About the creation of proton–antiproton pair at electron–positron collider in the energy range of $\psi(3770)$ mass

A.I. Ahmadov [a,c], Yu.M. Bystritskiy [a,*], E.A. Kuraev [a], P. Wang [b]

[a] *Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia*
[b] *Institute of High Energy Physic, Chinese Academy of Science, China*
[c] *Institute of Physics, Azerbaijan National Academy of Science, Baku, Azerbaijan*

Abstract

The process of electron–positron annihilation into proton–antiproton pair is considered within the vicinity of $\psi(3770)$ resonance. The interference between the pure electromagnetic intermediate state and the $\psi(3770)$ state is evaluated. It is shown that this interference is destructive and the relative phase between these two contributions is large ($\phi_0 \approx 250°$).
© 2014 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/3.0/). Funded by SCOAP³.

1. Introduction

Large statistics of J/ψ, $\psi(2S)$ and $\psi(3770)$ samples have been obtained in recent years by BEPCII/BESIII facility [1]. It provides the possibility to study many decay channels of J/ψ, $\psi(2S)$ and $\psi(3770)$ resonances. In a profound work, BESIII has measured the phase angle ϕ between the continuum and resonant amplitudes [2] and found two possible solutions, which are $\phi = (266.9 \pm 6.1 \pm 0.9)°$ or $\phi = (255.8 \pm 37.9 \pm 4.8)°$. This means that the strong decay
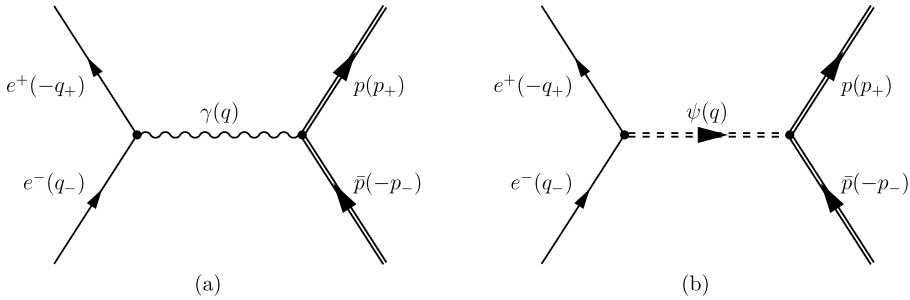
Fig. 1. Feynman diagrams of processes $e^+ + e^- \to \bar{p} + p$ in Born approximation and with the quarkonium $\psi(3770)$ intermediate state.

amplitude and electromagnetic decay amplitude are almost orthogonal. The BESIII data were taken as an energy scan in the vicinity of $\psi(3770)$. The data show some structure: clearly seen dip in the energy strip of size of the resonance $\psi(3770)$ width, which had been observed previously by CLEO Collaboration for some mesonic decay channels [3].

In this note we try to explain this rather specific behavior of the total cross section of process $e^+ e^- \to p\bar{p}$ in the energy range close to resonance $\psi(3770)$ creation.

In contrast to the channel $e^+ e^- \to \psi(3770) \to \mu^+ \mu^-$, in process of hadron creation (i.e. $e^+ e^- \to \psi(3770) \to \pi^+ \pi^-$, $\bar{p}p$, $\bar{n}n$), a QCD gluonic state contribution to the hadron (in particular nucleon) formfactor $\psi \to \bar{p}p$ is to be investigated. Besides the Breit–Wigner character of the amplitude, one must take into account the specific character of interaction of quarkonium to nucleon–antinucleon pair mediated through 3 gluon intermediate state and the final state interaction of the created nucleon pair.

The second effect is the final state interaction phase of amplitude which arises mostly from large distances (or soft exchanges of final stable hadrons). It has the same form for $\gamma^* \to \bar{p}p$ and for $\psi \to \bar{p}p$ vertexes and we can safely assume its cancellation in the interference of pure QED and quarkonium states.

On the contrary, the phase which arises from 3 gluon state can essentially affect on the Breit–Wigner character of pure QED final state.

It is the motivation of this paper to investigate the detailed behavior of the total cross section in the energy range within the mass of a narrow resonance $\psi(3770)$.

2. Born approximation

We consider two mechanisms of creation of a $p\bar{p}$ in electron–positron collisions (see Fig. 1)

$$e^+(q_+) + e^-(q_-) \to p(p_+) + \bar{p}(p_-). \tag{1}$$

One proceeds through virtual photon intermediate state (see Fig. 1(a)), leading to the contribution to matrix element

$$\mathcal{M}_B = \frac{4\pi\alpha}{s} G(s) J_\mu^e J^{p\mu}, \tag{2}$$

where lepton J_μ^e and proton J_μ^p currents have a form:

$$J_\mu^e = \bar{v}(q_+)\gamma_\mu u(q_-), \qquad J_\mu^p = \bar{u}(p_+)\gamma_\mu v(p_-),$$

and $G(s)$ is the model-dependent proton formfactor.

In the recent paper [4] the remarkable relation $F_1(\sqrt{s} \sim 2 \text{ GeV}) = 1$, $F_2(\sqrt{s} \sim 2 \text{ GeV}) = 0$ for proton formfactors near the threshold was obtained which meant, that proton in some environment near the $\sqrt{s} \sim 2$–3 GeV can be considered as a point-like particle. Assuming this facts and keeping in mind the closeness of the considered energy range to the $p\bar{p}$ threshold we put further $G(s) = 1$. The corresponding contribution to the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{4s} \left(2 - \beta^2 \sin^2 \theta \right), \qquad s = (q_+ + q_-)^2 = 4E^2, \qquad \beta^2 = 1 - \frac{m^2}{E^2}, \tag{3}$$

where m is the proton mass, $\sqrt{s} = 2E$ is the total energy in center of mass reference frame (cmf), E is the electron beam energy and the scattering angle θ is the cmf angle between the 3-momenta of the initial electron $\mathbf{q}_-$ and the created proton $\mathbf{p}_+$. The total cross section then

$$\sigma_B(s) = \frac{2\pi\alpha^2 \beta (3 - \beta^2)}{3s}. \tag{4}$$

3. The quarkonium $\psi(3770)$ contribution: three gluon vertex

The second mechanism (see Fig. 1(b)) describes the conversion of electron–positron pair to $\psi(3770)$ with the subsequent conversion to the proton–antiproton pair through three gluon intermediate state (see Fig. 2(a)).

For this aim we put the whole matrix element as

$$\mathcal{M} = \mathcal{M}_B + \mathcal{M}_\psi^{(3g)}, \tag{5}$$

where the contribution with $\psi(3770)$ intermediate state is

$$\mathcal{M}_\psi^{(3g)} = \frac{g_e}{s - M_\psi^2 + i M_\psi \Gamma_\psi} J_\nu^e J_{(3g)}^\nu. \tag{6}$$

Here we assumed that vertex $\psi \to e^+ e^-$ has the same structure as $\gamma \to e^+ e^-$, i.e.:

$$J_{\psi \to e^+ e^-}^\mu = g_e J_e^\mu, \tag{7}$$

and the constant g_e is defined via $\psi \to e^+ e^-$ decay ($g_e^2 = 12\pi \Gamma_{\psi \to e^+ e^-}/M_\psi$) thus giving $g_e = 1.6 \cdot 10^{-3}$ [5].

The current $J_{(3g)}^\nu$ which describes the transition of $\psi(3770)$ with momentum $q = 2p$ into proton–antiproton pair via three gluon intermediate state (see Fig. 2(a)). The gluons in Fig. 2(a) interact with colour degrees of freedom in the proton block. The dynamics of this block between gluon interaction vertexes is complicated, but for simplicity we approximate it with effective fermion propagator with proton mass. That thus gives:

$$J_{(3g)}^\nu = R(4\pi\alpha_s)^3 g_{col} \int \frac{d^4 k_1 \, d^4 k_2 \, d^4 k_3 (2\pi)^{-8}}{k_1^2 k_2^2 k_3^2 ((p_+ - k_1)^2 - m^2)((p_- - k_3)^2 - m^2)}$$
$$\times \delta(q - k_1 - k_2 - k_3) \left[\bar{u}(p_+) \hat{O}^\nu v(p_-) \right], \tag{8}$$

where α_s is the strong interaction coupling which is associated with each gluon line and $\hat{O}^\nu$ is

$$\hat{O}_\nu = \hat{O}_\nu^{\alpha\beta\gamma} \gamma_\alpha (\not{p}_+ - \not{k}_1 + m)\gamma_\beta(-\not{p}_- + \not{k}_3 + m)\gamma_\gamma \tag{9}$$
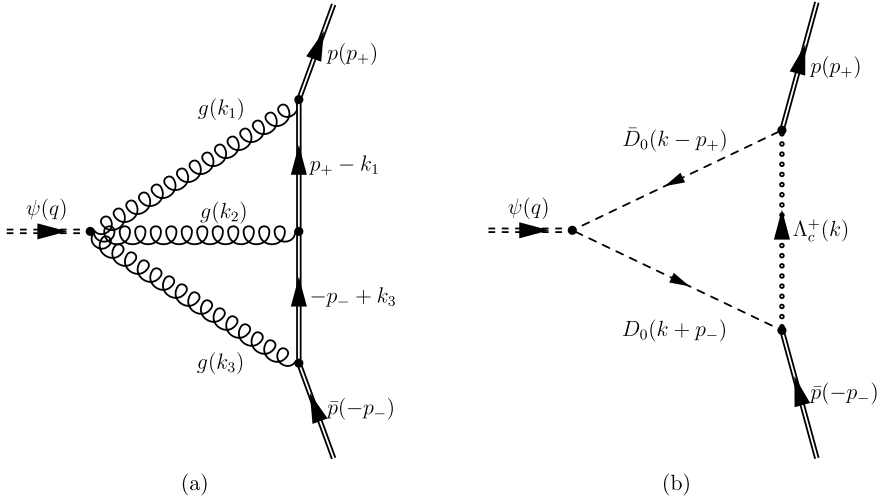
and

Fig. 2. Vertexes of $\psi(3770)$ transition into proton–antiproton pair.

$$\hat{O}_v^{\alpha\beta\gamma} = \text{Tr}\big[\hat{O}^{\alpha\beta\gamma}(\not{p}+M)\gamma_v(\not{p}-M)\big],$$
$$\hat{O}^{\alpha\beta\gamma} = \frac{\gamma_\gamma(-\not{p}+\not{k}_3+M)\gamma_\beta(\not{p}-\not{k}_1+M)\gamma_\alpha}{((p-k_3)^2-M^2)((p-k_1)^2-M^2)} + \text{permutations}, \tag{10}$$

where p and M are the 4-momentum and the mass of the charmed quark (antiquark) inside $\psi(3770)$ state and one must take into account the contributions from all gluon lines permutations. Color factor

$$g_{col} = \langle p|(3/4)d^{abc}t_a t_b t_c|p\rangle = 5/6 \tag{11}$$

describes the interaction of gluons with quarks of the proton. The quantity R is connected with wave function of $\psi(3770)$ and is derived in Appendix A.

Thus the contribution to the total cross section arising from the interference of relevant amplitudes has the form

$$\delta\sigma_{3g} = \frac{1}{8s}2\,\text{Re}\bigg[\sum_{\text{spins}}\int \mathcal{M}_B^*\mathcal{M}_\psi^{(3g)}\,d\Gamma_2\bigg], \tag{12}$$

where two-particle phase volume $d\Gamma_2$ is

$$d\Gamma_2 = \frac{d^3 p_+}{2E_+}\frac{d^3 p_-}{2E_-}\frac{1}{4\pi^2}\delta^4(q-p_+-p_-) = \frac{\beta}{16\pi}d\cos\theta, \tag{13}$$

and θ is again the angle between the directions of initial electron $\mathbf{q}_-$ and the produced proton $\mathbf{p}_+$.

To perform the summation over spin states we use the method of invariant integration [6]:

$$\sum_{\text{spins}}\int d\Gamma_2 J_\mu^{p*}J_\nu^{(3g)} = \frac{1}{3}\bigg(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\bigg)\int d\Gamma_2\sum_{\text{spins}}J_\lambda^{p*}J^{(3g)\lambda} = -\frac{2s\beta}{3\pi}Q, \tag{14}$$

where

$$Q = \frac{1}{4}\text{Tr}\big[(\not{p}_+ + m)\hat{O}_\lambda(\not{p}_- + m)\gamma_\lambda\big]. \tag{15}$$

Thus we get for the contribution to the total cross section

$$\delta\sigma_{3g} = \text{Re}\left(\frac{S_{3g}(s)}{s - M_\psi^2 + i M_\psi \Gamma_\psi}\right), \tag{16}$$

where

$$S_{3g}(s) = -\frac{\alpha}{24} g_e g_{col} R \alpha_s^3 \beta Z(\beta), \tag{17}$$

$$Z(\beta) = \frac{4}{\pi^5 s} \int \frac{d^4 k_1 d^4 k_2 d^4 k_3 \delta(2p - k_1 - k_2 - k_3)}{k_1^2 k_2^2 k_3^2 ((p_+ - k_1)^2 - m^2)((p_- k_2)^2 - m^2)} Q$$

$$= H(\beta) + i F(\beta), \tag{18}$$

where $H(\beta)$ and $F(\beta)$ are correspondingly real and imaginary part of vertex $\psi \to 3g \to p\bar{p}$, i.e. function $Z(\beta)$. Our approach consists in calculation of the s-channel discontinuity of $Z(\beta)$ with the subsequent restoration of real part $H(\beta)$ with the use of dispersion relation. For this aim we use the Cutkosky rule for gluon propagators

$$\frac{1}{(k_1^2 + i0)} \frac{1}{(k_2^2 + i0)} \frac{1}{(k_3^2 + i0)} \to (-2\pi i)^3 \delta(k_1^2) \delta(k_2^2) \delta(k_3^2). \tag{19}$$

This allows us integrate over phase volume of three gluon intermediate state as

$$d\Phi_3 = \frac{d^4 k_1 d^4 k_2 d^4 k_3}{(2\pi)^5} \delta(k_1^2) \delta(k_2^2) \delta(k_3^2) \delta^4(2p - k_1 - k_2 - k_3)$$

$$= (2\pi)^{-5} \frac{1}{8} dx_1 dx_2 d\Omega_1 d\Omega_2 \delta_c, \tag{20}$$

where

$$\delta_c = \delta(c - p(x)), \qquad p(x) = 1 - 2\frac{x_1 + x_2 - 1}{x_1 x_2},$$

$$x_i \equiv \frac{\omega_i}{E}, \qquad x_1 + x_2 + x_3 = 2,$$

and c is the cosine of the angle between directions $\mathbf{k_1}$ and $\mathbf{k_2}$. It is convenient to write the phase volume element in form

$$d\Phi_3 = \frac{s\pi^2}{8(2\pi)^5} dx_1 dx_2 d\gamma \theta(1 - x_1)\theta(1 - x_2)\theta(x_1 + x_2 - 1),$$

$$d\gamma = \frac{d\Omega_1 d\Omega_2}{4\pi^2} \delta_c = \frac{1}{\pi} \frac{dc_1 dc_2}{\sqrt{D}}, \qquad D = 1 - c_1^2 - c_2^2 - p^2(x) + 2c_1 c_2 p(x), \tag{21}$$

where $d\Omega_i$ is the phase volumes of the on mass shell gluons and $c_{1,2} \equiv \cos(\mathbf{p_+}, \mathbf{k_{1,2}})$. So we obtain s-channel discontinuity of Z in the form:

$$i\Delta_s Z = \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \int d\gamma \frac{Q_1}{C_1 C_2} = F(\beta), \tag{22}$$

where $C_1 = x_1(1 - \beta c_1)$ and $C_2 = x_2(1 + \beta c_2)$ and the integration over phase volume $d\gamma$ is performed in the kinematical region where $D > 0$. Explicit form of $i\Delta Z$ and Q_1 are given in
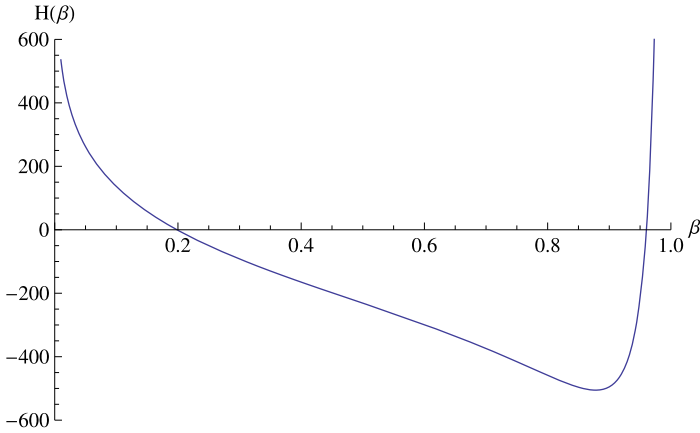
Fig. 3. The numerical estimation of the quantity $H(\beta)$ (see (18) and (24)) as a function of β.

Appendix B. The angular integration can be performed using the form of the phase volume given above and the set of integrals given in Appendix B.

As we are interested in the energy region close to the mass of resonance, we use some trick to restore the real part of Z by means of dispersion relations. For this aim we do a replacement

$$Z(s) \to \Psi(s) = \frac{M_\psi^2}{s} Z(s), \qquad \Psi(s) = Z(s) \frac{M^2}{E^2} = \frac{M^2}{E^2} \big(H(\beta) + i F(\beta) \big). \tag{23}$$

We use the Cauchy theorem (un-substracted dispersion relation) to obtain the real part:

$$H(\beta) = \mathcal{P} \frac{1}{\pi} \int\limits_0^1 \frac{d\beta_1^2}{\beta_1^2 - \beta^2} F(\beta_1)$$

$$= \frac{1}{\pi} \left\{ F(\beta) \ln \frac{1 - \beta^2}{\beta^2} + \int\limits_0^1 \frac{2\beta_1 d\beta_1}{\beta_1^2 - \beta^2} \big[F(\beta_1) - F(\beta) \big] \right\}. \tag{24}$$

The quantity $H(\beta)$ as a function of β is shown in Fig. 3.

4. The quarkonium $\psi(3770)$ contribution: D^0 mesons loop vertex

It is known that the main contribution to the decay width of $\psi(3770)$ arise from the OZI non-violating channels $\psi(3770) \to \bar{D}D$ [5]. However the contribution of $\bar{D}D$ state as an intermediate state converting to proton–antiproton is expected to be small. The main reason for this is the absence of charmed quarks inside a proton. In this section we will estimate the contribution of D mesons loop to the process of our interest by using only $D^0 \bar{D}^0$ loop in the vertex $\psi \to p\bar{p}$ (see Fig. 2(b)). The amplitude of the process (1) with the ψ intermediate state which converts via $D^0 \bar{D}^0$ loop into proton–antiproton we write in the form similar to (6):

$$\mathcal{M}_D = \frac{g_e}{s - M_\psi^2 + i M_\psi \Gamma_\psi} J_\nu^e J_D^\nu, \tag{25}$$

where current J_D^μ has a form:

$$J_D^\mu = \frac{g_{\psi DD}}{16\pi^2} \int \frac{d^4k}{i\pi^2} g_D\big((k-p_+)^2\big) g_D\big((k+p_-)^2\big)$$
$$\times \frac{[\bar{u}(p_+)\gamma_5(\slashed{k}+M_{\Lambda_c^+})\gamma_5 v(p_-)](2k+p_--p_+)^\mu}{(k^2-M_{\Lambda_c^+}^2)((k-p_+)^2-M_D^2)((k+p_-)^2-M_D^2)}, \tag{26}$$

where $g_{\psi DD}$ is the constant for vertex $\psi D^0 \bar{D}^0$ which can be estimated from the decay width $\Gamma_{\psi \to D^0 \bar{D}^0} = 0.26$ keV [5] which gives

$$g_{\psi DD} = \frac{4M_\psi \sqrt{3\pi \Gamma_{\psi \to D^0 \bar{D}^0}}}{(M_\psi^2 - 4M_D^2)^{3/4}} = 12.6. \tag{27}$$

The loop integral in (26) diverges in case of point-like particles. Usually one uses some formfactor to cut this divergency [7,8]. Following this tradition we use formfactors for the vertex $D^0 p \Lambda_c^+$ in the form [9]:

$$g_D(q^2) = \frac{2M_D^2 f_D}{m_u + m_c} \frac{g_{DN\Lambda}}{q^2 - M_D^2}, \tag{28}$$

where $f_D \approx 180$–200 MeV and quark masses we choose as $m_u \approx 280$ MeV and $m_c = 1.27$ GeV [5]. The constant $g_{DN\Lambda}$ was estimated in [10]:

$$g_{DN\Lambda} \approx 6.74. \tag{29}$$

Performing standard calculation of loop integral in (26) using Feynman trick to merge the denominators one can write the contribution $D\bar{D}$ intermediate state to the cross section in the form similar to (16) as:

$$\delta\sigma_D = \text{Re}\left(\frac{S_D(s)}{s - M_\psi^2 + iM_\psi \Gamma_\psi}\right), \tag{30}$$

where

$$S_D(s) = \frac{\alpha}{24\pi^2} g_e g_{DN\Lambda}^2 g_{\psi DD}\left(1 + \frac{2m^2}{s}\right)\sqrt{1 - \frac{4m^2}{s}} B_D(s), \tag{31}$$

$$B_D(s) = \left(\frac{2M_D^2 f_D}{m_u + m_c}\right)^2 \int\limits_0^1 dx \int\limits_0^{1-x} dy\, xy$$
$$\times \left\{\frac{1}{(d(s)+i\epsilon)^2} + \frac{2mx}{(d(s)+i\epsilon)^3}\frac{s-4m^2}{s+2m^2}\big(M_{\Lambda_c^+} - m(1-x)\big)\right\}, \tag{32}$$

$$d(s) = M_{\Lambda_c^+}^2 x + M_D^2(1-x) - m^2 x(1-x) - sy(1-x-y). \tag{33}$$

5. Discussion

In order to see the relative contribution of different mechanisms to the phase we will consider first the contribution of three gluons in the intermediate state. The total cross section then has a form
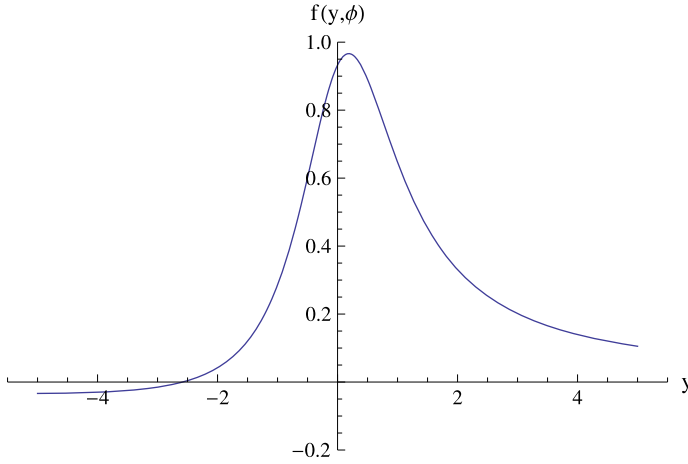
Fig. 4. The numerical estimation of the quantity $f(y, \phi)$ (see (35)) as a function of β.

$$\sigma(s) = \sigma_B(s) + \delta\sigma_{3g}(s), \qquad \frac{\delta\sigma_{3g}(s)}{\sigma_B(s)} = B(\beta)f(y, \phi), \tag{34}$$

where

$$B(\beta) = \frac{g_e g_{col} R \alpha_s^3}{32\alpha(3 - \beta^2)} \frac{P M_\psi}{\Gamma_\psi},$$

$$f(y, \phi) = \frac{y \cos \phi + \sin \phi}{y^2 + 1}, \qquad y = \frac{s - M_\psi^2}{M_\psi \Gamma_\psi}, \tag{35}$$

and the quantities P and ϕ are defined as

$$H + iF = Pe^{i\phi}, \qquad P = \sqrt{H^2 + F^2},$$

$$R = \frac{1}{9}\sqrt{\frac{2}{\pi}}\alpha_s^{3/2}, \qquad g_e = \sqrt{\frac{12\pi \Gamma_{ee}}{M_\psi}}, \qquad g_{col} = \frac{5}{6}. \tag{36}$$

The function $f(y, \phi)$ is shown in Fig. 4. At the point of $\psi(3770)$ resonance, $\beta = \beta_0 = 0.86$, we have both quantities F and H negative and thus the phase ϕ is equal to

$$\phi = \arctan\left(\frac{F(\beta_0)}{H(\beta_0)}\right) + 180° = 67° + 180° = 247°. \tag{37}$$

The ratio of the $B(\beta_0)$ to P is

$$B(\beta_0) = 3 \cdot 10^{-5} P, \quad P = 1396. \tag{38}$$

It is known that the main contribution to the width of $\psi(3770)$ arise from the OZI non-violating channels $\psi(3770) \to \bar{D}D$ [5]. However the contribution of $\bar{D}D$ state as an intermediate state converting to proton–antiproton is small. Main reason of it is the absence of charm quarks inside a proton. In order to demonstrate this we add the D-loop contribution $\delta\sigma_D$ from (30) to the cross section in (34), i.e.:

$$\sigma(s) = \sigma_B(s) + \delta\sigma_{3g}(s) + \delta\sigma_D(s), \tag{39}$$

and then, to calculate the phase ϕ, we need to use complete expressions for the amplitudes, i.e. $S_{3g}(s)$ from (17) and $S_D(s)$ from (31). This gives the following result for the phase:

$$\phi = \arctan\left(\frac{\text{Im}(S_{3g}(M_\psi^2)) + \text{Im}(S_D(M_\psi^2))}{\text{Re}(S_{3g}(M_\psi^2)) + \text{Re}(S_D(M_\psi^2))}\right) + 180°$$

$$= 81° + 180° = 261°, \tag{40}$$

and thus we conclude that D-meson loop contribution to the phase is rather small and the main contribution to the phase goes from three gluon intermediate state.

We should also notice that we did not evaluate the contribution of a square of amplitude with $\psi(3770)$ intermediate state. It is small compared with the contribution of interference of Born amplitude with the one with $\psi(3770)$ meson and will be estimated elsewhere. It does not exceed ten percents.

The quantities for phase ϕ in (37) and in (40) are in good agreement with recent experimental data for phase at BESIII Collaboration [2].

Acknowledgements

Appendix A. Vertex $\psi \to 3g$

To restore the quantity R from (8) we calculate the width of $\psi(3770)$ resonance decay into three gluons. Let us consider the conversion of the bound state with quantum numbers $J^{PC} = 1^{--}$ to three real massless vector bosons. Similar problem was solved years ago for the problem of ortho-positronium decay [11,12]. For the case of ortho-positronium Ops decay, we start from matrix element of the process:

$$Ops \to \gamma(k_1) + \gamma(k_2) + \gamma(k_3), \tag{A.1}$$

which has the form:

$$\mathcal{M}_{Ops} = A\frac{1}{m_e^4}O_\sigma^{\mu\nu\lambda}e_\mu(k_1)e_\nu(k_2)e_\lambda(k_3)\epsilon_\sigma(q), \tag{A.2}$$

with $e(k_i)$ and $\epsilon(q)$ are the polarization vectors of photons and the ortho-positronium respectively. The quantity A includes the information on the wave function of ortho-positronium. Operator

$$O_\sigma^{\mu\nu\lambda}e_\mu(k_1)e_\nu(k_2)e_\lambda(k_3) = \frac{1}{4}\text{Tr}\big[\hat{Q}(\not{p} + m_e)\gamma_\sigma(\not{p} - m_e)\big], \tag{A.3}$$

$$\hat{Q} = \frac{1}{x_1 x_3}\not{e}_3(-\not{p} + \not{k}_3 + m_e)\not{e}_2(\not{p} - \not{k}_1 + m_e)\not{e}_1 + \text{cyclic permutations}, \tag{A.4}$$

describes the electron loop. Using the amplitude (A.2) we obtain for the decay width

$$\Gamma_{Ops} = \frac{1}{12m_e}\int\sum_{\text{spins}}|\mathcal{M}_{Ops}|^2 \cdot \frac{m_e^2\pi^2}{(2\pi)^5}d^2x \cdot \frac{(4\pi\alpha)^3}{3!}$$
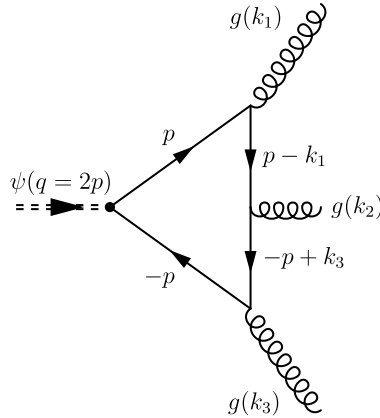
$$= \frac{64}{9}m_e A^2(\pi^2 - 9)\alpha^3. \tag{A.5}$$

Fig. 5. The diagram of $\psi(3770)$ decay into three gluons.

Comparing this value with the known result $\Gamma_{Ops} = (2m_e/(9\pi))(\pi^2 - 9)\alpha^6$ we conclude that

$$A = \frac{\alpha^{3/2}}{4\sqrt{2\pi}}. \tag{A.6}$$

Here we used the following formulae

$$\sum_{\text{spins}} \frac{1}{m_e^8} \left| O_\sigma^{\mu\nu\lambda} e_\mu(k_1) e_\nu(k_2) e_\lambda(k_3) \right|^2 = 256 Q(x),$$

$$Q(x) = \frac{1}{(x_1 x_2 x_3)^2} \left[x_1^2(1-x_1)^2 + x_2^2(1-x_2)^2 + x_3^2(1-x_3)^2 \right],$$

$$\int d^3x \, \delta(2 - x_1 - x_2 - x_3) Q(x) = \pi^2 - 9.$$

For the case of decay of $\psi(3770)$ to three gluons with the subsequent turning them to hadrons we define the amplitude in the form similar to (A.2) (see Fig. 5):

$$\mathcal{M}_{\psi \to 3g} = R(4\pi\alpha_s)^{3/2} \frac{1}{4} d^{abc} e_\mu^a(k_1) e_\nu^b(k_2) e_\lambda^c(k_3) \frac{1}{M^4} O_\sigma^{\mu\nu\lambda} \varepsilon^\sigma(q), \tag{A.7}$$

with $q = 2p$ and $\varepsilon(q)$ are the momentum and the polarization vector of $\psi(3770)$. The decay width then reads as:

$$\Gamma_{\psi \to 3g} = \frac{80}{27} M_\psi R^2 (\pi^2 - 9) \alpha_s^3. \tag{A.8}$$

And comparing this result with the known one [13,14]:

$$\Gamma_{\psi \to 3g} = \frac{160 M_\psi}{2187\pi} (\pi^2 - 9) \alpha_s^6, \tag{A.9}$$

we conclude that

$$R = \frac{1}{9} \sqrt{\frac{2}{\pi}} \alpha_s^{3/2} \approx 0.0146, \tag{A.10}$$

if one assumes that $\alpha_s \approx 0.3$. Note that both A and R are real.

Appendix B. Angular integrals

In this section we present the angular integrals which are relevant for the integration in (22):

$$\frac{1}{\pi} \int \frac{dc_1 dc_2}{C_1 C_2 \sqrt{D}} \{1; C_1; C_2; C_1 C_2; C_1^2; C_2^2; C_2^3; C_1^2 C_2; C_2^2 C_1\}$$
$$= \{J_{00}, J_{10}, J_{01}, J_{11}, J_{20}, J_{02}, J_{03}, J_{21}, J_{12}\}, \tag{B.1}$$

where

$$J_{00} = \frac{1}{x_1 x_2} I(x), \qquad J_{10} = \frac{1}{x_2} L, \qquad J_{01} = \frac{1}{x_1} L, \qquad J_{11} = 2,$$
$$J_{20} = \frac{x_1}{x_2} \big[L(1+p) - 2p \big], \qquad J_{02} = \frac{x_2}{x_1} \big[L(1+p) - 2p \big],$$
$$J_{21} = 2x_1, \qquad J_{12} = 2x_2,$$
$$J_{03} = \frac{x_2^2}{2x_1} \big[\big(1 + \beta^2 + 4p + (3 - \beta^2)p^2 \big) L + 2 \big(1 - 4p - 3p^2 \big) \big],$$

and

$$p = p(x) = 1 - \frac{2}{x_1 x_2}(x_1 + x_2 - 1),$$
$$C_1 = x_1(1 - \beta c_1), \qquad C_2 = x_2(1 + \beta c_2),$$
$$I(x) = \frac{2}{\sqrt{d}} \ln \frac{1 + \beta^2 p(x) + \sqrt{d}}{1 - \beta^2},$$
$$d = \big(1 + \beta^2 p(x) \big)^2 - \big(1 - \beta^2 \big) \big(1 - p(x)^2 \big),$$
$$L = \frac{1}{\beta} \ln \frac{1 + \beta}{1 - \beta}.$$

The explicit expression for $Q_1 = T_\lambda^{\alpha\beta\gamma} R_\lambda^{\alpha\beta\gamma}$ from (22) is

$$T_\lambda^{\alpha\beta\gamma} = \frac{1}{4} \text{Tr} \big[\hat{O}^{\alpha\beta\gamma} (\not{p} + M) \gamma_\lambda (\not{p} - M) \big],$$
$$R_\lambda^{\alpha\beta\gamma} = \frac{1}{4} \text{Tr} \big[(\not{p}_- - m) \gamma_\lambda (\not{p}_+ + m) \gamma_\alpha (\not{p}_+ - \not{k}_1 + m) \gamma_\beta (-\not{p}_- + \not{k}_2 + m) \gamma_\gamma \big],$$

where

$$\hat{O}^{\alpha\beta\gamma} = \frac{1}{x_1} \bigg[\frac{1}{x_2} \gamma_\beta(-\not{p} + \not{k}_2 + M) \gamma_\gamma + \frac{1}{x_3} \gamma_\gamma(-\not{p} + \not{k}_3 + M) \gamma_\beta \bigg] (\not{p} - \not{k}_1 + M) \gamma_\alpha$$
$$+ \frac{1}{x_2} \bigg[\frac{1}{x_3} \gamma_\gamma(-\not{p} + \not{k}_3 + M) \gamma_\alpha + \frac{1}{x_1} \gamma_\alpha(-\not{p} + \not{k}_1 + M) \gamma_\gamma \bigg] (\not{p} - \not{k}_2 + M) \gamma_\beta$$
$$+ \frac{1}{x_3} \bigg[\frac{1}{x_2} \gamma_\beta(-\not{p} + \not{k}_2 + M) \gamma_\alpha + \frac{1}{x_1} \gamma_\alpha(-\not{p} + \not{k}_1 + M) \gamma_\beta \bigg] (\not{p} - \not{k}_3 + M) \gamma_\gamma.$$

After calculation of traces and simplifications one gets

$$Q_1 = \frac{32}{x_1 x_2 x_3} \big\{ P_{00} + C_1 P_{10} + C_2 P_{01} + C_1 C_2 P_{11} + C_1^2 P_{20}$$
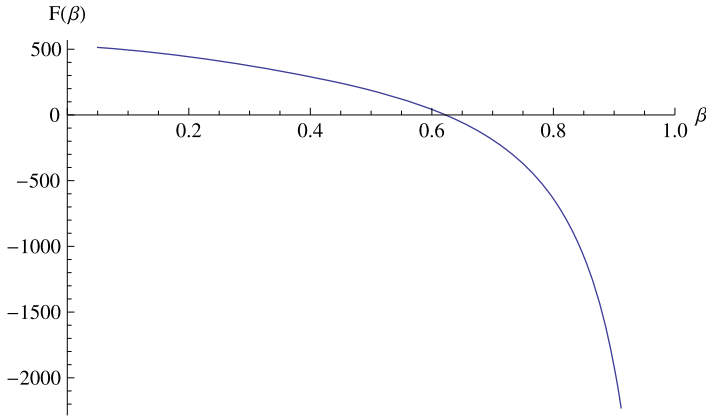$$+ C_2^2 P_{02} + C_2^3 P_{03} + C_1^2 C_2 P_{21} + C_1 C_2^2 P_{12} \big\}, \tag{B.2}$$

Fig. 6. The numerical estimation of the quantity $F(\beta)$ (see (B.3)) as a function of β.

where coefficients P_{ij} have a form

$$
\begin{aligned}
P_{00} ={}& -4(-1+x_1)^2\big(-4+x_1+b(-1+b+x_1)\big) \\
& -4\big(17+b^2(-2+x_1)-26x_1+8x_1^2+b(-1+x_1)(1+2x_1)\big)x_2 \\
& -4\big(-17+9x_1+b(4+b+x_1)\big)x_2^2+8(-2+b)x_2^3; \\
P_{10} ={}& 4(-1+x_1)^2-2\big(8+x_1(-11-3b+6x_1)\big)x_2+4(1+b-5x_1)x_2^2-4x_2^3, \\
P_{01} ={}& 2\big(10-8x_1-3x_1^2+2x_1^3+2(-2+x_1)(4+3x_1)x_2+2(3+x_1)x_2^2 \\
& +b\big[x_1^2-2(-2+x_2)(-1+x_2)+2x_1(1+x_2)\big]\big), \\
P_{11} ={}& 2\big(-4+x_1+x_1^2+(5+x_1)x_2-2b(-1+x_1+x_2)\big), \\
P_{20} ={}& 2x_2(-2-b+3x_1+3x_2), \\
P_{02} ={}& -2\big(2(-1+x_2)+x_1\big(b+2(-1+x_1+x_2)\big)\big), \\
P_{21} ={}& 2(2-x_1-x_2), \qquad P_{12}=2x_1, \qquad P_{03}=2x_1,
\end{aligned}
$$

here we use the notation that $b=\beta^2$. The contribution to imaginary part $F(\beta)$ then reads as

$$
\begin{aligned}
F(\beta) = \int\limits_0^1 dx_1 \int\limits_{1-x_1}^1 dx_2 \frac{32}{x_1 x_2 x_3} \{ & P_{00}J_{00}+P_{10}J_{10}+P_{01}J_{01} \\
& + P_{11}J_{11}+P_{20}J_{20}+P_{02}J_{02}+P_{03}J_{03}+P_{21}J_{21}+P_{12}J_{12} \}.
\end{aligned} \tag{B.3}
$$

Numerically the quantity $F(\beta)$ as a function of β is presented in Fig. 6.

References

[1] BESIII Collaboration, M. Ablikim, et al., Phys. Lett. B 710 (2012) 594, arXiv:1202.2048.
[2] BESIII Collaboration, M. Ablikim, et al., arXiv:1403.6011, 2014.
[3] CLEO Collaboration, G. Adams, et al., Phys. Rev. D 73 (2006) 012002, arXiv:hep-ex/0509011.
[4] R.B. Ferroli, S. Pacetti, A. Zallo, Eur. Phys. J. A 48 (2012) 33, arXiv:1008.0542.
[5] Particle Data Group, J. Beringer, et al., Phys. Rev. D 86 (2012) 010001.
[6] E. Byckling, K. Kajantie, Particle Kinematics, John Wiley and Sons, London, New York, Sydney, Toronto, 1973.

 [7] Y.-J. Zhang, G. Li, Q. Zhao, Phys. Rev. Lett. 102 (2009) 172001, arXiv:0902.1300.
 [8] X. Liu, B. Zhang, X.-Q. Li, Phys. Lett. B 675 (2009) 441, arXiv:0902.0480.
 [9] L. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. 127 (1985) 1.
[10] F. Navarra, M. Nielsen, Phys. Lett. B 443 (1998) 285, arXiv:hep-ph/9803467.
[11] I. Pomeranchuk, Dokl. Akad. Nauk SSSR 60 (1948) 213.
[12] A. Ore, J. Powell, Phys. Rev. 75 (1949) 1696.
[13] T. Appelquist, H. Politzer, Phys. Rev. Lett. 34 (1975) 43.
[14] W. Kwong, P.B. Mackenzie, R. Rosenfeld, J.L. Rosner, Phys. Rev. D 37 (1988) 3210.