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Numerical simulations for the pricing of options in jump diffusion markets

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Abstract In this paper we find numerical solutions for the pricing problem in jump diffusion markets. We utilize a model in which the underlying asset price is generated by a process that consists of a Brownian motion and an independent compensated Poisson process. By risk neutral pricing the option price can be expressed as an expectation. We simulate the option price numerically using the Monte Carlo method.

1. Introduction

Options are financial derivative products that give the right, but not the obligation, to engage in a future transaction on some underlying financial instrument. For instance, a European call option on a financial underlying asset S-with price $(S_t)_{t\in[0,T]}$ is a contract between two agents (buyer and seller) which gives the holder the right to

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buy S at a pre-specified future time T (the expiration date) for an amount K (called the strike). Moreover, at the signature of the contract ($t = 0$) the buyer pays an amount of money called the premium and at the expiration date ($t = T$) he obtains the payoff $h(S_T) = \max(S_T - K, 0) = (S_T - K)^+$. Naturally, two questions arise herein: (i) what price should the seller charge for the option? (known as the pricing problem); (ii) which self-financing strategy should the seller use to secure a wealth equals to the payoff at $t = T$? (known as the hedging problem).

Most of the works on modeling financial derivatives assume that the underlying asset prices S is a continuous process. For instance, in the pioneer work of Black and Scholes (1973) financial asset prices are modeled by the Brownian motion. One of the shortcoming of this model is that, it does not consider the random jumps which can occur in the prices at any time. However, the international financial crisis has shown the importance of adding jumps to financial modeling for stock prices. Unlike the continuous case, models with jumps allow for the possibility that at any moment, a financial price can suddenly decrease (or increase) and attain a significant lower (or higher) value in a negligible time.

Indeed, many researchers have studied financial models with jumps (Bellamy and Jeanblanc, 2000; Dritschel and Protter, 1999; El-Khatib and Privault, 2003; Jeanblanc and Privault, 2002; Merton, 1976), but the issue has not been resolved because of some theoretical complications. Thus, these models generate incomplete markets where the contingent claim (payoff) can impossibly be hedged.

In this paper we study the pricing problem for an underlying asset price with jumps which is governed by the following stochastic differential equation:

$$\frac{dS_t}{S_t} = r_t dt + \sigma_t [a_t dW_t + b_t d(N_t - \lambda_t t)], \quad t \in [0, T], \quad S_0 \text{ is given} > 0, \qquad (1.1)$$

where r, σ, a, b are deterministic functions such that $1 + \sigma_t b_t > 0$. Here $(N_t)_{t\in[0,T]}$ is a Poisson process with deterministic intensity λ and $(W_t)_{t\in[0,T]}$ is a Brownian motion. Note that the process M defined by $M_t := N_t - \lambda_t t$ for $t \in [0,T]$ is the compensated process associated to N. We consider a market with two assets: the risky asset S given by the Eq. (1.1) to which is related a European call option and a risk-free asset given by

$$dA_t = r_t A_t dt, \quad t \in [0, T], \quad A_0 = 1.$$

We work on a probability space $(\Omega, \mathcal{F}, P)$. $(M_t)_{t\in[0,T]}$ and $(W_t)_{t\in[0,T]}$ are independent and we denote by $(\mathcal{F}_t)_{t\in[0,T]}$ the filtration generated by $(N_t)_{t\in[0,T]}$ and $(W_t)_{t\in[0,T]}$. We assume that (1.1) is the price of the asset under the risk-neutral probability P. Recall that a stochastic process is a function of two variables the time $t \in [0,T]$ and the event $\omega \in \Omega$, but in the literature it is common to write S_t, while it means $S_t := S_t(\omega)$. The same interpretation is true for W_t, N_t and M_t or any other stochastic process in this paper. To the authors knowledge, it is impossible to find an explicit formula for the solution of the pricing problem. However, the premium can be determined and expressed in the following expectation form (see Harrison and Kreps, 1979; Harrison and Pliska, 1981)

$$C := e^{-\int_0^T r_s ds} E_P\left[(S_T - K)^+\right], \tag{1.2}$$

where E_P denotes the expected value in a risk-neutral world. Here P is called the equivalent martingale measure. Note that, when $b = 0$ Eq. (1.1) is reduced to the well-known continuous case

$$S_t = S_0 \exp\left(\int_0^t a_s \sigma_s dW_s + \int_0^t \left(r_s - \frac{1}{2}a_s^2\sigma_s^2\right) ds\right).$$

Therefore, the expectation in (1.2) can be calculated by integrating over the normal distribution which gives the same pricing formulas as in the Black and Scholes paper (Black and Scholes, 1973). However if $a \neq 0$ and $b \neq 0$, the expectation function cannot be calculated to have an explicit formula because the random variable S_T does not have a known probability density. To surmount this problem, we use Monte Carlo techniques to simulate the premium. The Monte Carlo method is a very effective tool to simulate the prices of financial derivatives that do not have closed explicit formulas. The use of this method in options pricing was initiated by Boyle (1977). Since then it has been used by many researchers in finance. In this paper, we compute the premium and the price of the option at any time t, $0 \leqslant t \leqslant T$ using the Monte Carlo method.

The remainder of the paper is organized as follows. Some theoretical results are presented in Section 2. In Section 3, we perform numerical simulations for the premium. Concluding remarks are given in Section 4.

2. Theoretical results

First we solve Eq. (1.1) using the following lemma (Itô's formula, see Protter, 1990).

Lemma 1. *Let f, g, and k be three adapted processes such that*

$$\int_0^t |f_s| ds < \infty, \quad \int_0^t |g_s|^2 ds < \infty, \quad \text{and} \quad \int_0^t \lambda_s |k_s| ds < \infty$$

and let $X = (X_t)_{t\in[0,T]}$ be the process defined by

$$dX_t = f_t dt + g_t dW_t + k_t dM_t.$$

We have for any function $F \in \mathcal{C}^{1,2}([0,T] \times \mathbb{R})$

$$F(X_t, t) = F(X_0, 0) + \int_0^t g_s \partial_x F(X_{s^-}, s) dW_s + \sum_{s \leqslant t} (F(X_s, s) - F(X_{s^-}, s))$$
$$+ \int_0^t \left(\partial_s F(X_s, s) + (f_s - k_s\lambda_s)\partial_x F(X_{s^-}, s) + \frac{1}{2} g_s^2 \partial_{xx}^2 F(X_{s^-}, s)\right) ds.$$

Which can be written in the following form (see for example Exercise 1.3.1 page 13 in JeanBlanc (2007) for pure jump processes)

$$F(X_t,t)=F(X_0,0)+\int_0^t g_s\partial_x F(X_{s^-},s)dW_s+\int_0^t[F(X_{s^-}+k_s,s)-F(X_{s^-},s)]dM_s$$
$$+\int_0^t\Big[\partial_s F(X_s,s)+(f_s-k_s\lambda_s)\partial_x F(X_{s^-},s)+\frac{1}{2}g_s^2\partial_{xx}^2F(X_{s^-},s)$$
$$+\lambda_s(F(X_{s^-}+k_s,s)-F(X_{s^-}),s)\Big]ds. \qquad (2.1)$$

Lemma 2. *The underlying asset price S at time t, $0\leqslant t\leqslant T$, is then given by*

$$S_t=S_0\exp\left(\int_0^t a_s\sigma_s dW_s+\int_0^t\left(r_s-\lambda_s b_s\sigma_s-\frac{1}{2}a_s^2\sigma_s^2\right)ds\right)\times\prod_{k=1}^{k=N_t}(1+\sigma_{T_k}b_{T_k}). \qquad (2.2)$$

Proof. Applying Itô's formula (2.1) with $F(S_t,t)=\ln S_t$, we obtain

$$\ln S_t=\ln S_0+\int_0^t\sigma_s a_s S_s\partial_x\ln S_{s^-}dW_s+\int_0^t[\ln(S_{s^-}+\sigma_s b_s S_{s^-})-\ln S_{s^-}]dM_s$$
$$+\int_0^t\Big[\partial_s\ln S_s+(r_sS_s-\sigma_s b_s S_s\lambda_s)\partial_x\ln S_s+\frac{1}{2}\sigma_s^2a_s^2S_s^2\partial_{xx}^2\ln S_s$$
$$+\lambda_s(\ln(S_{s^-}+\sigma_s b_s S_{s^-})-\ln S_{s^-})\Big]ds,$$
$$=\ln S_0+\int_0^t\sigma_s a_s S_s\frac{1}{S_s}dW_s+\int_0^t[\ln(S_{s^-}(1+\sigma_s b_s))-\ln S_{s^-}](dN_s-\lambda_s ds)$$
$$+\int_0^t\left[\frac{r_sS_s-\sigma_s b_s S_s\lambda_s}{S_s}+\frac{-\sigma_s^2a_s^2S_s^2}{2S_s^2}+\lambda_s(\ln(S_{s^-}(1+\sigma_s b_s))-\ln S_{s^-})\right]ds,$$
$$=\ln S_0+\int_0^t\sigma_s a_s dW_s+\int_0^t\left(r_s-\sigma_s b_s\lambda_s-\frac{1}{2}\sigma_s^2a_s^2\right)ds+\int_0^t\ln(1+\sigma_s b_s)dN_s,$$

which is equivalent to (2.2). □

In the following lemma we provide a closed formula for the option prices when the coefficients of the model (r, σ, a, b and λ) are constants.

Lemma 3. *If r, σ, a, b and λ are constants then the price of the European option with stock price governed by (1.1) has the following form*

$$C=S_0e^{-\lambda b\sigma}\sum_{i=0}^{i=\infty}\frac{((1+\sigma b)\lambda T)^i}{i!}e^{(-\lambda T)}\Phi\left(d_2^i\right)-Ke^{-rT}\sum_{i=0}^{i=\infty}\frac{(\lambda T)^i}{i!}e^{(-\lambda T)}\Phi\left(d_1^i\right), \qquad (2.3)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and

$$d_1^i=\frac{\ln\left(\frac{S_0(1+\sigma b)^i}{K}\right)+\left(r-\lambda b\sigma-\frac{1}{2}a^2\sigma^2\right)T}{a\sigma\sqrt{T}},\quad d_2^i=d_1^i+a\sigma\sqrt{T}.$$

Proof. Using Eq. (2.2), the stock price at maturity can be simplified as

$$S_T = S_0 \exp\left(a\sigma W_T + \left(r - \lambda b\sigma - \frac{1}{2}a^2\sigma^2\right)T\right) \times (1+\sigma b)^{N_T}.$$

Then, the premium can be written as

$$\begin{aligned}
C &= e^{-rT}E_P[(S_T - K)^+] = E_P\left[(e^{-rT}S_T - e^{-rT}K)^+\right] \\
&= \sum_{i=0}^{i=\infty} E_P\left[(e^{-rT}S_T - e^{-rT}K)^+ | N_T = i\right] P(N_T = i) \\
&= \sum_{i=0}^{i=\infty} E_P\left[\left(S_0(1+\sigma b)^i e^{\left(a\sigma\sqrt{T}\frac{W_T}{\sqrt{T}} - \left(\lambda b\sigma + \frac{1}{2}a^2\sigma^2\right)T\right)} - e^{-rT}K\right)^+\right] \frac{(\lambda T)^i}{i!} e^{(-\lambda T)} \\
&= \sum_{i=0}^{i=\infty} \left(\int_{-\infty}^{\infty} \left(S_0(1+\sigma b)^i e^{(a\sigma\sqrt{T}x - (\lambda b\sigma + \frac{1}{2}a^2\sigma^2)T)} - e^{-rT}K\right)^+ \right. \\
&\left. \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx\right) \frac{(\lambda T)^i}{i!} e^{(-\lambda T)} = \frac{1}{\sqrt{2\pi}} S_0 \sum_{i=0}^{i=\infty} \int_{I^i} \left(e^{\left(a\sigma\sqrt{T}x - \left(\lambda b\sigma + \frac{1}{2}a^2\sigma^2\right)T\right)}\right) \\
& e^{-\frac{x^2}{2}} dx \frac{((1+\sigma b)\lambda T)^i}{i!} e^{(-\lambda T)} - \frac{1}{\sqrt{2\pi}} \sum_{i=0}^{i=\infty} \int_{I^i} e^{-rT} K e^{-\frac{x^2}{2}} dx \frac{(\lambda T)^i}{i!} e^{(-\lambda T)} \\
&= \frac{1}{\sqrt{2\pi}} S_0 e^{-\lambda b\sigma} \sum_{i=0}^{i=\infty} \int_{x \geqslant -d_1^i} e^{-\frac{(x - a\sigma\sqrt{T})^2}{2}} dx \frac{((1+\sigma b)\lambda T)^i}{i!} e^{(-\lambda T)} \\
&- e^{-rT}K \sum_{i=0}^{i=\infty} \frac{1}{\sqrt{2\pi}} \int_{x \geqslant -d_1^i} e^{-\frac{x^2}{2}} dx \frac{(\lambda T)^i}{i!} e^{(-\lambda T)} \\
&= S_0 e^{-\lambda b\sigma} \sum_{i=0}^{i=\infty} \frac{((1+\sigma b)\lambda T)^i}{i!} e^{(-\lambda T)} \Phi\left(d_2^i\right) - K e^{-rT} \sum_{i=0}^{i=\infty} \frac{(\lambda T)^i}{i!} e^{(-\lambda T)} \Phi\left(d_1^i\right),
\end{aligned}$$

where $I^i = \{x | S_0(1+\sigma b)^i e^{(a\sigma\sqrt{T}x - (\lambda b\sigma + \frac{1}{2}a^2\sigma^2)T)} \geqslant e^{-rT}K\} = \left[-d_1^i, \infty\right)$ and

$$d_1^i = \frac{\ln\left(\frac{S_0(1+\sigma b)^i}{K}\right) + \left(r - \lambda b\sigma - \frac{1}{2}a^2\sigma^2\right)T}{a\sigma\sqrt{T}}, \quad d_2^i = d_1^i + a\sigma\sqrt{T}.$$

Here $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. □

3. Numerical computing of option prices

In this section we discuss the simulation of the premium (1.2) using the Monte Carlo method. The main steps are summarized below (Hull, 2005):

Step 1. Simulation of S_T We select an integer $L > 0$, then we simulate $S_T[i]$ for $i \in \{1,\ldots,L\}$.

Step 2. Monte Carlo solution for the premium The simulation of the premium via the Monte Carlo method involves the following steps:

- For each path $S_T[i]$, compute the payoff $\max(S_T[i] - K, 0)$.
- Calculate the mean of the resulting payoffs $\frac{1}{L}\left(\sum_{i=1}^{i=L}\max(S_T[i] - K, 0)\right)$.
- Estimate the price of the option by discounting the mean payoff at the risk-free rate $\frac{1}{L}\left(\sum_{i=1}^{i=L}\max(S_T[i] - K, 0)\right)e^{-\int_0^T r_s ds}$.

In the proceeding subsections, we give the details of the above steps.

3.1. Simulation of S_T

We seek L realizations of S_T:

$$S_T(\omega_1),\ldots,S_T(\omega_i),\ldots,S_T(\omega_L),$$

where $\omega_1,\ldots,\omega_i,\ldots,\omega_L$ are chosen randomly from Ω. We follow the following algorithm:

- Simulate N trajectories for $(S_t)_{t\in[0,T]}$:

  $$(S_t[1])_{t\in[0,T]},\ldots,(S_t[i])_{t\in[0,T]},\ldots,(S_t[L])_{t\in[0,T]},$$

  where $(S_t[i])_{t\in[0,T]}$ is a simulation of $(S_t(\omega_i))_{t\in[0,T]}$ and $i \in \{1,\ldots,L\}$.
- For each $i \in \{1,\ldots,L\}$, take the value of $(S_t[i])_{t\in[0,T]}$ at the terminal time: $S_T[i]$.

First, we select an integer $H > 0$, then we discretize the time interval $[0, T]$ into steps $t_j = j\Delta t$, $j = 0,1,\ldots,H$ of identical duration $\Delta t = \frac{T}{H}$:

$$S_{t_0}[i],\ldots,S_{t_j}[i],\ldots,S_{t_H}[i]$$

and thus we get L approximations of $S_T : S_{t_H}[1],\ldots,S_{t_H}[i],\ldots,S_{t_H}[L]$.

Let i be fixed in $\{1,\ldots,L\}$. We start by simulating a trajectory $(W_t[i])_{t\in[0,T]}$ of the Brownian motion and a trajectory $(N_t[i])_{t\in[0,T]}$ and then we use Eq. (2.2) to find the approximation $S_{t_H}[i]$ of S_T. We implement the following steps:

1. *Simulation of the Brownian motion and the Brownian integral.* We simulate $(W_{t_j}[i])_{j=0,1,\ldots,H}$ noting the fact that the Brownian motion fulfills:

   $$W_{t_0}[i] = 0,$$

   $$W_{t_j}[i] = W_{t_{j-1}}[i] + \sqrt{\Delta t}Z_j[i], \quad j = 1,\ldots,H,$$

where $Z_j[i]$ follows a normal distribution $N(0,1)$. We simulate $2L$ uniform random variable $U_j[i]$ and $V_j[i]$, and we use the Box–Muller method $Z[j] = \sqrt{-2\log(U_j[i])}\cos(2\pi V_j[i])$. Then, the integral $\int_0^T a_t\sigma_t dW_t$ in Eq. (2.2) is approximated by

$$\int_0^T a_t \sigma_t dW_t[i] = \sum_{j=1}^{j=H} a_{t_j} \sigma_{t_j} (W_{t_j}[i] - W_{t_{j-1}}[i]).$$

2. *Simulation of the Poisson Process and the Poissonian part*. Regarding the Poisson process, we simulate first the jump times $(T_k)_{k \geqslant 0}$ of $(N_t)_{t \in [0,T]}$ with intensity λ by $(T_{N_{t_j}}[i])_{j=0,\ldots,H}$. We are using the following properties of the Poisson process:

$$T_{N_{t_0}}[i] = 0,$$
$$T_{N_{t_j}}[i] = T_{N_{t_{j-1}}}[i] + \text{ExpLaw}(\lambda), \quad j = 1, \ldots, H,$$

where ExpLaw is an exponential random variable which can be written as $\text{ExpLaw}(\lambda) = \frac{-1}{\lambda} \log(urand())$ and $urand()$ is a uniform random variable. A trajectory of the Poisson process $N_{t_j}[i],\ j = 0, \ldots, H$ is then determined by using:

$$N_{t_0}[i] = 1,$$
$$N_{t_j}[i] = \sum_{k=0}^{k=j} 1_{\{T_k[i] \leqslant t_j\}}, \quad j = 1, 2, \ldots, H.$$

The Poissonian part $\prod_{k=1}^{k=N_T}(1 + b_{T_k}\sigma_{T_k})$ in Eq. (2.2) is approximated by $\prod_{k=1}^{k=N_{t_H}[i]}(1 + b_{T_k[i]}\sigma_{T_k[i]})$.

3.2. Monte Carlo solution for the premium

We have from the previous subsections L realizations for S_T, so we can apply the Monte Carlo method to compute the premium numerically using

$$\frac{1}{L}\left(\sum_{i=1}^{i=L} \max(S_T[i] - K, 0)\right) e^{-\int_0^T r_s ds}. \tag{3.1}$$

3.2.1. Reduction of the variance

To reduce the computational time we reduce the variance by using the antithetic variable method. This technique consists of computing two values of the premium C. The first value C_1 is calculated as described above and the second value C_2 is calculated similarly as C_1 with changing the sign of all the random samples from the standard normal distribution. Then C is obtained by taking the average of C_1 and C_2.

The standard error of the estimate premium is then $\frac{s_C}{\sqrt{L}}$, where s_C is the standard deviation of the estimate premium and L is the number of trials. A 95% confidence interval for the premium is

$$\mu_C - \frac{1.96 s_C}{\sqrt{L}} < C < \mu_C + \frac{1.96 s_C}{\sqrt{L}},$$

where μ_C is the mean of the estimated premium.

Now, we present the numerical results of the premium by the Monte Carlo simulation when $T = 1$, $\lambda_t = 0.01t + 3$, $r_t = 0.01(2 + 8\sin(\pi t) + \cos(\frac{\pi t}{2}))$, $a_t = 0.1$, $b_t = 0.3$, $\sigma_t = \sqrt{t} + 0.1$, $S_0 = 7$ and $K = 7.5$. Notice that, the parameters T and λ are used to simulate trajectories for the Brownian motion and for the Poisson process with number of realizations $H = 500$ (see Fig. 1). Then, we simulate trajectories for the stock price at terminal time $T = 1$ with $H = 500$ (see Fig. 2) and for the premium with number of realizations $L = 500$ (see Fig. 3). It is found that, the

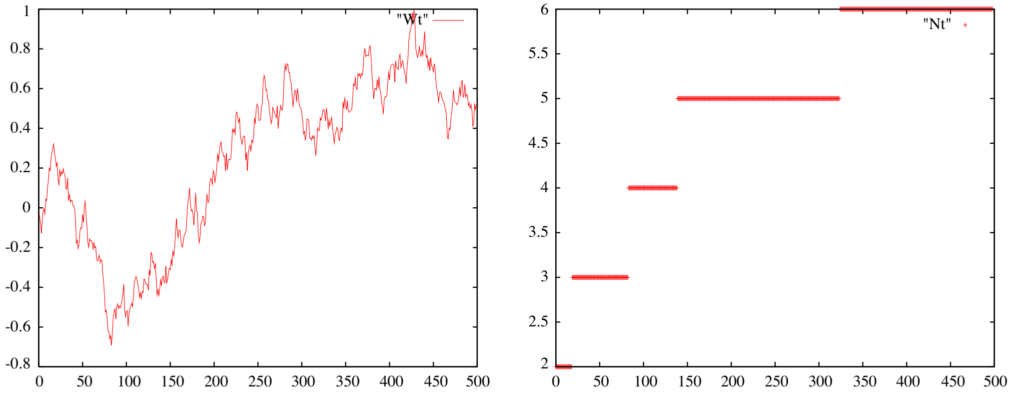

Figure 1 Brownian motion W_t (left) and Poisson process N_t (right).

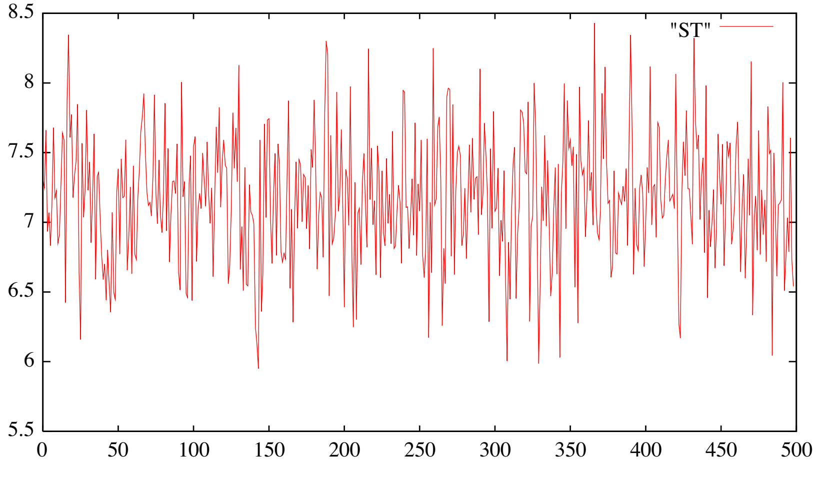

Figure 2 Realizations of the asset price for $T = 1$, $\lambda t = 0.01t + 3$, $r_t = 0.1 + \sin(\frac{\pi t}{2})$, $a_t = 0.25$, $b_t = 0.3$, $\sigma_t = \sqrt{t} + 0.1$, and $S_0 = 7$.

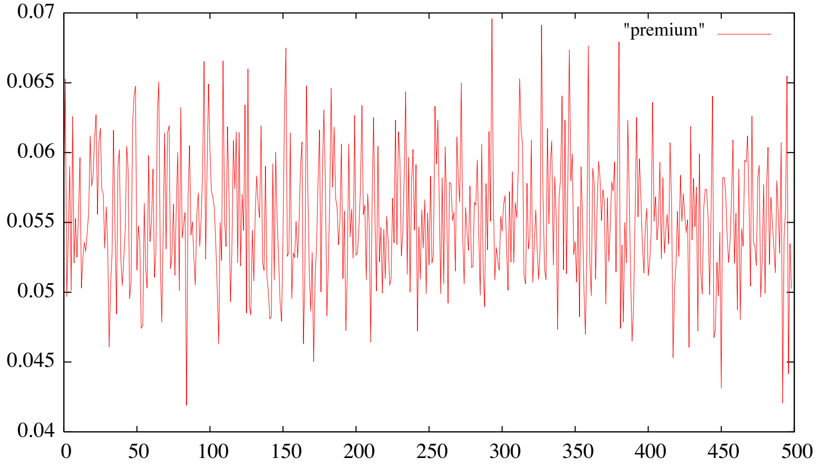

Figure 3 Realizations of the premium for $T = 1,\ \lambda_t = 0.01t + 3,\ r_t = 0.01\left(2 + 8\sin(\pi t) + \cos\left(\frac{\pi t}{2}\right)\right),\ a_t = 0.1,\ b_t = 0.3,\ \sigma_t = \sqrt{t} + 0.1,\ S_0 = 7$ and $K = 7.5$.

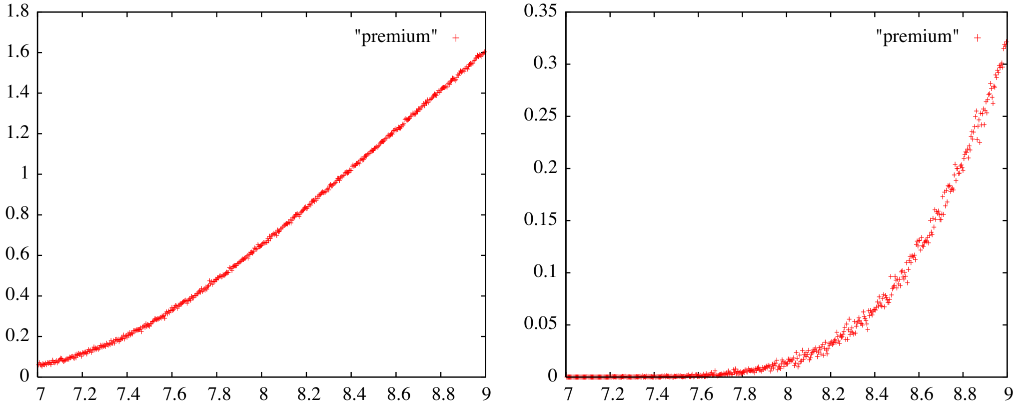

Figure 4 The premium (vertical) as a function of the stock price at $t = 0$ (horizontal), for strike $= 7.5$ (left) and strike $= 9$ (right).

standard error of the estimate premium is 0.237×10^{-3}. A 95% confidence interval for the premium is therefore given by $5.469 \times 10^{-2} < C < 5.562 \times 10^{-2}$.

We also provide the premium as a function of the stock price at $t = 0$ for two different values of the strike $K = 7.5$ and $K = 9$ with number of realizations $L = 500$, see Fig. 4.

4. Conclusion

In this paper, a jump diffusion model is considered for option pricing. The pricing problem for such a model does not have a closed formula since the market is incomplete. However, since it imitates financial crashes, it is a more realistic approach. The price of a European option is simulated numerically by using the Monte Carlo method.

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