DYNAMIC STABILITY OF HINGELESS AND BEARINGLESS ROTORS IN FORWARD FLIGHT†

BRAHMANANDA PANDA and INDERJIT CHOPRA
Center for Rotorcraft Education and Research, Department of Aerospace Engineering, University of Maryland, College Park, MD 20742, U.S.A.

Abstract—The aeroelastic stability of flap bending, lead-lag bending and torsion of hingeless and bearingless rotor blades in forward flight is examined, using a finite element formulation based on Hamilton’s principle. The hingeless blade is idealized as an elastic beam, and is discretized into beam elements. Each beam element consists of fifteen nodal degrees of freedom. Between the elements there is a continuity of displacement and slope for lag and flap bending, and a continuity of displacement for twist and axial deflection. For a bearingless rotor blade the flexbeam, the torque tube and the main blade are assumed as elastic beams, and these are discretized into beam elements. Quasisteady strip theory is used to evaluate the aerodynamic forces, and the unsteady aerodynamic effects are introduced approximately through a dynamic wake induced inflow modelling. The natural vibration characteristics of a rotating blade are calculated from the finite element equations. The blade finite element response equations are transformed to the model space in the form of a few normal mode equations. These nonlinear response equations containing periodic terms are solved iteratively using Floquet theory. The periodic perturbation equations linearized about the nonlinear response position are solved for stability using Floquet transition matrix theory as well as constant coefficient approximation in the fixed reference frame. Results are presented for both stiff-in-plane and soft-in-plane blade configurations. Stability results are also obtained for a bearingless blade configuration consisting of single flexbeam with a wrap-around type torque tube and the pitch links located, one on the leading edge and the other on the trailing edge of the torque tube. The effects of several parameters on the blade stability are examined, including, the blade modelling, lag stiffness, dynamic inflow, constant coefficient approximation and forward speed.

NOMENCLATURE

\[ a \] lift curve slope
\[ [A], [A] \] matrix in first order equations in rotating and fixed system respectively
\[ C \] blade chord
\[ C_a \] blade section drag coefficient
\[ C_l \] blade section lift coefficient
\[ C_m \] blade section moment coefficient
\[ [C] \] damping matrix in response equations
\[ C_r \] thrust coefficient, \( T/\pi p R^2 \)
\[ C_w \] weight coefficient, \( W/\pi p R^4 \)
\[ f \] equivalent drag are of helicopter
\[ \{F_{nl}\} \] nonlinear force vector in response equation
\[ h \] distance of hub from helicopter e.g.
\[ [k] \] stiffness matrix in response equations
\[ k, k \] coefficients in Drees model
\[ L_u, L_v, L_w \] aerodynamic forces in \( u, v \) and \( w \) directions, respectively
\[ m \] mass per unit length of blade
\[ m_o \] reference mass per unit length
\[ [M] \] matrix in response equations
\[ n \] number of elements
\[ N_o \] number of blades
\[ R \] rotor radius
\[ R_s \] structural coupling parameter
\[ T \] rotor thrust force
\[ u, v, w \] elastic displacements in the \( x, y, z \) directions, respectively
\[ U_r, U_t, U_o \] blade section radial, tangential and normal velocities
\[ W \] helicopter gross weight
\[ x, y, z \] blade orthogonal coordinates
\[ \alpha \] blade section angle of attack
\[ \omega_k \] real part of \( k \)th characteristic exponent
\[ \beta \] precone
\[ \gamma \] blade lock number, \( pa CR^{1/4} \)
\[ \rho \] air density
\[ \delta \] perturbation quantity
\[ \delta T, \delta V \] variation of kinetic and strain energies, respectively
\[ \delta W \] virtual work done due to aerodynamic loads

BRAHMANANDA PANDA and INDERJIT CHOPRA

INTRODUCTION

In recent years, the rotorcraft design trends are tending toward hingeless blade configurations because of their better maintenance characteristics and more control power. With a hingeless blade, the flap and lag hinges are eliminated and the blade is stiffer structurally, particularly, its flap mode stiffness. A bearingless rotor is a special example of hingeless rotor where even pitch bearing is eliminated. With the stiffer blades, there is a transfer of larger dynamic forces from the blade to the shaft and the blades are more susceptible to aeroelastic instabilities.

The objective of the present paper is to examine aeroelastic stability of flap bending, lead-lag bending and torsion of hingeless and bearingless rotors in forward flight using finite element formulation.

The aeroelastic stability of rotor blades in hover and forward flight has been investigated by several researchers (see recent reviews[1-3]). Because of the complexity of the phenomena involving nonlinear structural aerodynamic and inertial forces, different researchers studied the problem with varying levels of assumptions and modelling approximations. The simplest form of a rotor blade representation is to assume the blade to be rigid and restrained by bending springs at hinges located near the root end of the blade. Many researchers examined the aeroelastic stability of this simple model. For example, Peter[4] and Kaza and Kvaternik[5] investigated the aeroelastic stability of this model undergoing two degree flap and lag motions in forward flight. Recently, the authors[6] examined the aeroelastic stability of this simple model undergoing three degree flap, lag and torsion motions in forward flight. It was shown that the inclusion of the torsion degree of motion has an important influence on the blade stability. A better representation of a hingeless blade is to treat it as an elastic beam undergoing flap bending, lead-lag bending and torsional deflections. Many researchers examined the aeroelastic stability of this model in hover and a few authors investigated the stability of this model even in forward flight. For example, Friedmann and Kottapalli[7] and Johnson[8] have investigated the aeroelastic stability of an elastic blade in forward flight using classical modal approach.

For rotor blades with complex root geometries as in the case with bearingless blades, it becomes difficult to apply the classical modal method. The finite element method which has been extensively used in the structural analysis problems, has a great potential to solve complex blade dynamics problems. The method is very flexible to different constraint conditions, and also quite natural with nonuniform properties. Sivaneri and Chopra[9] made a finite element formulation based on Hamilton’s principle to investigate the aeroelastic stability of an elastic blade in hover. This analysis was developed for a blade consisting of a single-load-path structure and could analyze conventional articulated and hingeless blades. Later on Sivaneri and Chopra[10] extended their finite element formulation to multiple-load-structure to study the aeroelastic stability of a bearingless blade in hover. Chopra[11] made a satisfactory correlation of analytical stability results calculated using this finite element analysis with the experimental data for some selected bearingless configurations. Straub and Friedman[12] used a Galerkin-type finite element formulation of flap bending and lag bending blade to study the aeroelastic stability of a hingeless blade in forward flight.

In the present paper, the finite element formulation is extended to investigate the aeroelastic stability of hingeless and bearingless blades in forward flight. Again, the finite element theory is based on energy principles (Hamilton). The blade is assumed to be an elastic beam and is discretized into beam elements. Each element consists of fifteen nodal degrees of freedom.
Stability of hingeless and bearingless rotors in forward flight

Between elements, there is a continuity of flap bending displacement \( w \) and slope \( w' \), leadlag bending displacement \( v \) and slope \( v' \), axial displacement \( u \) and geometric twist \( \phi \). In an element, there are two internal nodes for axial displacement and one internal node for twist. The formulation is developed for a general blade with nonuniform properties, also including pretwist and precone and with chordwise offsets of the center of mass, aerodynamic center and tension center from the elastic axis.

The aerodynamic loads in forward flight are obtained using quasisteady strip theory. For steady inflow calculation a constant as well as linear variation of inflow (Drees) is used.

The vehicle trim solution is determined iteratively from the vehicle overall nonlinear equilibrium equations, three force equations (vertical, longitudinal and lateral) and two moment equations (pitching and rolling). This is called a propulsive trim solution and it determines the pilot-control settings as well as the vehicle orientation for a prescribed flight condition. There are many other forms of trim solutions available in literature\[8\], and the approximations involved with these solutions are discussed in Ref. 6.

The blade time-dependent response solution is calculated from the blade nonlinear periodic equations. First, the natural vibration characteristics of the rotating blade are calculated about its undeflected position from the finite element equations. Then, the blade finite element response equations are transformed to the modal space in the form of a few (about six) normal mode equations. These nonlinear equations containing periodic terms are solved iteratively using Floquet theory\[13\]. A somewhat similar type of quasilinearization procedure is used by Friedmann and Kottapalli[7]. The present solution retains all harmonics of modes considered, and it is calculated in the rotating reference frame. Another popular method, harmonic balancing\[14\] is widely used by researchers because of its simplicity but the solution procedure becomes heavy if higher harmonics are to be retained.

For stability solution, the perturbation equations of motion are linearized about the blade equilibrium position and these equations contain periodic terms. First, the natural vibration characteristics of the rotating blade are calculated about its mean deflected position from the finite element equation. Then the blade perturbation equations are transformed to a few (about six) normal mode equations. For the stability solution the unsteady effects can be important and these are introduced in an approximate manner here through a dynamic inflow modelling. The effect of dynamic inflow model on blade stability in forward flight has been examined earlier for a simple blade model. For flap-lag model (two degrees)[14] as well as flap-lag-torsion model (three degrees)\[6\], the influence of dynamics inflow appears quite appreciable.

The linearized periodic normal mode equations are solved using three different approaches. The first approach is to analyze the stability of the blade in the rotating reference frame using Floquet transition matrix theory. This approach is applicable if the inflow is assumed to be steady. The second and the third approach analyze the stability of rotor in the hub fixed reference frame. The blade equations in the rotating frame are transformed to the fixed reference frame numerically using Fourier coordinate transformation. In the second approach, the rotor equations in fixed frame are solved using Floquet transition matrix theory. Since the rotor equations in fixed frame contains few selected periodic terms, in the third approach a constant coefficient approximation is used, by cancelling out the periodic terms altogether and then solving the resulting constant coefficient equations. The last two approaches are used when dynamic inflow model is included. In the present formulation, the dynamic inflow model of Ref. 15 is used. In all the three approaches an eigenanalysis is made and the nature of eigenvalues explain the stability of the blades.

A similar procedure is used to obtain the response and stability solution of a bearingless rotor. The bearingless configuration adopted for analysis consists of a flexbeam with a wrap-around type torque tube (Fig. 1). The torsionally soft flexbeam generally extends from the hub to about 15%-40% of the blade radius, where it is connected to the main blade. The pitch control to the blade is applied through a torsionally stiff torque tube by rotating it with the pitch link which elastically twists the flexbeam. In the present paper the pitch links are situated both at the leading as well as at the trailing edge of the torque tube. The flexbeam, the torque tube and the main blade are assumed as elastic beams and these are discretized into beam elements. The displacement and compatibility conditions at the clevis, between the inboard beams and
the outboard beams are satisfied. From the vehicle trim solution, the rotor thrust and the pitch controls are calculated, and with these then the pitch link schedule along the azimuth is calculated iteratively, using response solutions. For response calculations, the pitch link is modelled by an equivalent vertical force and the boundary conditions are let free at the torque tube end. Then using normal mode equations the nonlinear response is calculated. For stability solution, the perturbed equations of motion are linearized about the response solution, and for this the boundary constraints due to pitch link are introduced. The response and stability solution procedure for a bearingless blade is quite identical to that for a hingeless blade.

The effects of several parameters on blade stability are examined, including, blade modelling, lag stiffness, dynamic inflow, constant coefficient approximation and forward speed.

**FORMULATION**

The formulation details are given in Refs. 9 and 10. The rotor blade is treated as an elastic beam rotating at constant angular velocity $\Omega$. Figure 2 shows the undeformed and deformed positions of the blade. The rectangular coordinate system $x$, $y$, $z$ is attached to the undeformed blade which is at a precone angle of $\beta_p$ and the $x$-axis coincides with the elastic axis. A point $P$ on the undeformed elastic axis undergoes displacements $u$, $v$, $w$ in the $x$, $y$, $z$ directions, respectively and occupies the position $P'$ on the deformed axis: $u$ is the axial deflection, $v$ the

![Fig. 2. Blade coordinate systems and deflections.](image-url)
lead-lag deflection and $w$ the flap deflection. Then the blade undergoes a rotation $\theta$, about the deformed elastic axis:

$$\theta_i = \theta + \phi$$

and

$$\dot{\phi} = \phi - \int_0^r v'' w' \ dx,$$  \hspace{1cm} (1)

where $\theta$ is the pretwist, $\phi$ is the geometric twist, and $\phi$ is the elastic twist due to torsion.

The formulation is based on Hamilton’s principle:

$$\int_{t_i}^{t_f} \left( \delta U - \delta T - \delta W \right) dt = 0,$$ \hspace{1cm} (2)

where $\delta U$, $\delta T$ and $\delta W$ are, respectively, the variation of strain energy, the variation of kinetic energy and the virtual work done by external forces. These energy expressions are made independent of the time derivatives of virtual displacements, $\delta u$, $\delta v$ and $\delta w$, and $\delta \phi$, and hence equation (2) can be written as

$$\delta U - \delta T - \delta W = 0.$$ \hspace{1cm} (3)

Hodges and Dowell[17] have given expression for $\delta U$ and $\delta T$ for moderate slopes. The expression for $\delta W$ is

$$\delta W = \int_0^r \left( L_u \delta U + L_v \delta V + L_w \delta W + M_\phi \delta \psi \right) dx,$$ \hspace{1cm} (4)

where $L_u$, $L_v$, $L_w$ and $M_\phi$ are aerodynamic force distribution along the length of the blade in the axial, lead-lag and torsion directions, respectively, and virtual rotation $\delta \psi$ is

$$\delta \psi = \delta \phi + w' \delta v'.$$ \hspace{1cm} (5)

The aerodynamic forces in forward flight are distributed along the length of the blade and are obtained based on a quasisteady strip theory approximation. These forces are periodic and depend on the azimuth position of the blade. Forces of noncirculatory origin are also included. In the present analysis the strip theory is based on sections in the deformed frame. As a result, forces obtained in the deformed frame are transformed to the undeformed frame. The velocity vector for a blade section is

$$\mathbf{V} = U_x \hat{i}_x + U_y \hat{i}_y + U_z \hat{i}_z,$$ \hspace{1cm} (6)

where

\begin{align*}
U_x &= -\dot{u} + v' \eta_r \cos \theta_1 + \dot{w} \eta_r \sin \theta_1 + \Omega (v + \eta_r \cos \theta_1) - \lambda \Omega R \beta_\rho + \Omega R \mu \cos \psi, \\
U_y &= -\dot{v} + \phi \eta_r \sin \theta_1 - \Omega (x + u - v' \eta_r \cos \theta_1 - w' \eta_r \sin \theta_1), \\
U_z &= -\dot{w} + \dot{\phi} \eta_r \cos \theta_1 - \Omega \beta_\rho (v + \eta_r \cos \theta_1) - \lambda \Omega R - \mu \Omega R \beta_\rho \cos \psi,
\end{align*}

where $\eta_r$ is the $\eta$ coordinate of the three-quarter chord point.

The radial velocity $U_x$, tangential velocity $U_y$, and normal velocity $U_z$ can be obtained by coordinate transformation from deformed frame to undeformed frame.

$$\begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} = -[T] \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix},$$ \hspace{1cm} (7)
The transformation matrix $[T]$ is defined in Hodges et al.\cite{Hodges18}. For steady induced inflow, a linear variation in inflow is used and is related to the rotor thrust as

$$\lambda = \mu \tan \alpha + \frac{C_T}{2(\mu^2 + \lambda^2)^{1/2}} (1 + k_1 \cdot x \cos \psi + k_2 \cdot x \sin \psi).$$  

(8)

where $k_1$ and $k_2$ are obtained from Drees model as

$$k_1 = \frac{4}{3} \left[ (1 - 1.8 \mu^2) \sqrt{1 - (\lambda/\mu)^2} - \lambda/\mu \right],$$

$$k_2 = -2 \mu.$$

For hover the $k_1$ and $k_2$ become zero.

**Finite element discretization**

The blade is discretized into beam elements and each element consists of two end nodes and three internal nodes with a total of fifteen degrees of freedom (Fig. 3). Each of the end nodes has six degrees of freedom $u$, $v$, $v'$, $w$, $w'$ and $\phi$. There are two internal nodes for $u$ and one internal node for $\phi$. Between elements there is a continuity of displacement and slope for lag and flap bending, and a continuity of displacement for twist and axial deflection. The distributions of deflections over an element are represented in terms of nodal displacements and shape functions. Within the element, there is a cubic polynomial distribution for $u$, $v$, and $w$ and a quadratic polynomial for $\phi$. The element forces are obtained applying Hamilton’s principle.

$$\int_{t_1}^{t_2} \sum_{i=1}^{n} (\delta U_i - \delta T_i - \delta W_i) \, dt = 0,$$

(9)

where $\delta U_i$, $\delta T_i$, $\delta W_i$ are, respectively, the strain energy, kinetic energy and virtual work contribution of the $i$th element. The assembly of $n$ elements yields the nonlinear periodic equations of motion in terms of nodal displacements.

For bearingless rotors the flexbeam(s), the torque tube and the main outboard blade are all discretized into a number of beam elements. During the assembly of element matrices the displacement compatibility conditions at the junction of main blade, torque tube and flexbeam are satisfied. These are

$$u_i = u_f = u_b,$$

$$v_i = v_f = v_b,$$

$$v'_i = v'_f = v'_b,$$

$$w_i = \eta_i \phi_i = w_f = \eta_f \phi_f = w_b,$$

$$w'_i = w'_f = w'_b,$$

$$\phi_i = \phi_f = \phi_b,$$

(10)

where subscripts $i$, $f$ and $b$ respectively represent torque tube, flexbeam and blade. The finite element structural formulation details can be seen in Ref. 10. The assembly of $n$ elements yields the equations of motion in terms of nodal displacements as

$$[M(q)][\dot{\ddot{q}}] + [C(q, \dot{\psi})][\dot{q}] + [K(q, \dot{\psi})][q] = \{Q_{na}(q, \dot{\psi})\}.$$  

(11)

These equations expressed in rotating reference frame contain nonlinear and periodic terms. After the assembly of element matrices the geometric boundary constraints are applied on global equations. For hingeless rotors the blade is cantilevered at the root end ($x = 0$), which means $u$, $v$, $v'$, $w$, $w'$ and $\phi$ are zero. For bearingless configurations, the boundary constraints are applied to the flexbeam as well as the torque tube. The flexbeam is cantilevered at the root.
which again means $u$, $v$, $v'$, $w$, $w'$ and $\phi$ are zero; whereas for the torque tube the constraint conditions very much depend upon the location of the pitch link. For the present calculations the pitch link is assumed rigid and the blade pitch is accomplished through the vertical displacement of pitch link. The torque tube is generally very stiff torsionally, so any change of blade pitch through pitch link displacement results in nearly a rigid body pitch displacement for the torque tube and an elastic twist distribution for the flexbeam. Since the pitch link is assumed to be rigid, there is no axial elastic deflection of pitch link. It is however possible to include the spring stiffness of pitch link easily as shown in Ref. 10. The pitch link joint with the torque tube is assumed to be a pin, and also the axial and lag displacements are freely permitted. For example, with bearingless configurations with one pitch link located at the leading edge and another at the trailing edge of the torque tube, the flap displacement is constrained zero. The resulting global equations of motion are nonlinear and contain periodic coefficients. The solution of these large number of finite element equations for response and stability of blade is a very involved task. In fact, it becomes more complicated with the inclusion of the dynamic inflow modelling. Therefore there is a need to reduce the number of equations which can be manageable for computation and also can make a good estimate of the solution. This key simplification is achieved by converting these equations in the modal space in the form of normal mode equations. In the present paper the structural finite element equations are solved to obtain natural vibration characteristics about some mean deflected position. For this purpose the Coriolis damping terms are also neglected. The resulting equations become

$$\{M_s\}\ddot{\{q\}} + \{K_s\}\{q\} = 0,$$  \hspace{1cm} (12)

where the inertia matrix $\{M_s\}$ and the stiffness matrix $\{K_s\}$ are symmetric. These equations are solved as an algebraic eigenvalue problem using Jacobi Method. For the blade response solution, the normal mode equations are obtained using rotating vibration characteristics about its undeflected position (zero pitch). For stability solution, the normal mode equations are obtained using rotating vibration characteristics about the blade's mean deflected position.

SOLUTION PROCEDURE

An aeroelastic stability analysis of a blade in forward flight consists of three major phases; vehicle trim, blade steady response and blade stability. These three phases of analysis are inherently coupled. A complete coupled solution is very involved, and it is therefore a common practice to uncouple these phases and study them separately. It is possible to achieve a certain amount of coupling between these phases through an iterative process. The first step of the analysis is to determine the vehicle trim solution.
Vehicle trim solution

The vehicle trim solution in forward flight involves the calculations of pilot-control settings as well as the vehicle orientation for a prescribed flight condition. The propulsive trim parameters are determined from the satisfaction of three forces equilibrium equations (vertical, longitudinal and lateral) and two moments equilibrium equations (pitch, roll). Only the flap dynamics is used for obtaining the solution. The solution is calculated iteratively from nonlinear vehicle equilibrium equations (large angles). The offsets of the fuselage center of gravity from the hub axes are included. For steady inflow a linear variation of inflow (Drees) is used in the analysis. The details of this propulsive trim procedure are given in Ref. 6.

Blade response solution

The blade response solution involves the determination of time dependent blade position at different azimuth locations. To obtain the steady response of the blade the global equations in terms of nodal displacements are transformed to normal mode equations. To reduce computation time the elemental structural and aerodynamic matrices are directly transformed to normal mode equations. For this transformation the rotating, elastic models for zero blade pitch are used. The final equations of motion in modal space can be written as

\[
[M_r(\psi)]\{\ddot{x}\} + [C_r(\psi)]\{\dot{x}\} + [K_r(\psi)]\{x\} = \{F_{nl}(\psi, \{x\}, \{\dot{x}\})\},
\]

where the inertia matrix \([M(\psi)]\), damping matrix \([C(\psi)]\) and stiffness matrix \([K(\psi)]\) contain periodic terms. The actual nodal deflections \(\{q\}\) can be obtained from normal mode coordinates \(\{x\}\) using the transformation

\[
\{q\} = [\Phi]\{x\},
\]

where \([\Phi]\) consists of free vibration modes. The coupled normal mode equations (13) obtained in rotating reference frame can be rewritten

\[
\{\dot{Y}\} - [A(\psi)]\{Y\} = [G(\psi, \{Y\})],
\]

where \(\{Y\}\) is the state variable vector involving \(2m\) states, and the \(m\) is the number of modes used in the analysis. The above coupled nonlinear periodic equations are solved using an iterative procedure based upon Floquet theory[13]. The detailed numerical procedure to calculate the initial condition for numerical integration of the nonlinear periodic equations of motion is described in Ref. 6. Using these initial conditions the blade response at any azimuth position is calculated by numerical integration of eqns (15) by Runge-Kutta scheme. The response solution obtained in rotating reference frame contains all harmonics for flap, lag and torsion deflections. The blade response solution at various azimuth steps are stored for stability analysis.

For bearingless rotor blade, a torque is applied at the torque tube end through the pitch link to obtain the desired pitch angle. The pitch setting depends on the rotor thrust requirement and can be calculated from the vehicle trim solution. For the desired pitch, one would like first to make an estimate of pitch link displacement. An efficient way is to replace the pitch link with an equivalent force and calculate the steady blade response of the blade with free boundary conditions at the torque tube end. The pitch link force however depends on the pitch link displacement as well as the flight conditions; and its initial estimate is made using the first few vibration modes. The level of the pitch link force is adjusted iteratively until a desired pitch angle is obtained. This approach results in a converged solution with a few natural modes. The other nonlinear response calculations for bearingless blades are identical to those for hingeless blades.

The calculated response solution contains all harmonics for flap, lag and torsion deflections and is subsequently used for aeroelastic stability analysis.

Aeroelastic stability solution

For stability analysis the blade perturbation equations of motion are linearized about the equilibrium position. These equations are transformed to the modal space using free vibration
characteristics of the blade about its mean deflected position. Therefore as first step the free vibration characteristic of the blade are calculated about its nonlinear deflected position. The perturbation equations in the form of normal mode equations are

\[ \{\delta \dot{Y}\} = [A(\{Y\}, \{\dot{Y}\}, \psi)]\{\delta Y\}. \tag{16} \]

where \(\{Y\}, \{\dot{Y}\}\) are the blade equilibrium position in the rotating reference frame and \(\{\delta Y\}, \{\delta \dot{Y}\}\) are the perturbation states. These linearized equations are solved for stability using Floquet transition matrix theory. Here the eigenvalues of transition matrix of \(A\) can be written in characteristic exponents form.

\[ \lambda_i = \alpha_i + i\omega_i. \tag{17} \]

The real and imaginary parts of \(\alpha_i\) represent the blade damping and frequency, respectively. The criterion for the mode to be stable is when \(\alpha_i < 0\). The above approach is applicable if the inflow is assumed to be steady.

For the perturbation solution unsteady aerodynamics effect can be important and these are introduced in an approximate manner through a dynamic inflow modelling. With the inclusion of dynamic inflow it is convenient to analyze blade stability in the fixed reference frame. For dynamic inflow modelling, the inflow is perturbed about the steady inflow \(\lambda\),

\[ \lambda = \lambda_0 + \delta \lambda. \tag{18} \]

where \(\delta \lambda\) is the perturbed inflow component. A linear variation of perturbed inflow is used

\[ \delta \lambda = \delta \lambda_0 + \delta \lambda_1 x \cos \psi + \delta \lambda_1 y \sin \psi. \tag{19} \]

The dynamic inflow components \(\delta \lambda_0, \delta \lambda_1, \delta \lambda_2\) are related to rotor perturbation aerodynamic forces and moments

\[ \{m\} \{\delta \lambda\} + \{l^{-1}\} \{\delta \lambda\} = \{\delta F\}. \tag{20} \]

where

\[ \{\delta F\} = \sum_{i=1}^{N} \left\{ \begin{array}{c} \delta C_T \\ \delta C_m \\ \delta C_m \end{array} \right\}. \]

and

\[ \{\delta \lambda\} = \left\{ \begin{array}{c} \delta \lambda_0 \\ \delta \lambda_1 \end{array} \right\}. \]

The \(\delta C_T, \delta C_m, \delta C_m\) are the perturbed thrust, roll moment and pitch moment and these are obtained for the \(ith\) blade. The \(\{m\}\) and \(\{l^{-1}\}\) matrices used here have been taken from Ref. 15 and these are based on actuator disk theory. The transformations of the blade equations from the rotating reference frame to the fixed reference frame are carried out numerically using Fourier coordinate transformation. The formulation details for the inclusion of dynamic inflow in the fixed reference frame are presented in Ref. 6. The blade equations of motion along with dynamic inflow model can be represented as

\[ \{\delta \dot{Y}_r\} = [A_F(\psi, \{Y\}, \{\dot{Y}\})] \{\delta Y_r\}. \tag{21} \]

These equations are solved for stability using both Floquet transition matrix theory and constant coefficient approximation.

The above procedure is applied to both hingeless and bearingless rotors. For both classes
of rotors, the natural vibration characteristics of the blade are calculated about the mean deflected position. However with the bearingless configurations the boundary constraints for the perturbation solution are somewhat modified from those of response solution. The pitch link is assumed rigid and it constrains the displacement of the torque tube.

RESULTS AND DISCUSSION

Hingeless rotor

The aeroelastic stability is examined for a four-bladed hingeless rotor with Lock number \( \gamma = 5.0 \), solidity ratio, \( \sigma = 0.05 \), chord to radius ratio \( C/R = 0.039 \) and zero precone. The fuselage center of gravity lies on the shaft axis and is assumed to be at a distance 0.2\( R \) below the rotor center. The aerofoil characteristics used are

\[
\begin{align*}
C_t &= 5.7\alpha, \\
C_d &= 0.01, \\
C_n &= -0.02.
\end{align*}
\]

The helicopter drag coefficient in terms of flat plate area ratio \( f/\pi R^2 \) of 0.01 is used. The uniform blade properties selected for the stability analysis of a hingeless rotor blade are given in Table 1. The stiffnesses \( E I, m, G J, \) and the inertial parameters \( k_m, k_n, k_t \) are chosen such that the rotating frequencies correspond to given values. The first rotating natural frequencies in flap bending and torsion are 1.14/rev and 4.94/rev respectively. Two different lag bending frequencies are used; 0.58/rev for soft-inplane and 1.5/rev for stiff-inplane rotor. A thrust level \( C_a/\sigma = 0.1 \) is used for the stability calculation.

First, results were calculated for some selected configurations to make comparison with those of other authors for identical conditions. As an example the response and stability results in forward flight obtained in rotating reference frame for hingeless blades were quite comparable to those in Refs. 7 and 12.

For numerical results, a convergence study was conducted to determine time steps needed in one revolution for time integration (Runge–Kutta) used in Floquet theory. It was concluded that 120 time steps are quite adequate for well converged response as well as stability solutions. It was also observed that four finite elements and about six normal modes (two flap, two lag, two torsion) were sufficient to obtain a converged stability solution. For stability results the first lag bending mode damping is presented only. The flap and torsion modes are comparatively high damped modes and are not presented here.

The propulsive trim control settings \( \theta_0, \theta_1, \theta_2, \) and \( \alpha_{HP} \) in hub plane are presented in Fig. 4 for different forward speeds. The solution is calculated iteratively from nonlinear vehicle equilibrium equations (large angles). For steady induced inflow a linear distribution model (Drees) as well as a constant inflow model are used.

Figures 5(a)–(c) present time dependent equilibrium position of blade for one complete revolution for a stiff-inplane rotor. The results are obtained for three different advance ratios, 0, 0.2 and 0.4. The advance ratio \( \mu \) of zero represents hover condition and the advance ratio of 0.4 represents a high forward speed condition. The response solution in terms of flapwise bending, chordwise bending and torsion deflections is calculated iteratively from nonlinear blade equations (rotating frame) using the Floquet theory. The response solution contains first as well

<table>
<thead>
<tr>
<th>Table 1. Hingeless blade structural properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( EI/m_\omega \Omega R^2 ) = 0.0130</td>
</tr>
<tr>
<td>( EI/m_\omega \Omega R^2 ) = 0.1669 (stiff)</td>
</tr>
<tr>
<td>( = 0.0153 ) (soft)</td>
</tr>
<tr>
<td>( GJ/m_\omega \Omega R^2 ) = 0.00566</td>
</tr>
<tr>
<td>( K_{\omega_1}/R ) = 0.0</td>
</tr>
<tr>
<td>( K_{\omega_2}/R ) = 0.025</td>
</tr>
<tr>
<td>( K_{\omega_2}/K_{\omega_1} ) = 1.0</td>
</tr>
<tr>
<td>( m/m_\omega ) = 1.0</td>
</tr>
</tbody>
</table>
as higher harmonics. In the figures the blade tip deflections are presented. With increasing forward speeds, the vibratory amplitude as well as the mean level of response change. The peak-to-peak amplitude increases for all three bending modes (flap, lag and torsion) with increasing advance ratios. The azimuth positions at which peaks occur are different for three bending modes. However, the occurrence of peak for any one of the bending mode (say flap) is only slightly changed with advance ratio. Figure 6(a)–6(c) shows the blade response solution for a soft-inplane rotor for an advance ratio $\mu$ of 0.4. For comparison, the linear response solution (dotted) is also presented. There are some differences between linear and nonlinear solutions. Comparing the response results for stiff-inplane and soft-inplane rotors, one finds that there are changes in both sets of results. For example, for flap bending the peak-to-peak amplitude has gone up by 40% for soft-inplane rotor. Also the maximum blade tip deflection (up) takes place at the advancing side of the disk for soft-inplane rotor whereas it takes place on the retreating side for the stiff-inplane rotor. For lag mode bending, the peak-to-peak response amplitude for soft-inplane rotor is reduced to about one-seventh the value of that of stiff-inplane rotor. There is however less influence on torsion response amplitude due to change in blade inplane frequency.

Figures 7 and 8 show the lag mode stability results, respectively, for stiff-inplane and soft-inplane rotors. On the figures, the damping in terms of negative real part of the eigenvalue $\alpha_i$ in rotating frame is plotted. The $(-\alpha_i)$ of zero represents the zero damping condition, a boundary line for dynamic instability. A positive value of $( - \alpha_i)$ means that damping is positive and the blade lag mode is stable from dynamic instability. The results shown here are calculated without the inclusion of the dynamic inflow. The full line shows the results for an elastic blade model whereas the dotted line shows the results for a simple hinged model consisting three degrees of motion (rigid flap, rigid lag and feather). The simple model results are obtained for identical flap, lag and torsion frequencies (rotating) and also assuming full blade structural coupling, $R_s = 1$ (rigid hub) (Ref. 6). The simple model predicts stability trends reasonably well at a much smaller computational cost. To predict accurately the stability results one needs to adopt an elastic blade model. The stiffer the lag stiffness, the more the need to treat the blade as a hingeless elastic blade.

Figures 9 and 10 show the damping of low-frequency cyclic lag mode, respectively, for stiff-inplane and soft-inplane rotors. On the figure, the damping in terms negative of real part of the eigenvalue in the fixed reference frame is plotted. Again the negative value of $( - \alpha_i)$ will make the blade unstable. The low frequency lag mode is a regressing mode for the stiff-
Fig. 5. Steady blade response for stiff out-of-plane hinges under (a) \( \mu = 0 \), \( \mu = 0.2 \), and (b) \( \mu = 0.4 \), showing (a) fixed-end deflection, (b) free-deflection, and (c) moment deflection.
Fig. 6. Steady blade response for soft-inplane hingeless rotor ($C_{l}/	au = 0.1, \nu = 0.58$). (a) Flap-bending deflection, (b) lag-bending deflection and (c) torsion deflection.
Fig. 7. Effect of elastic blade modelling on lag mode stability for stiff-in-plane hingeless rotor ($C = 0.1$, $v_0 = 1.14$).

Fig. 8. Effect of elastic blade modelling on lag mode stability for stiff-in-plane hingeless rotor ($C = 0.1$, $v_0 = 0.58$, steady inflow).
Stability of hingeless and bearingless rotors in forward flight

Fig. 9. Damping of low-frequency cyclic lag mode for a stiff-in-plane hingeless rotor ($C_{i}/\sigma = 0.1$, $v_s = 1.14$).

Fig. 10. Damping of low-frequency cyclic lag mode for a soft-in-plane hingeless rotor ($C_{i}/\sigma = 0.1$, $v_s = 0.58$).
inplane rotor and it is a progressive mode for the soft-inplane rotor. Three sets of results are shown and these are respectively represent dynamic inflow Floquet results (full line), dynamic inflow constant coefficient approximation results (big dots) and steady inflow results (small dots). The constant coefficient approximation is again quite satisfactory for low advance ratios ($\mu < 0.3$). The dynamic inflow is important for low forward speeds ($\mu < 0.1$), in fact neglecting it overestimates the blade stability. For stiff-inplane rotors, the dynamic inflow appears important even at high forward speeds ($\mu > 0.3$). Another interesting point with stiff-inplane rotors is that higher forward speeds ($\mu > 0.35$), the lagmode stability sharply degrades, and somewhat similar observations were made by other authors.

**Bearingless rotors**

Numerical results for stability are also calculated for bearingless blade configurations in forward flight. The blade is respresented by a flexbeam with a wrap-around type torque tube

<table>
<thead>
<tr>
<th>Element</th>
<th>$l/R$</th>
<th>Flapwise $EI/mΩ^2R^4$</th>
<th>Chordwise $EI/mΩ^2R^4$</th>
<th>Torsion $GJ/mΩ^2R^2$</th>
<th>Mass $m_m$</th>
<th>Torsion Inertia $k_Ω^2 R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.367</td>
<td>.0055</td>
<td>.1501</td>
<td>.0029</td>
<td>1.0</td>
<td>.00091</td>
</tr>
<tr>
<td>2</td>
<td>0.436</td>
<td>.0055</td>
<td>.1501</td>
<td>.0029</td>
<td>1.0</td>
<td>.00091</td>
</tr>
<tr>
<td>3</td>
<td>0.113</td>
<td>.00158</td>
<td>.0052</td>
<td>.00021</td>
<td>.299</td>
<td>.000029</td>
</tr>
<tr>
<td>4</td>
<td>0.085</td>
<td>2.099</td>
<td>2.099</td>
<td>9.15</td>
<td>72.6</td>
<td>.0346</td>
</tr>
<tr>
<td>5</td>
<td>0.0564</td>
<td>4.257</td>
<td>4.257</td>
<td>1.815</td>
<td>7.65</td>
<td>.0020</td>
</tr>
<tr>
<td>6</td>
<td>0.0564</td>
<td>4.257</td>
<td>4.257</td>
<td>1.815</td>
<td>7.65</td>
<td>.0020</td>
</tr>
</tbody>
</table>

![Graph](image)

Fig. 11. Damping of low-frequency cyclic lag mode for a soft-inplane bearingless rotor ($C_1/σ = 0.05, ν_c = 0.74$).
(Fig. 1). The pitch links are pinned to the torque tube and are located, one at the leading edge and another at the trailing edge. For calculations of results, the blade is discretized into six elements: two elements for main blade, two elements for flexbeam and two elements for torque tube. Six modes (two flap bending, two lag bending and two torsion modes) are used for blade response as well as stability calculations.

The nondimensional structural properties of elements are given in Table 2. The rotating natural frequencies in flap bending, lag bending and torsion are 0.74/rev, 1.14/rev and 3.11/rev, respectively. The model rotor characteristics are Lock number $\gamma = 5.9$, solidity ratio $\sigma = 0.03$, three bladed and zero precone. The airfoil characteristics used are

$$C_r = 5.73\alpha,$$
$$C_d = 0.0079 + 1.79\alpha^2,$$
$$C_m = 0.$$

First, the blade response and stability results were calculated for bearingless rotor model in hover ($\mu = 0$) and these were compared with those of Ref. 11. The comparison between the two sets of results appeared quite satisfactory. Then the blade stability analysis in forward flight was performed.

Figure 11 shows the damping of low-frequency cyclic lag mode for a bearingless rotor blade for different advance ratio. The results show that with increase in forward speed the lag damping increases, and hence the blade becomes more stable. The effect of dynamic inflow on lag mode stability is destabilizing for this soft-inplane rotor. There is an appreciable influence of dynamic inflow at large advance ratios which is not the case with hingeless blades which show no response to dynamic inflow at high advance ratios ($\mu > 0.3$). Again the constant coefficient approximation is quite satisfactory with low advance ratios ($\mu < 0.25$).

**CONCLUSIONS**

This paper presents the successful application of finite element method based on Hamilton's principle to determine the response and dynamic stability of hingeless and bearingless rotor blades in forward flight. The blade undergoing flap bending, lead-lag bending and torsion motion is discretized into a number of beam elements and each element consists of fifteen degrees of freedom. The convergence study showed that four beam elements were sufficient to obtain satisfactory response and stability solutions for a hingeless rotor blade, whereas six elements (two for blade, two for flexbeam and two for torque tube) were needed to obtain satisfactory solution for a bearingless rotor blade. The nonlinear steady blade response along the azimuth was calculated successfully solving nonlinear periodic blade equations in the modal space using an iterative procedure based on Floquet theory. For response normal mode equations, six rotating modes (two flap, two lag and two torsion) appeared adequate for hingeless as well as bearingless blades. The vibratory amplitude and the mean level of response depends upon the advance ratio as well as blade structural stiffness. The blade stability is calculated from linearized perturbation equations containing periodic terms using Floquet transition matrix theory as well as constant coefficient approximation in the fixed reference frame. With the inclusion of dynamic inflow, it is needed to solve the stability in the fixed reference frame. Comparing the stability results of elastic blade model with those of spring restrained rigid model, it is seen that for accurate results one needs to adopt an elastic blade modelling, in particular for stiff-inplane rotors. The stability results are obtained for both soft-inplane and stiff-inplane rotors. The effect of dynamic inflow on blade stability is quite important for hingeless as well as bearingless rotors. For soft-inplane hingeless rotors, the influence of dynamic inflow is negligible at high advance ratio ($\mu > 0.3$). For stiff-inplane rotors the lag mode stability degrades at high advance ratios ($\mu > 0.35$). The constant coefficient approximation is satisfactory for low advance ratios ($\mu < 0.25$) for both hingeless and bearingless blade configurations.

**Acknowledgement**—This research work was supported by the Army Research Office under Contract No. DAAG-29-83K0002; Technical Monitor Dr. Robert Singleton.
REFERENCES