



## Radiative scaling neutrino mass and warm dark matter

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### ABSTRACT

A new and radical scenario of the simple 2006 model of radiative neutrino mass is proposed, where there is no seesaw mechanism, i.e. neutrino masses are not inversely proportional to some large mass scale, contrary to the prevalent theoretical thinking. The neutral singlet fermions in the loop have masses of order 10 keV, the lightest of which is absolutely stable and the others are very long-lived. All are components of warm dark matter, which is a possible new paradigm for explaining the structure of the Universe at all scales.

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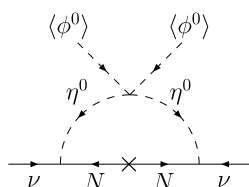


Fig. 1. One-loop generation of scotogenic Majorana neutrino mass.

Neutrino mass and dark matter are two important issues in physics and astrophysics. In 2006, it was proposed [1] that they are in fact intimately related in a model of radiative Majorana neutrino mass in one loop, where the particles appearing in the loop are odd under an exactly conserved  $Z_2$  symmetry [2], thus allowing them to be dark-matter candidates. The model is simplicity itself. It extends the minimal standard model of particle interactions to include three neutral singlet fermions  $N_k$  and a second scalar doublet  $\eta = (\eta^+, \eta^0)$ , all of which are odd under the aforementioned  $Z_2$ , whereas all standard-model particles are even. Thus the observed neutrinos  $\nu_i$  are forbidden to couple to  $N_k$  through the standard-model Higgs doublet  $\Phi = (\phi^+, \phi^0)$ , but the new interactions  $h_{ik}(\nu_i \eta^0 - l_i \eta^+) N_k + H.c.$  as well as the mass terms  $(M_k/2) N_k N_k + H.c.$  are allowed. Hence Majorana neutrino masses are generated in one loop as shown in Fig. 1. This mechanism has been called “scotogenic”, from the Greek “scotos” meaning darkness. Because of the allowed  $(\lambda_5/2)(\Phi^\dagger \eta)^2 + H.c.$  interaction,  $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$  is split so that  $m_R \neq m_I$ . The diagram of Fig. 1 can be computed exactly [1], i.e.

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} \times \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right]. \quad (1)$$

A good dark-matter candidate is  $\eta_R$  as first pointed out in Ref. [1]. It was subsequently proposed by itself in Ref. [3] and studied in detail in Ref. [4]. The  $\eta$  doublet has become known as the “inert” Higgs doublet, but it does have gauge and scalar interactions even if it is the sole addition to the standard model. It may enable the scalar sector of this model to have a strong electroweak phase transition [5–7], thus allowing for successful baryogenesis. Because of the  $\lambda_3(\Phi^\dagger \Phi)(\eta^\dagger \eta)$  interaction, it may also contribute significantly to  $H \rightarrow \gamma\gamma$  [8,9].

The usual assumption for neutrino mass in Eq. (1) is

$$m_I^2 - m_R^2 \ll m_I^2 + m_R^2 \ll M_k^2, \quad (2)$$

in which case

$$(\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k} \left[ \ln \frac{M_k^2}{m_0^2} - 1 \right], \quad (3)$$

where  $m_0^2 = (m_I^2 + m_R^2)/2$  and  $m_R^2 - m_I^2 = 2\lambda_5 v^2$  ( $v = \langle \phi^0 \rangle$ ). This scenario is often referred to as the radiative seesaw. There is however another very interesting scenario, i.e.

$$M_k^2 \ll m_R^2, m_I^2. \quad (4)$$

Neutrino masses are then given by

$$(\mathcal{M}_\nu)_{ij} = \frac{\ln(m_R^2/m_I^2)}{16\pi^2} \sum_k h_{ik} h_{jk} M_k. \quad (5)$$

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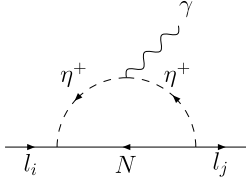


Fig. 2. Radiative decay of  $l_i \rightarrow l_j \gamma$ .

This simple expression is actually very extraordinary, because the prevalent theoretical thinking on neutrino masses is that they should be inversely proportional to some large mass scale, coming from the dimension-five operator [10]

$$\mathcal{L} = \frac{f_{ij}}{2\Lambda} (v_i \phi^0 - l_i \phi^+) (v_j \phi^0 - l_j \phi^+) + H.c., \quad (6)$$

whereas Eq. (5) is clearly not of this form, unless of course  $|m_R^2 - m_I^2| = 2|\lambda_5|v^2 \ll m_{R,I}^2$ , which is what all previous applications of Eq. (1) assume. However, there is no need for  $\lambda_5$  to be small. For example, let  $m_R = 240$  GeV,  $m_I = 150$  GeV, then  $\lambda_5 = 0.58$  and  $\ln(m_R^2/m_I^2) = 0.94$ . Eq. (5) also allows neutrino masses to be of order 0.1 eV and  $M_k$  of order 10 keV, with  $h_{ik}^2$  of order  $10^{-3}$ . If  $M_k = 0$ , then  $N_k$  may be assigned  $L = -1$  and  $L$  would be conserved. Thus  $M_k \neq 0$  corresponds to the breaking of  $L$  to  $(-1)^L$  and  $M_k$  could be naturally small, as compared to the electron mass which preserves  $L$ . Without the  $Z_2$  symmetry, the canonical seesaw mechanism [11], i.e.  $m_\nu = -m_D^2/M$ , would require the  $\nu N$  Dirac mass  $m_D$  to be of order 30 eV and the  $\nu - N$  mixing, i.e.  $m_D/M$ , to be of order  $3 \times 10^{-3}$ . This would render  $N$  unacceptable as a dark-matter candidate. As it is, with  $Z_2$  and  $\eta$ , there is no  $m_D$  and no  $\nu - N$  mixing. There is also no seesaw. Each neutrino mass is simply proportional to a linear combination of  $M_k$  according to Eq. (5). Hence their ratio is just a scale factor and small neutrino masses are due to this “scaling” mechanism. Note that the interesting special case where only  $M_1$  is small has been considered previously [12,13].

If  $\eta^\pm$ ,  $\eta_R$ ,  $\eta_I$  are of order  $10^2$  GeV, the interactions of  $N_k$  with the neutrinos and charged leptons are weaker than the usual weak interaction, hence  $N_k$  may be considered “sterile” and become excellent candidates of warm dark matter, which is a possible new paradigm for explaining the structure of the Universe at all scales [14,15]. However, unlike the usual sterile neutrinos [16] which mix with the active neutrinos, the lightest  $N_k$  here is absolutely stable. This removes one of the most stringent astrophysical constraints on warm dark matter, i.e. the absence of galactic X-ray emission from its decay, which would put an upper bound of perhaps 2.2 keV on its mass [17], whereas Lyman- $\alpha$  forest observations (which still apply in this case) impose a lower bound of perhaps 5.6 keV [18]. Such a stable sterile neutrino (called a “scotino”) was already discussed recently [19] in a left-right extension of the standard model, but the present proposal is far simpler. Conventional left-right models where the  $SU(2)_R$  neutrinos mix with the  $SU(2)_L$  neutrinos have also been studied [20–22].

The diagram of Fig. 1 is always accompanied by that of  $l_i \rightarrow l_j \gamma$  as shown in Fig. 2. For  $\mu \rightarrow e \gamma$ , this branching fraction is given by [23]

$$B(\mu \rightarrow e \gamma) = \frac{\alpha}{768\pi} \frac{|\sum_k h_{\mu k} h_{ek}^*|^2}{(G_F m_{\eta^+}^2)^2}. \quad (7)$$

Using the experimental upper bound [24] of  $2.4 \times 10^{-12}$ , this implies

$$m_{\eta^+} > 310 \text{ GeV} \left( \left| \sum_k h_{\mu k} h_{ek}^* \right| / 10^{-3} \right)^{1/2}. \quad (8)$$

Note that  $m_{\eta^+}$  of order 300 GeV is possible for electroweak baryogenesis. Together, this would imply that  $B(\mu \rightarrow e \gamma)$  is required by this model to be just below the present bound. As for the muon anomalous magnetic moment, it is given by [25]

$$\Delta a_\mu = -\frac{m_\mu^2}{96\pi^2 m_{\eta^+}^2} \sum_k |h_{\mu k}|^2. \quad (9)$$

Hence Eq. (8) implies

$$|\Delta a_\mu| < 1.23 \times 10^{-13} \left( \sum_k |h_{\mu k}|^2 / \left| \sum_k h_{\mu k} h_{ek}^* \right| \right), \quad (10)$$

much below the experimental uncertainty of  $6 \times 10^{-10}$ .

Since  $N_k$  are assumed light, muon decay also proceeds at tree level through  $\eta^+$  exchange, i.e.  $\mu \rightarrow N_\mu e \bar{N}_e$ . The inclusive rate is easily calculated to be

$$\Gamma(\mu \rightarrow N_\mu e \bar{N}_e) = \frac{(\sum_k |h_{\mu k}|^2)(\sum_k |h_{ek}|^2) m_\mu^5}{6144\pi^3 m_{\eta^+}^4}. \quad (11)$$

Since  $N_\mu$  and  $\bar{N}_e$  are invisible just as  $\nu_\mu$  and  $\bar{\nu}_e$  are invisible in the dominant decay  $\mu \rightarrow \nu_\mu e \bar{\nu}_e$  (with rate  $G_F^2 m_\mu^5 / 192\pi^3$ ), this would change the experimental value of  $G_F$ . However, their ratio  $R$  is very small. Using Eq. (8),

$$R < 2.5 \times 10^{-8} \left( \sum_k |h_{\mu k}|^2 \right) \left( \sum_k |h_{ek}|^2 \right) / \left| \sum_k h_{\mu k} h_{ek}^* \right|^2, \quad (12)$$

much below the experimental uncertainty of  $10^{-5}$ .

Whereas the lightest scotino, called it  $N_1$ , is absolutely stable,  $N_{2,3}$  will decay into  $N_1$  through  $\eta_R$  and  $\eta_I$ . The decay rate of  $N_2 \rightarrow N_1 \bar{\nu}_i \nu_j$  is given by

$$\begin{aligned} \Gamma(N_2 \rightarrow N_1 \bar{\nu}_i \nu_j) &= \frac{|h_{i2} h_{j1}^*|^2}{256\pi^3 M_2} \left( \frac{1}{m_R^2} + \frac{1}{m_I^2} \right)^2 \\ &\times \left( \frac{M_2^6}{96} - \frac{M_1^2 M_2^4}{12} + \frac{M_1^6}{12} - \frac{M_1^8}{96 M_2^2} + \frac{M_1^4 M_2^2}{8} \ln \frac{M_2^2}{M_1^2} \right). \end{aligned} \quad (13)$$

Let  $M_2 - M_1 = \Delta M$  be small compared to  $M_2$ , then

$$\Gamma(N_2 \rightarrow N_1 \bar{\nu}_i \nu_j) = \frac{|h_{i2} h_{j1}^*|^2 (\Delta M)^5}{1920\pi^3} \left( \frac{1}{m_R^2} + \frac{1}{m_I^2} \right)^2. \quad (14)$$

As an example, let  $\Delta M = 1$  keV,  $|h_{i2} h_{j1}^*|^2 = 10^{-6}$ ,  $m_R = 240$  GeV,  $m_I = 150$  GeV, then  $\Gamma = 6.42 \times 10^{-50}$  GeV, corresponding to a decay lifetime of  $3.25 \times 10^{17}$  y, which is much longer than the age of the Universe, i.e.  $13.75 \pm 0.11 \times 10^9$  y. This means that  $N_{1,2,3}$  may all be components of dark matter today. Note that  $N_2 \rightarrow N_1 \gamma$  is now possible with  $E_\gamma \simeq \Delta M$ , but since  $\Delta M$  may be small, say 1 keV, whereas  $M_{1,2,3} \sim 10$  keV, the tension between galactic X-ray data and Lyman- $\alpha$  forest observations is easily relaxed.

The decay rate of  $N_2 \rightarrow N_1 \gamma$  is given by [26]

$$\begin{aligned} \Gamma(N_2 \rightarrow N_1 \gamma) &= \frac{\alpha |\sum_i h_{i2} h_{i1}^*|^2 M_2^3 (M_2^2 + M_1^2)}{4096\pi^4 m_{\eta^+}^4} \left( 1 - \frac{M_1^2}{M_2^2} \right)^3 \\ &\simeq \frac{\alpha |\sum_i h_{i2} h_{i1}^*|^2 M_2^3 (\Delta M)^3}{256\pi^4 m_{\eta^+}^4}. \end{aligned} \quad (15)$$

Note that it is proportional to  $|\sum_i h_{i2} h_{i1}^*|^2$  which may be very much suppressed relative to  $(\sum_i |h_{i2}|^2)(\sum_i |h_{i1}|^2) \sim 10^{-6}$ .

The effective  $N\bar{N} \rightarrow \bar{l}l, \nu\bar{\nu}$  interactions are of order  $h^2/m_\eta^2 \sim 10^{-8} \text{ GeV}^{-2}$ , hence they remain in thermal equilibrium in the early Universe until a temperature of a few GeV. Their number density  $n_N$  is given by [27]

$$\frac{n_N}{n_\gamma} = \left( \frac{43/4}{g_{dec}^*} \right) \left( \frac{2}{11/2} \right) \frac{3/2}{2}, \quad (16)$$

where  $g_{dec}^* = 16$  in this model, counting  $N_{1,2,3}$  in addition to photons, electrons, and the three neutrinos. Their relic abundance at present would then be [27]

$$\Omega_N h^2 \simeq \frac{115}{16} \left( \frac{\sum_i M_i}{\text{keV}} \right). \quad (17)$$

For  $\sum_i M_i \sim 30 \text{ keV}$ , this would be  $1.9 \times 10^3$  times the measured value [28] of  $0.1123 \pm 0.0035$ . The usual solution to this problem is to invoke a particle which decouples after  $N_1$  and decays later as it becomes nonrelativistic, with a large release of entropy. It is a well-known mechanism and has been elaborated recently [16,20,22,29,30] in the context of warm dark matter.

Another solution is to assume that the reheating temperature of the Universe is below a few GeV, so that  $N_i$  are not thermally produced. Instead, they come from the decay of a scalar singlet  $S$  with the allowed interaction  $SN_i N_j$ . To accomplish this, consider  $m_S = 2 \text{ GeV}$ . Assume that  $S$  decouples as it becomes nonrelativistic with an annihilation cross section times relative velocity of about  $10^{-5} \text{ pb}$ . If  $S$  is stable, this would correspond to a very large relic density; but  $S$  decays to  $NN$ , so the actual relic density (i.e. that of  $N$ ) is reduced by the factor  $2M/m_S \simeq 10^{-5}$ . Since  $\langle \sigma v \rangle$  is inversely proportional to relic density, this makes it effectively 1 pb, and yields the correct observed dark-matter relic density of the Universe.

The interactions of  $S$  with itself and the particles of the scotogenic model are given by

$$-\mathcal{L}_{int} = \frac{1}{3} \mu_1 S^3 + \mu_2 S (\Phi^\dagger \Phi) + \mu_3 S (\eta^\dagger \eta) + \frac{1}{2} f_{ij} S N_i N_j + H.c. \\ + \frac{1}{4} \lambda_2 S^4 + \frac{1}{2} \lambda_3 S^2 (\Phi^\dagger \Phi) + \frac{1}{2} \lambda_4 S^2 (\eta^\dagger \eta). \quad (18)$$

Assume  $\mu_{1,2}$  to be negligible, so that  $S - H$  mixing may be ignored, where  $H$  is the physical Higgs boson with a mass of about 125 GeV. Assume  $f_{ij} < 10^{-4}$ , so that  $N_i$  does not enter into thermal equilibrium through its interaction with  $S$  below a few GeV. However, the interaction  $\sqrt{2} \lambda_3 \nu H S^2$  will allow  $S$  to thermalize because  $H$  couples to quarks and leptons. For  $\lambda_3 \sim 10^{-3}$ ,  $\langle \sigma v \rangle \sim 10^{-5} \text{ pb}$  may be obtained.

Consider now the phenomenology of this model at the Large Hadron Collider (LHC). The decay rate of  $H \rightarrow SS$  is given by

$$\Gamma(H \rightarrow SS) = \frac{\lambda_3^2 v^2}{4\pi m_H} \sim 0.02 \text{ MeV}, \quad (19)$$

compared to the expected total width of about 4.3 MeV in the standard model. Since  $S$  decays into  $NN$ , this appears as an invisible decay of  $H$ . However, this branching fraction is less than 0.5 percent, so it will be very difficult to check. As for the extra scalar particles  $\eta^\pm, \eta_R, \eta_I$ , they may be produced at the LHC through

their electroweak gauge interactions. Once produced, the decay  $\eta^+ \rightarrow l_i^+ N_j$  may be observed, but the decays  $\eta_{R,I} \rightarrow \nu_i N_j$  are invisible. If kinematically allowed,  $\eta^+ \rightarrow \eta_{R,I} W^+$  and  $\eta_R \rightarrow \eta_I Z$  are possible signatures. The case  $m_I > m_R$  with  $m_I - m_R < m_Z$ , so that  $\eta_I \rightarrow \eta_R + \text{virtual } Z \rightarrow \eta_R + \mu^+ \mu^-$  has already been studied in some detail [31].

In conclusion, the scotogenic model [1] of neutrino mass has been shown to admit a solution where there is no seesaw mechanism and  $N_{1,2,3}$  may have masses of about 10 keV. They are suitable as components of warm dark matter for explaining the structure of the Universe at all scales [14,15]. Since  $N_1$  is absolutely stable, whereas  $N_{2,3} \rightarrow N_1 \gamma$  is suppressed with  $E_\gamma \simeq \Delta M$ , the galactic X-ray upper bound of perhaps 2.2 keV on its mass [17] is avoided. It will also not be detected in terrestrial experiments. On the other hand, since this model requires an extra scalar doublet, it may be tested at the Large Hadron Collider.

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