Novel mechanical behavior of periodic structure with the pattern transformation

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Abstract In periodic cellular structures, novel pattern transformations are triggered by a reversible elastic instability under the axial compression. Based on the deformation-triggered new pattern, periodic cellular structures can achieve special mechanical properties. In this paper, the designed architecture materials which include elastomer matrixes containing empty holes or filled holes with hydrogel material are modeled and simulated to investigate the mechanical property of the periodic materials. By analyzing the relationship between nominal stress and nominal strain of periodic material, and the corresponding deformed patterns, the influence of geometry and shapes of the holes on the mechanical property of architecture material is studied in more details. We hope this study can provide future perspectives for the deformation-triggered periodic structures. © 2013 The Chinese Society of Theoretical and Applied Mechanics. [doi:10.1063/2.1305407]

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Periodic cellular structure has been widely investigated for special mechanical properties through generating novel pattern transformation. When this type of structure is compressively loaded beyond the critical value, the periodic structure can exhibit the pattern switching to a new configuration owing to its local elastic instabilities. Actually, initial porosity of cellular structures, the arrangement of the holes, the load orientation, and many other factors play important roles on the pattern switching. In recent years, numerous researchers have studied the unique properties of the porous elastic cellular structures with pattern transformation through theoretical, numerical, and experimental approaches. From previous studies, it can be found that the pattern transformation also occurs in periodic structures at wide range scale, which can be used in the design and fabrication of phononic crystals, shape-memory hysteresis for photonic switching, and some tunable hydrophobic surfaces. Especially, biomedical and nanotechnology applications are exciting examples of potential end uses of cellular materials. Recent interest in fauna and flora systems has revealed soft cellular structures of astonishing complexity. For example, these porous material structure underpin an vivid structural color ranging from iridescent blue butterflies to fruit skin. Based on the deformation-triggered pattern, the periodic cellular structure can display a special mechanical property, such as negative Poisson’s ratio (i.e., auxetic material). The material with negative Poisson’s ratio is not common in nature, and usually occurs in man-made architecture material, such as honeycomb structures, re-entrant structures, and bucklilall. Recently, the ability to synthetically produce periodic structures through micro-fabrication processes, interference lithography, and thermodynamically driven self-assembly has created new opportunities to mimic natural structures. Among the man-made auxetic material, various deformation-triggered periodic structures have been designed. It is imperative to study the novel mechanical properties of these periodic structures, especially the influence of local instabilities on global mechanical properties.

This paper focuses on the mechanism of deformation of periodic elastomeric cellular structures and the composite gel material structures. First, we investigate the effects of the hole shapes in elastomer matrix on the pattern switching for the periodic elastomeric cellular structure. The models include periodic elastomer structures with circular holes, quadrate holes, and hexagon holes. The porosity of the elastomeric cellular structures has the same value of 0.59. Then we develop the composite gel material, in which the arrays of hydrogel particles or hydrogel rods are filled in the periodic elastomeric cellular structures. Mullin et al. experimentally studied the mechanical properties of the similar novel composite material by combining the silicone rubber samples with jelly filling each of the holes. They pointed out that a much softer material needs to be placed in the holes to achieve pattern switch for the composite material. Based on Liu et al.’s work on the incremental modulus of gel, we plug the hydrogel materials into the porous elastomers. It is found that the designed periodic composite gel materials still undergo the pattern transformation, while gel inclusions affect the critical strains and stresses of the switching. The numerical simulations of periodic elastomeric cellular structures and composite gel material under compressed load are both carried out utilizing the nonlinear finite element code ABAQUS.

In the study, the specific configurations of various periodic cellular structures are modeled as illustrated in Fig. 1. The porosity in these four types of peri-

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is also 100mm\(\times\)10mm in vertical and horizontal directions. The size of the model in Fig. 2(b) is 40mm\(\times\)40mm, comprising square arrays of square holes with the diameter of 8.67mm and the center-to-center spacing of 10mm in vertical and horizontal directions. The size of model in Fig. 2(b) is also 100mm\(\times\)100mm, comprising square holes with the length of 7.7mm and the center-to-center spacing of 10mm. The models of Figs. 1(c) and 1(d) have the same size of 100mm\(\times\)100mm, comprising hexagon holes with the length of 4.77mm and the center-to-center spacing of 10mm. While, the load orientations in these two identical models are different due to the two symmetry axes in hexagon.

Besides, the specific combinations of two materials are modeled as illustrated in Fig. 2. The size of Fig. 2(a) is 40mm\(\times\)40mm, comprising square arrays of circular gel inclusions with the diameter of 8.67mm and the center-to-center spacing of 10mm in vertical and horizontal directions. The size of model in Fig. 2(b) is 40mm\(\times\)40mm, comprising 10mm\(\times\)10mm square arrays of square gel inclusions with the length of 7.7mm and the center-to-center spacing of 10mm. The volume fractions of gel in the two types of composite materials also have the same value of 0.59.

It is realized that the results of finite-sized models are necessarily influenced by boundary conditions at both loaded and traction free edges. To eliminate the boundary condition effects, the periodic material and composite material can be represented as a periodic array of representative volume elements (RVEs). Consequently, periodic boundary conditions are imposed on all cell boundaries. In the study, two different materials are adopted, i.e., polymethyl methacrylate (PMMA) for porous matrix material and hydrogel for inclusion. For the periodic cellular structures, with various shapes of holes. The porosities in four models are the same value of 0.59. The size of the model in Fig. 1(a) is 100mm\(\times\)100mm, comprising circular holes with the diameter of 8.67mm and the center-to-center spacing of 10mm in vertical and horizontal directions. The size of the model in Fig. 1(b) is also 100mm\(\times\)100mm, comprising square holes with the length of 7.7mm and the center-to-center spacing of 10mm. The models of Figs. 1(c) and 1(d) have the same size of 100mm\(\times\)100mm, comprising hexagon holes with the length of 4.77mm and the center-to-center spacing of 10mm. While, the load orientations in these two identical models are different due to the two symmetry axes in hexagon.

The strain energy \(W\) for for neo-Hookean material is the function of the deformation gradient \(F\). The energy form in plane strain is

\[
W = \frac{\mu}{2}[(I - 3) - 2 \ln J] + \frac{\kappa}{2}(J - 1)^2, \tag{1}
\]

where \(I = F_{ij} F_{ij}\) and \(J = \det(F)\) are invariants of the deformation gradient \(F\), \(\mu\) and \(\kappa\) are, respectively the initial shear and bulk moduli of the solid at zero strain. The PMMA is modeled as nearly incompressible, characterized by \(\kappa/\mu = 50\), \(\nu = 0.49\). From the early studies, the initial Young’s modulus is given as \(E = 3.25\) MPa, so that \(\mu = E/(2(1 + \nu)) = 1.1\) MPa.

The inclusion material is gel material, with a strain energy \(W(F)\) given by

\[
W = \frac{1}{2} k T (I - 3 - 2 \ln J) + \frac{k T}{\nu} \left((J - 1) \ln \frac{J}{J - 1} + \frac{\chi}{J}\right), \tag{2}
\]

where \(k T\) is equal to \(4 \times 10^{-21}\) J at room temperature. A representative value of the volume per molecule is \(\nu = 10^{-28}\) m\(^3\), and \(N\) is the number of polymer chains divided by the volume of the dry polymer. The two dimensionless material parameter \(N T\) and \(\chi\) are chosen appropriately in the numerical examples below. We will take the values \(N T = 10^{-2}\) and \(\chi = 0.1\). When periodic elastomeric cellular solid is compressed, the porous material undergoes an instable transformation at a critical point. Similar instabilities also trigger the transformation to the new configuration in the novel composite gel material. The pattern transformation in the periodic cellular structures and the composite gel materials are captured. Figure 3 shows the deformation patterns after instable transformation for different hole shapes at the nominal strain of 0.1. The color contours display the von-Mises stress under pattern transformation. From Figs. 3(a) and 3(b), it can be observed that the circular holes of the periodic cellular structure bifurcate into ellipses in vertical and horizontal directions alternatively. While the linear structure of the porous material with square holes changes into sinusoidal shape in the vertical and horizontal directions. However, from Figs. 3(c) and 3(d),
it can be found that due to the different loading axes, holes in hexagon type I transform into flat hexagons of horizontal direction and rectangles of vertical direction, contrary to the new configuration of hexagon type II. The maximum value of the von-Mises stresses appear at the slit tip of the ellipse, the peak of sinusoidal shape and edges of flat hexagons, respectively.

The composite gel materials can be regarded as periodic cellular structures with gel inclusions. For the composite gel materials, we find that at first, the composite materials do not easily transform to new configurations and the shapes of each gel inclusion keep the same. However, when the compressive load is beyond the critical value, composite materials bifurcate and gel inclusions collapse into different shapes. The patterns at the nominal strain of 0.2 are shown in Fig. 4. We should note that the inclusions filled into the matrix is much softer than the matrix material, otherwise the cellular structure will not lead to any pattern switching. As expected, it can be found that the pattern transformation in composite material switches later than the periodic cellular material, largely owing to that gel inclusions suppress the motivation of the matrix domains.

The curves of the nominal stress versus nominal strain for four types of porous materials and composite gel material are shown in Figs. 5 and 6, respectively. Figure 6 also provides a direct comparison between the matrix material without gel inside and the one with gel.

From Fig. 5, we can see that the behavior of the periodic cellular structure is characterized by an initial linear elastic behavior with a sudden change to a plateau stress. The pattern transformations occur at relatively different nominal strains, although all the periodic cellular structures have the same value of porosity. Through comparing the two cellular structures with different arrangements of hexagon holes, the model under the compressive load along the axis of hexagonal edge in Fig. 3(c) leads to a earlier pattern transformation switching than the that along diagonal loading as shown in Fig. 3(d). Figure 5 shows that the pattern transformation depends on the shapes of the holes in periodic cellular structures.

From Fig. 6, we can find that the behavior of the
composite material is characterized by an initial almost linear elastic behavior with a sudden change to a different elastic relationship, which is totally different from the former porous material. The pattern transformation occurs much later in the composite material with the square gel inside than that with the circular gel inside, though both models have the same value of gel fraction.

As mentioned earlier, the porous materials with arrays of holes in different shapes have negative Poisson’s ratio when the pattern switching takes place.\(^1\)\(^-\)\(^3\)\(^-\)\(^6\) The simulated Poisson’s ratios are plotted as a function of nominal strain as shown in Fig. 7 for various shapes of holes. At initial stage of deformation, the Poisson’s ratios of the porous material are much close due to the same porosity. When pattern transformation induced by the instability occurs, the Poisson’s ratios start to decrease. It should be noted that the decreasing of Poisson’s ratio is more complex and the change of the values depends on many factors.

In conclusion, we study the periodic cellular structures with various shapes of holes and develop the novel composite material by filling the gel inclusions into the periodic elastomeric cellular structures. Numerical simulations of the porous material and the designed composite gel material are carried out to investigate the mechanical properties. The behavior of the composite material is characterized by initial almost linear elastic behavior with a sudden change to a different elastic relationship. It can be found that the internal structure with holes in different shapes greatly affects mechanical characteristics of porous material, especially for the pattern switching, so do the various shapes of gel inclusions on the composite gel material. Although the practical application-based work is still a long way in the future for our designed composite gel material, we hope this study can provide future perspectives for periodic structures, as well as how their properties can be optimized and predicted.

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