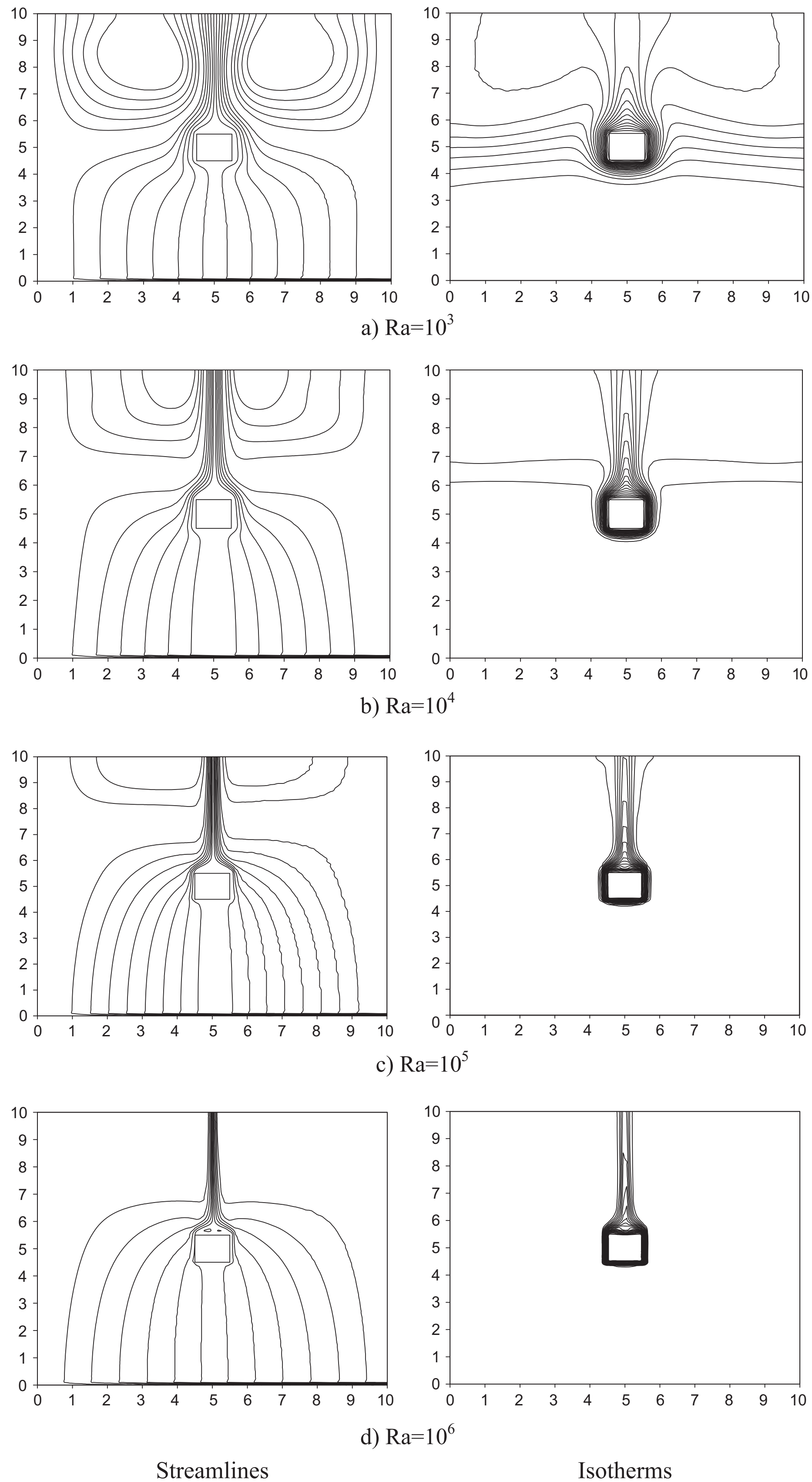

Figure 2 (a–d) Streamlines and Isotherms for $Pr = 0.7$, Ra from 10^3 to 10^6.

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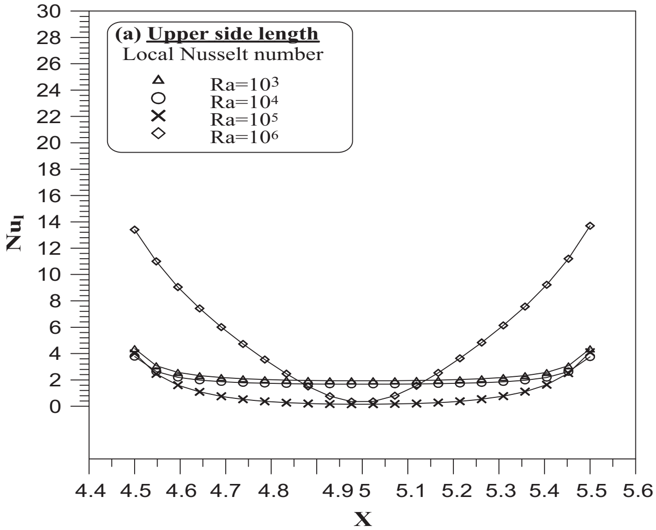
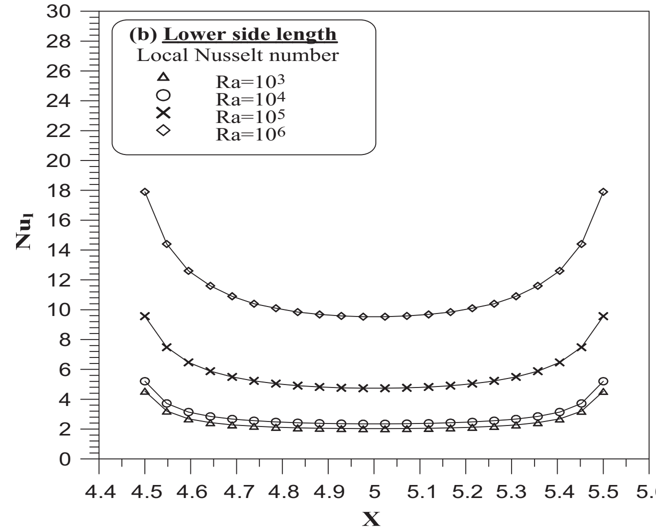
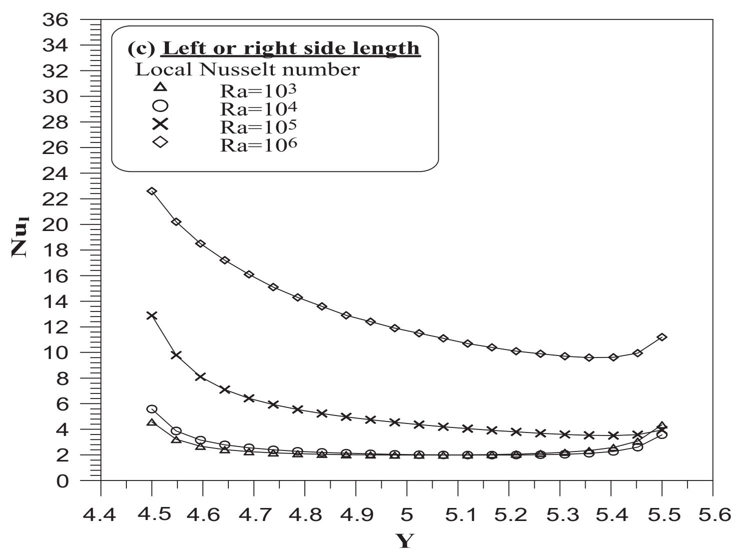


(a) Upper side length



(b) Lower side length



(c) Left or right side length

Figure 3 (a–c) Effect of *Ra* on local Nusselt number.

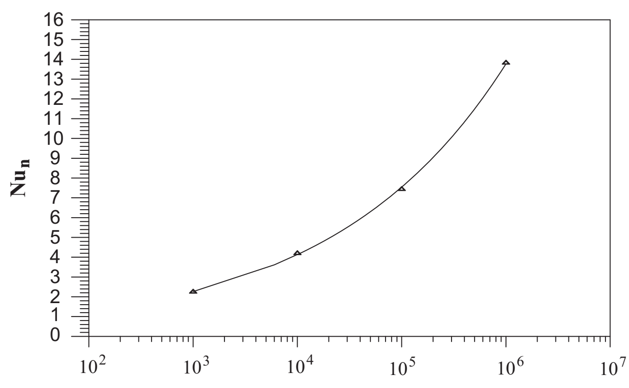


Figure 4 Effect of *Ra* on average Nusselt number.

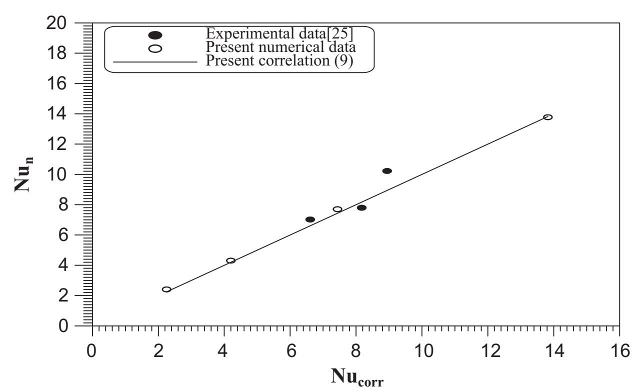


Figure 5 Correlation of numerical results.

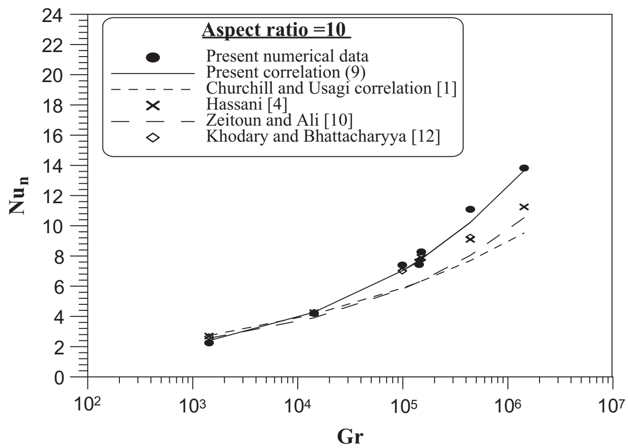


Figure 6 Comparison between present and previous data.

The streamlines indicate that the fluid, around the square cylinder and far from it, is entrained toward the square cylinder and forced to flow vertically upward above the cylinder. Also, the isotherms around the cylinder show a vertical narrow plume.

Increasing the Rayleigh number from 10^3 to 10^6, speeds up the flow toward the cylinder and the plume becomes thinner. For the same case, the isotherms shrink around the cylinder, as Ra was increased.

3.2. Local Nusselt number

The local Nusselt number distribution along all sides of the square cylinder was affected by increasing the Rayleigh number from 10^3 to 10^6 for natural convection as shown in Fig. 3(a)–(c). The figure shows that, the local Nusselt number along all sides of the square cylinder except the upper side was increased as Ra increased. For upper side, increasing Ra from 10^3 up to 10^5 decreased the local Nusselt number. A further increase in Ra to 10^6 led to a large increase in local Nusselt number as shown in figure.

The value of the local Nusselt number is always higher at the edges of the horizontal sides of the cylinder. For vertical sides, Nu_l is higher at the bottom edge and gradually decreases along the vertical side.

3.3. Average Nusselt number

Fig. 4 shows the average Nusselt number for natural convection, Nu_n plotted versus Rayleigh number from 10^3 to 10^6 for $Pr = 0.7$. As shown in figure, the average Nusselt number increases as Ra increases. However, a higher increase in Nu_n is noticed for high Ra. For $10^5 < Ra < 10^6$, the slope reaches a value of 0.27 which indicates a laminar boundary layer regime.

4. Correlations and comparison with previous work

The heat transfer data are collected in one correlation as suggested by Oosthuizen and Bishop [2]. The Least Squares method is used to get the optimum values of the correlation constants. This is given as follows:

$$Nu_{corr} = 0.384Gr^{0.252} = 0.42Ra^{0.252} \tag{9}$$

The present numerical results for natural convection are shown in Fig. 5. The experimental measurements from one of the authors [25] are also plotted. The maximum deviation is 14.33% and the average deviation is 5.73% while the standard deviation is about 0.017.

4.1. Comparison with previous work

Present correlation (9) is compared with present and previous work in Fig. 6. The comparison is limited to available data in the range $1.428 \times 10^3 < Gr \leqslant 1.428 \times 10^6$.

For natural convection, Fig. 6 shows comparison between present work and each of Churchill and Usagi [1], Hassani [4], Zeitoun and Ali [10] and Khodary and Bhattacharyya [12]. The present correlation (9) shows a very good agreement in the low range $10^3 < Gr < 2 \times 10^4$. For $Gr > 2 \times 10^4$ the present correlation shows a good agreement with [4,12]. However, the present correlation overestimates Nu_n w.r.t. data from [1,10]. The maximum deviation is 31.13% with an average deviation of 13.41%.

5. Conclusions

Laminar natural convection heat transfer from an isothermal horizontal square cylinder to air ($Pr = 0.7$) is numerically investigated. Streamlines and isotherms are generated to describe the flow around the square cylinder. The effect of Ra was also investigated numerically for natural convection and duct aspect ratio of 10 in the range ($10^3 \leqslant Ra \leqslant 10^6$). The results are presented as local Nusselt number distributions along the sides of the square cylinder, and average Nusselt numbers for different values of the Rayleigh number. The results are also presented in a simple correlation. The following conclusions can be extracted:

1. The streamlines indicate that the fluid, around the square cylinder and far from it, was entrained toward the cylinder and forced to flow vertically upward. Also, the isotherms around the cylinder show a vertical narrow plume. Increasing Ra from 10^3 to 10^6 increased the mass entrained and the plume became thinner than before.
2. The local Nusselt number distributions along left and right sides of the cylinder are identical which ensures the accuracy of the numerical solution. The values of the local Nusselt numbers increased with Ra on all sides except the upper one. These values are higher at the left and right corners and the lower side gives higher values than the upper side.
3. The average Nusselt number increases with increasing Ra from 10^3 to 10^6.

References

[1] S.W. Churchill, R. Usagi, A general expression for the correlation of rates of transfer and other phenomena, AIChE J. 18 (1972) 1121–1128.
[2] P.H. Oosthuizen, M. Bishop, An experimental study of mixed convection heat transfer from square cylinders, in: AIAA 22nd Thermophysics Conference. 8–10 June 1987/Honolulu, Hawaii, 1987, pp. 1–7.

[3] P. H Oosthuizen, M. Bishop, A numerical study of mixed convective heat transfer from square cylinders, in: Proceedings of the Eleventh Canadian Congress of Applied Mechanics. 31 May, 4 June 1987/Edmanton, 1987.

[4] A.V. Hassani, Natural convection heat transfer from cylinders of arbitrary cross section, ASME, J. Heat Transfer 114 (1992) 768–773.

[5] S.B. Clemes, K.G.T. Hollands, A.P. Brunger, Natural convection heat transfer from long horizontal isothermal cylinders, Trans. ASME J. Heat Transfer 116 (1994) 96–104.

[6] M.M. Yovanovich, Natural convection from isothermal convex bodies: simple models for bounds on body-gravity function, in: AIAA 34th Thermophysics Conference. 19–22 June 2000/ Denver, Co, 2000.

[7] S. Mahmud, D.P. kumar, N. Hyder, Laminar natural convection around an isothermal square cylinder at different orientations, Int. Commun. Heat Transfer 29 (7) (2002) 993–1003.

[8] E. Radziemska, W.M. Lewandowski, Natural convection heat transfer from isothermal cuboids, Int. J. Heat Mass Transfer 46 (2003) 2169–2178.

[9] C.O. Popiel, J. Wojtkowiak, Experimental free convective heat transfer from side walls of a vertical square cylinder in air, Exp. Thermal Fluid Sci. 29 (2004) 1–8.

[10] O. Zeitoun, M. Ali, Natural convection heat transfer from isothermal horizontal rectangular ducts, Alexandria Eng. J. 44 (5&6) (2005) 695–704.

[11] D.A. Kumar, A. Dalal, A numerical study of natural convection around a square, horizontal, heated cylinder placed in an enclosure, Int. J. Heat Mass Transfer 49 (2006) 4608–4623.

[12] K. Khodary, T.K. Bhattacharyya, Optimum natural convection from square cylinder in vertical channel, Int. J. Heat Fluid Flow 27 (2006) 167–180.

[13] R.W. Davis, E.F. Moore, L.P. Purtell, Numerical–experimental study of confined flow around rectangular cylinders, Phys. Fluids 27 (1) (1984) 46–59.

[14] T. Igarashi, Heat transfer from a square prism to an air stream, Int. J. Heat Mass Transfer 28 (1) (1985) 175–181.

[15] T. Igarashi, Fluid flow and heat transfer around rectangular cylinders (the case of a width/height ratio of a section of 0.33∼1.5), Int. J. Heat Mass Transfer 30 (5) (1987) 893–901.

[16] R.J. Goldstein, S.Y. Yoo, M.K. Chung, Convection mass transfer from a square cylinder and its base plate, Int. J. Heat Mass Transfer 33 (1) (1990) 9–18.

[17] S.Y. Yoo, R.J. Goldstein, M.K. Chung, Effects of angle of attack on mass transfer from a square cylinder and its base plate, Int. J. Heat Mass Transfer 36 (2) (1993) 371–381.

[18] M. Breuer, J. Bernsdorf, T. Zeiser, F. Durst, Accurate computations of the laminar flow past a square cylinder based on two different methods: lattice- Boltzmann and finite-volume, Int. J. Heat Fluid Flow 21 (2000) 186–196.

[19] J.L. Rosales, A. Ortega, J.A.C. Humphery, A numerical simulation of the convection heat transfer in confined channel flow past square cylinders: comparison of inline and offset tandem pairs, Int. J. Heat Mass Transfer 44 (2001) 587–603.

[20] S. Turki, H. Abbassi, S.B. Nasrallah, Effect of the blockage ratio on the flow in a channel with a built – in square cylinder, Comput. Mech. 33 (2003) 22–29.

[21] A. Sharma, V. Eswaran, Heat and fluid flow across a square cylinder in the two–dimensional laminar flow regime, Numer. Heat Transfer 45 (3) (2004) 247–269, part A.

[22] A.K. Dhiman, R.P. Chhabra, V. Eswaran, Steady flow across a confined square cylinder: effects of power-law index and blockage ratio, J. Non-Newton. Fluid Mech. 148 (2008) 141–150.

[23] A.K. Dhiman, R.P. Chhabra, V. Eswaran, Steady mixed convection across a confined square cylinder, Int. Commun. Heat Mass Transfer 35 (2008) 47–55.

[24] S.V. Patankar, Numerical Heat Transfer and Fluid Flow, Mc Graw-Hill, New York, 1980.

[25] A.R. Moussa, Laminar mixed convection from an isothermal horizontal square cylinder (Ph.D. thesis), Mech. Eng. Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt, 2009.