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The median function on graphs with bounded profiles☆

Kannan Balakrishnan[a], Manoj Changat[b], Sandi Klavžar[c,*,1]

[a] Department of Computer Applications, Cochin University of Science and Technology, Cochin-22, India
[b] Department of Futures Studies, University of Kerala, Trivandrum, India
[c] Department of Mathematics and Computer Science, FNM, University of Maribor, Gosposvetska 84, 2000 Maribor, Slovenia

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Abstract

The median of a profile $\pi = (u_1, \ldots, u_k)$ of vertices of a graph G is the set of vertices x that minimize the sum of distances from x to the vertices of π. It is shown that for profiles π with diameter θ the median set can be computed within an isometric subgraph of G that contains a vertex x of π and the r-ball around x, where $r > 2\theta - 1 - 2\theta/|\pi|$. The median index of a graph and r-joins of graphs are introduced and it is shown that r-joins preserve the property of having a large median index. Consensus strategies are also briefly discussed on a graph with bounded profiles.

Keywords: Consensus; Median function; Local property; Median graph; Consensus strategy

1. Introduction

The idea of consensus is present in many different fields, for instance in economics, sociology, and biology; we refer to [3] for a general mathematical formalization of the consensus theory. The general situation can be frequently presented as a group of clients that wish to achieve a consensus by some rational process which can in turn be modeled with consensus functions on some discrete structure. The most studied discrete structures in this respect are posets (see, for instance, [4,9]) and graphs.

A natural way of achieving a consensus on a graph is by means of a median function. The special case of median functions when the profile is the whole vertex set of a (vertex-weighted) graph has been extensively studied, see [2,14] and the references therein. On the other hand, a profile in a graph is often small with respect to the whole graph. For instance, it can consist of as few as three vertices of which two are at distance two [12]. In such cases the median of the profile might be found without processing the entire graph, and many algorithms which do not work globally may

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* Corresponding author. Fax: 386 2 23 55 605.
E-mail addresses: bkannan@cusat.ac.in (K. Balakrishnan), changat@vsnl.com, mchangat@gmail.com (M. Changat), sandi.klavzar@uni-mb.si (S. Klavžar).
1 The author is also with the Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana.

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be locally feasible for such profiles. Hence it seems reasonable to consider profiles with bounded diameter and we initiate such studies in this paper.

In the next section we introduce the concepts and definitions needed in this paper. In particular, the majority strategy is described. In the subsequent section we consider profiles π with bounded diameter and show that then the median of π can be obtained locally, either in a properly bounded isometric subgraph (Theorem 2) or in an induced subgraph that contains π (Theorem 3).

Mulder [12] proved that the majority strategy produces the median of π in G for all π if and only if the corresponding graph is median. (In fact, this is also equivalent to the fact that the majority strategy produces the median of π in G, for all π of length 3.) For another characterization of median graphs in terms of the median function see [1]. Due to this central role of median graphs in the graph theoretical consensus theory we introduce in Section 4 a median index of a graph. (For graphs that are locally hypercubes see [7].) We also introduce a d-join of graphs and prove that the d-join of graphs with median indices at least d is a graph with the same property (Theorem 7).

We conclude the paper with a discussion of known consensus strategies related to situations with bounded profiles and propose two new strategies that could be useful in the location theory.

2. Preliminaries

All graphs considered in this paper are simple and connected.

The *distance* $d_G(u, v)$, or briefly $d(u, v)$, between two vertices u and v in a graph G is defined as the number of edges on a shortest u, v-path. A subgraph H of a graph G is an *isometric subgraph* if $d_H(u, v) = d_G(u, v)$ for all vertices u, v in H. The *diameter* $\mathrm{diam}(G)$ of a graph G is $\max_{u,v \in V(G)} d(u, v)$.

A *profile* π on a graph G is a finite sequence of vertices of G. (Sequences are taken in order to enable possible repetitions.) For a vertex x of G, let $D(x, \pi) = \sum_{v \in \pi} d(x, v)$, where d is the usual shortest path distance. Then x is a *median vertex for* π if $D(x, \pi)$ is minimum. The *median function* M is the function that for each profile π on G returns the set of its median vertices $M(\pi, G)$. The set $M(\pi, G)$ is called the *median* of π in G. The median function is also known as the *median procedure*, see [10,11].

For a graph G, a vertex u of G and a set of vertices $X \subseteq V(G)$ we will write $d(u, X) = \min\{d(u, x) \mid x \in X\}$. The distance $d(x, \pi)$ between the vertex x and the profile π is defined analogously. The *diameter of a profile* π, $\mathrm{diam}(\pi)$, is $\max_{u,v \in \pi} d(u, v)$.

The *k-ball* at a vertex x of a graph G is $B_k(x) = \{v \in V \mid d(x, v) \leq k\}$. We will also use the same notation to denote the subgraph of G induced by $B_k(x)$.

A connected graph G is a *median graph* if, for every triple u, v, w of vertices, there exists a unique vertex x, called the median of u, v, w, such that x lies simultaneously on shortest paths joining u and v, v and w, and u and w. We refer to [8] for a survey on median graphs including their role in location theory.

Let T be a tree. Then it is not difficult to observe that for a given profile π, we can find $M(\pi, T)$ by starting in an arbitrary vertex and moving in the tree to the majority of π. In [12] Mulder extended this approach in the following way.

Let uv be an edge of a connected graph G, and let π be a profile in G. Set π_{uv} to be the subprofile of π consisting of all elements of π nearer to u than to v. Then the *majority strategy* for π reads as follows.

Input: A connected graph G and a profile π in G.
Output: $X \subseteq V(G)$.

1. Start at an initial vertex u of G.
2. If v is a neighbor of u with $|\pi_{uv}| \geq |\pi|/2$, then move to v; move to a vertex already visited twice only if there is no other choice.
3. Stop when either we are stuck at a vertex v (i.e. $|\pi_{wv}| < |\pi|/2$, for all neighbors w of v) or we have visited vertices at least twice, and, for each vertex v visited at least twice and each neighbor w of v, either w is also visited twice or $|\pi_{wv}| < |\pi|/2$.
4. Let X consist of the single vertex where we get stuck or of all vertices visited at least twice.

If the majority strategy produces for π the same set X from any initial vertex, then we say that it *produces* X for π. Mulder [12] proved:

Theorem 1. *Let G be a connected graph. Then the following conditions are equivalent.*

(i) *G is a median graph.*
(ii) *The majority strategy produces $M(\pi, G)$, for all π.*
(iii) *The majority strategy produces $M(\pi, G)$, for all π of length 3.*

3. The median function on profiles with bounded diameter

In this section we consider profiles with bounded diameter. We obtain conditions on the containment of the profile in some isometric or induced subgraph which guarantee that we can act locally. We begin by showing that the median of a profile π in G with $\operatorname{diam}(\pi) = \theta$ can be obtained by restricting to a relatively small isometric subgraph of G.

Theorem 2. *Let θ be a nonnegative integer, G a connected graph, π a profile in G with $\operatorname{diam}(\pi) = \theta$, and $x \in \pi$. Let H be an isometric subgraph of G containing $B_r(x)$ where r is a fixed integer satisfying*

$$r > 2\theta - 1 - \frac{2\theta}{|\pi|}.$$

Then $M(\pi, H) = M(\pi, G)$.

Proof. Note first that if $\theta = 0$ then $\pi = (x)$ and hence clearly $M(\pi, H) = M(\pi, G) = \{x\}$. Assume in the rest that $\theta > 0$, and let $r > 2\theta - 1 - \frac{2\theta}{|\pi|}$.
Claim: $M(\pi, G) \subseteq B_r(x)$.

Let $w \in \pi$. Then

$$D(w, \pi) = \sum_{\substack{u \in \pi \\ u \neq w}} d(w, u) \leq (|\pi| - 1)\theta.$$

Let $w' \in G \setminus B_r(x)$. Then $d(w', x) \geq r + 1$ while for any other $w \in \pi$, $w \neq x$, we infer

$$r + 1 \leq d(w', x) \leq d(w', w) + d(w, x) \leq d(w', w) + \theta,$$

therefore $d(w', w) \geq r + 1 - \theta$. Thus for $w' \in G \setminus B_r(x)$ we have:

$$\begin{aligned} D(w', \pi) &\geq r + 1 + (|\pi| - 1)(r + 1 - \theta) \\ &= r|\pi| + |\pi| - \theta|\pi| + \theta \\ &> \Big(2\theta - 1 - \frac{2\theta}{|\pi|}\Big)|\pi| + |\pi| - \theta|\pi| + \theta \\ &= (|\pi| - 1)\theta \\ &\geq D(w, \pi). \end{aligned}$$

This proves the claim.

Since H is an isometric subgraph of G containing $B_r(x)$ we have $D_H(w, \pi) = D_G(w, \pi)$ for any $w \in V(H)$. Hence by the above argument, $M(\pi, H) \subseteq B_r(x)$. As $B_r(x)$ is a subgraph of H we conclude that $M(\pi, H) = M(\pi, G)$. $\square$

Note that if $B_r(x)$ is an isometric subgraph of G for some vertex x of π then we may set $H = B_r(x)$ in Theorem 2. This holds in particular for any tree T and any vertex from a profile in T.

In the previous theorem the graph H to which a computation can be restricted must be isometric. In some cases it might not be easy to find such a subgraph (or it might also not exist), hence we next wish to drop the isometry assumption. This can be done by extending the corresponding balls.

Theorem 3. *Let G, π, θ, x and r be as in Theorem 2. Let H be an induced subgraph of G containing $B_{r+\theta}(x)$. Then $M(\pi, H) = M(\pi, G)$.*

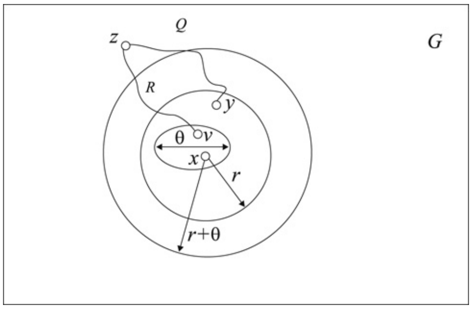

Fig. 1. Situation from the proof.

Proof. Note that in order to establish the claim in the proof of Theorem 2 the isometry assumption of H is not used. Therefore, also in the induced case, $M(\pi, G) \subseteq B_r(x)$. Thus it suffices to prove that the distance between a vertex $v \in \pi$ and a vertex $y \in B_r(x)$ is unaltered in the induced subgraph $B_{r+\theta}(x)$. This distance cannot increase because every shortest path from v to y in $B_r(x)$ is also a path in $B_{r+\theta}(x)$. Also in $B_{r+\theta}(x)$ we have $d(v, y) \le d(v, x) + d(x, y) \le r + \theta$.

Consider an arbitrary v, y-path P that does not lie completely in $B_{r+\theta}(x)$. Then P is a concatenation of the subpath Q from y to a vertex z outside $B_{r+\theta}(x)$ and the subpath R from z to v; see Fig. 1.

Since $y \in B_r(x)$ and $z \notin B_{r+\theta}(x)$ we have that $|Q| \ge \theta + 1$. Moreover, since

$$\theta + r + 1 \le d(x, z) \le d(x, v) + d(v, z) \le \theta + d(v, z)$$

we infer that $|R| \ge d(v, z) \ge r + 1$. Hence P is of length at least $r + \theta + 2$ and thus P is not a geodesic. We conclude that $M(\pi, G)$ is the same as $M(\pi, H)$. □

Theorems 2 and 3 can be applied in all situations in which it is possible to detect some previously studied structure in the vicinity of a profile. As an example of such a result we state:

Corollary 4. *Let $\theta \ge 1$ and let π be a profile in a connected graph G with $\operatorname{diam}(\pi) \le \theta$. If x is a vertex of π such that $B_{3\theta}(x)$ is a tree, then $M(\pi, G)$ is either a single vertex or a path.*

Proof. Goldman [5] proved that the median of a profile in a tree is either a single vertex or a path. Now we just apply Theorem 3. □

4. Locally median graphs

As already mentioned, median graphs form one of the central graph classes in the graph theoretical consensus theory. We therefore introduce the following concepts.

Let v be a vertex of a graph G. Then the *median index of* v, $\mathrm{mx}_G(v)$, is the largest integer $k \le \operatorname{diam}(G)$ such that $B_j(v)$ is a median graph for $0 \le j \le k$. The *median index of* G, $\mathrm{mx}(G)$, is the minimum of the median indices of the vertices of G. For instance, $\mathrm{mx}(C_n) = \lfloor n/2 \rfloor - 1$, while for a tree T, $\mathrm{mx}(T) = \operatorname{diam}(T)$. G is said to be *locally p-median* if its median index is p.

Proposition 5. *Let $\theta \ge 0$ and let G be a connected graph. Let π be a profile in G with $\operatorname{diam}(\pi) \le \theta$, and let v be an element of π with $\mathrm{mx}_G(v) \ge 3\theta$. If x is a vertex of G with $d(x, \pi) \le \theta$, then the majority strategy started from x produces $M(\pi, G)$.*

Proof. Clearly, all the vertices of π belong to $B_\theta(v)$. Since $d(x, \pi) \le \theta$, we have $x \in B_{2\theta}(v)$ and all shortest paths from x to π lie in $B_{3\theta}(v)$. As $B_{2\theta}(v)$ is an induced subgraph of $B_{3\theta}(v)$, Theorem 3 implies that $M(\pi, G)$ is contained in $B_{2\theta}(v)$. Since $B_{3\theta}(v)$ is a median graph, Theorem 1 implies that starting from x, the majority strategy finds the median of π in $B_{3\theta}(v)$. Moreover, for any vertex y outside $B_{3\theta}(v)$, $D(y, \pi) > D(x, \pi)$, therefore no move to an outside vertex can be made by the majority strategy. So the same moves will be made in the original graph G and hence the result. □

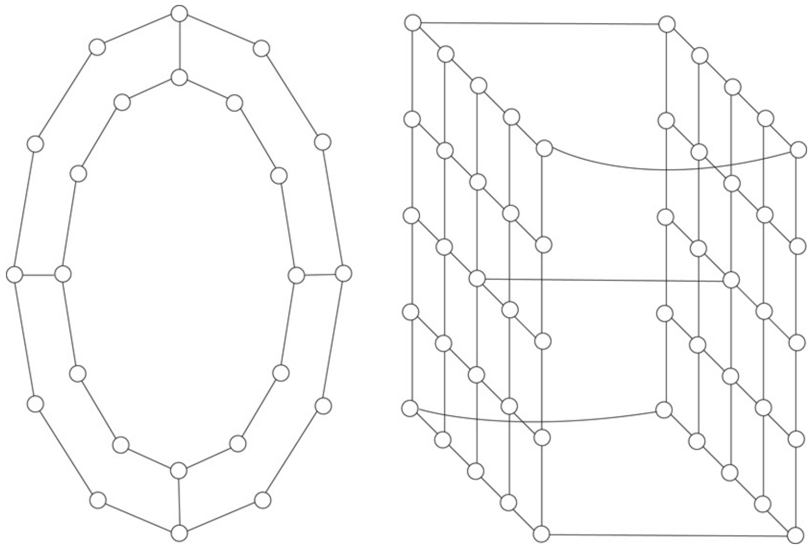

Fig. 2. A 3-join and a 4-join.

It is therefore desirable to have graphs with large median indices. To construct large graphs with this property we introduce a graph operation called the d-join of graphs.

Let d be an arbitrary positive integer. By a *d-distance sequence* in a graph G we mean a finite sequence S of distinct vertices of G such that for any two vertices u, v of S, $d(u, v) \geq d$. Clearly any permutation of a d-distance sequence is also a d-distance sequence. Let G_1 and G_2 be graphs and let S_1 and S_2 be d-distance sequences of equal lengths in G_1 and G_2 respectively. Then the *d-join* of G_1 and G_2 with respect to S_1 and S_2 is the graph obtained from the disjoint union of G_1 and G_2 by joining the corresponding vertices in S_1 and S_2 by edges.

The d-join construction is illustrated in Fig. 2. The left graph is a 3-join of C_{12} with itself, the right graph is a 4-join of $P_5 \,\Box\, P_5$ with itself. (Recall that the *Cartesian product* $G \,\Box\, H$ of two graphs has the vertex set $V(G) \times V(H)$ where the vertex (g, h) is adjacent to (g', h') whenever $gg' \in E(G)$ and $h = h'$, or $g = g'$ and $hh' \in E(H)$, see [6].)

To show that the d-join operation preserves large median index we first recall the following – part of the folklore – result.

Lemma 6. *A connected graph G is a median graph if and only if every block of G is median.*

We can now state the main theorem of this section.

Theorem 7. *Let $d \geq 1$ and let G_1 and G_2 be graphs with $\mathrm{mx}(G_1) \geq d$ and $\mathrm{mx}(G_2) \geq d$. Let G_3 be a d-join of G_1 and G_2, then $\mathrm{mx}(G_3) \geq d$.*

Proof. Let G_3 be the d-join of G_1 and G_2 with respect to the d-distance sequences $S = (s_1, \ldots, s_k)$ and $T = (t_1, \ldots, t_k)$. Consider an arbitrary vertex $x \in V(G_3)$. We need to show that $B_i(x)$ is a median graph for $1 \leq i \leq d$.

We first show that $B_d(x)$ is a median graph. Assume without loss of generality that $x \in G_1$ and for all $1 \leq i \leq k$ define

$$T_i(x) = \{w \in V(G_2) \mid d(x, w) = d(x, t_i) + d(t_i, w) \leq d\}.$$

That is, $T_i(x)$ consists of those vertices of G_2 that can be reached from x via t_i along a shortest path of length at most d.

Note that

$$B_d(x) = (B_d(x) \cap G_1) \cup \bigcup_{i=1}^{k} T_i(x).$$

Claim: For any $1 \leq i \neq j \leq k$, $T_i(x) \cap T_j(x) = \emptyset$.

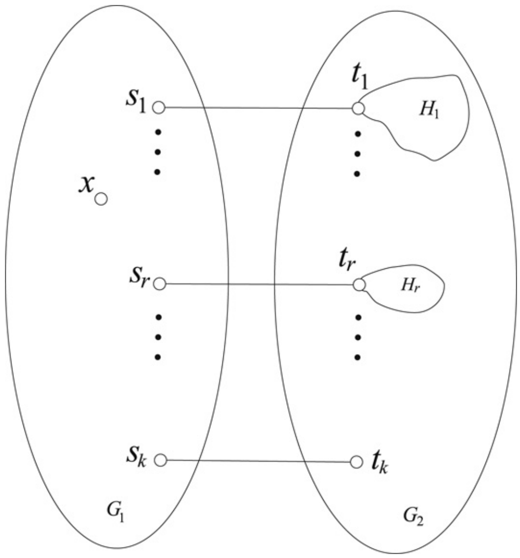

Fig. 3. The d-neighborhood of x.

Suppose on the contrary that there exists a vertex $y \in T_i(x) \cap T_j(x)$. Then, by the definition of $T_i(x)$ and $T_j(x)$,

$$d(x, y) = d(x, t_i) + d(t_i, y) = d(x, s_i) + 1 + d(t_i, y) \le d$$

and

$$d(x, y) = d(x, t_j) + d(t_j, y) = d(x, s_j) + 1 + d(t_j, y) \le d.$$

Summing these two inequalities we get

$$d(x, s_i) + d(x, s_j) + d(y, t_i) + d(y, t_j) \le 2d - 2. \tag{1}$$

On the other hand, having in mind that G_3 is a d-join, we infer that

$$d(s_i, x) + d(x, s_j) \ge d(s_i, s_j) \ge d$$

and

$$d(t_i, y) + d(y, t_j) \ge d(t_i, t_j) \ge d\,.$$

This gives

$$d(x, s_i) + d(x, s_j) + d(y, t_i) + d(y, t_j) \ge 2d\,. \tag{2}$$

Since inequalities (1) and (2) are in contradiction the claim is proved.

$B_d(x) \cap G_1$ is the d-neighborhood of x in G_1. Since $\mathrm{mx}(G_1) \ge d$ it follows that $B_d(x) \cap G_1$ induces a median graph.

Assume first that $B_d(x) \cap G_2 = \emptyset$. Then $B_d(x) = B_d(x) \cap G_1$ and hence we conclude that $B_d(x)$ is a median graph in this case.

Suppose $B_d(x) \cap G_2 \ne \emptyset$. Let H_i, $1 \le i \le k$, be the subgraph of G_2 induced by the vertices from $T_i(x)$. Clearly, if $d(x, t_i) > d$ then H_i is the empty graph. We may assume without loss of generality that for some $r \ge 1$, precisely the subgraphs $H_1, \ldots, H_r$ are not empty. The situation is shown in Fig. 3. Suppose $d(x, s_i) = a_i$, $1 \le i \le r$. Then $V(H_i) = B_{d-a_i-1}(t_i) \cap G_2$ and because $\mathrm{mx}(G_2) \ge d$ we also find out that H_i is a median graph for $i = 1, \ldots, r$. Lemma 6 now implies that $B_d(x)$ induces a median graph also in the case when $B_d(x) \cap G_2 \ne \emptyset$.

Finally, the structure of $B_i(x)$, where $1 \le i < d$, is analogous to the structure of $B_d(x)$, hence by the same arguments as above $B_i(x)$ induces a median graph. We conclude that $\mathrm{mx}(G_3) \ge d$. $\square$

Let G_{kn} be the n-join of two copies of the cycle C_{kn}, where $2 < k \leq n$. That is, the two cycles are connected with k edges such that an n-join is constructed. The case $k = 3$ and $n = 4$ is shown in Fig. 2. Since $\text{mx}(C_{kn}) = \lfloor kn/2 \rfloor - 1 \geq n$, Theorem 7 implies that $\text{mx}(G_{kn}) = n$.

For another example consider the Cartesian product $G_n = P_{2n+1} \,\Box\, P_{2n+1}$, where $n \geq 1$. Select the four vertices of degree 2 and the central vertex of G_n for a $2n$-distance sequence of G_n and let H_n be the $2n$-join of two copies of G_n. (H_2 is shown in Fig. 2.) Then $\text{mx}(H_n) = 2n$ by Theorem 7.

5. Consensus strategies

In the preliminaries we have described the majority strategy that searches for the median set of a profile in an arbitrary graph. The consensus criteria of the strategy is to move, if we are at v and w is a neighbor of v, from v to w whenever

$$|\pi_{wv}| \geq \frac{1}{2}|\pi|.$$

Modifying this consensus criteria, two other well-known strategies are obtained:

- *Condorcet*: move from v to w whenever $|\pi_{vw}| \leq \frac{1}{2}|\pi|$.
- *Plurality*: move from v to w whenever $|\pi_{vw}| \leq |\pi_{wv}|$.

It is easy to observe that in the case of bipartite graphs the majority strategy, the condorcet strategy, and the plurality strategy coincide. This observation together with Theorem 2 yields the following result for graphs that are bipartite in a vicinity of a profile.

Proposition 8. *Let $\theta \geq 0$ and let π a profile in G with* $\text{diam}(\pi) \leq \theta$. *If v is a vertex of π such that the induced subgraph $B_{3\theta}(v)$ is bipartite, then the majority strategy, the condorcet strategy, and the plurality strategy coincide on $B_\theta(v)$.*

We say that a strategy is *effective* provided that if we start the strategy from a vertex of the profile π in a graph G, then the strategy necessarily produces $M(\pi, G)$. Now, if the majority strategy is effective then G must be bipartite in a vicinity of the profile. More precisely:

Proposition 9. *Let $\theta \geq 0$. If for each profile π with* $\text{diam}(\pi) \leq \theta$ *the majority strategy is effective, then G does not contain any odd cycle of length less than $2\theta + 3$.*

Proof. The lemma is obvious for $\theta = 0$ since the profile contains a single vertex, and every single vertex graph is trivially bipartite.

Now assume that $\theta \geq 1$. We first prove that G is triangle-free. Assume that G contains a triangle u, v, w. Consider the profile $\pi = (u, v, w)$. Then $D(x, \pi) = 2$ for x in π and $D(x, \pi) \geq 3$ for x outside π. So $M(\pi, G) = \{u, v, w\}$. If we apply majority strategy starting at u, we find that we are stuck at u and we do not get all of $M(\pi, G)$. Hence G has to be triangle-free.

Assume that G contains an odd cycle of length less than $2\theta + 3$. Let C be a minimal odd cycle in G of length $t < 2\theta + 3$. Then C is an isometric cycle in G. Take any vertex u of C and let v and w be vertices on C at a distance t from u. Now we have $D(v, \pi) = D(w, \pi) = t + 1$. Take any vertex x distinct from v and w. Since G is triangle-free, x cannot be adjacent to both v and w, say $d(x, w) \geq 2$. Due to the triangle inequality, we have $d(x, u) + d(x, v) \geq t$. Hence $D(x, \pi) \geq t + 2$, therefore $M(\pi, G) = \{v, w\}$. We apply the majority strategy from initial position v with respect to π. Let x be any neighbor of v. If $x = w$, only x is nearer to x than v. If $x \neq w$, then only u could be nearer to x than v. Hence we do not move to x, so that we are stuck at v. Again we do not get all of $M(\pi, G)$. $\square$

To conclude the paper we propose two additional consensus strategies, the idea arising from artificial intelligence [13]. In these strategies the consensus criteria is to move from v to w provided that:

- *Hill Climbing*: move from v to w whenever $D(w, \pi) \leq D(v, \pi)$.
- *Steepest Ascent Hill Climbing*: move from v to w whenever $D(w, \pi) \leq D(v, \pi)$ and $D(w, \pi)$ is minimum among all neighbors of v.

It seems that these strategies could offer new insights into the consensus theory.

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