

The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction

Estimation of Cable Tension Using Measured Natural Frequencies

H. NAM^{1a} and N. T. NGHIA^{2b}

¹Faculty of Civil Engineering, HCMC University of Technology, Vietnam

²Faculty of Civil Engineering, University of Transport and Communications, Vietnam

Abstract

Estimating cable tension in cable-stayed bridges or in external tendons is essential for regular inspection and assessment of those structures. Vibration measurements provide a solution, however, may not be accurate in cases parameters such as amount of sag and flexural rigidity of cable are significant. In this study, the characteristic equation for vibration of the most general case of a cable, where both the sag and flexure in the cable are taken into account, is analytically derived. After that by considering proper simplifying assumptions of small flexural rigidity parameter, asymptotic forms of that equation are obtained. It renders a practically applicable procedure to estimate cable tension using measured natural frequencies. The developed procedure is verified by realistic data of a cable stayed bridge in Vietnam.

Keywords: cables, tension, sag, flexure, natural frequency.

1. INTRODUCTION

In Vietnam nowadays more and more new infrastructure projects at the national level render more and more bridges built. Significant projects can be listed as suspension bridge Thuận Phước of 450-m main span, and cable stayed bridges: Mỹ Thuận (350-m main span, 24-m deck width, 116-m pylon height); Cần Thơ (550, 26, 165); Rạch Miễu (270, 16, 95); Bãi Cháy (435, 16, 90); Kiên (200, 16.7, 79.5); Bình (260, 22.5, 101.6); and very recently Phú Mỹ in Ho Chi Minh City (380, 16, 90), shown in Figure 1. As a result of the strong demand and rapid development of infrastructure system, structural health monitoring

^a Corresponding author, Presenter: Email: nam@hcmut.edu.vn

^b Email: nghiant.hnuct@gmail.com

(SHM) certainly becomes an important area of research within the civil engineering community. The need to reduce costs, improve the reliability, and accelerate the process of inspecting these important structures becomes evident. Except Mỹ Thuận Bridge which was very first long-span bridge in Vietnam, the other bridges all concern on the implementation a SHM scheme.

Estimating cable tension in cable-stayed bridges or in external tendon is essential for regular inspection and assessment of those structures. Cable tension can be known from the lift-off tests using hydraulic jacks but it is too expensive and hazardous to do. Vibration measurements provide a better solution, however, may not be accurate in cases parameters such as the amount of sag and flexural rigidity of the cable are significant. In previous studies, a semi-empirical or numerical approach has been utilized to find the tension force in stay cables. Notable is the work of Zui et al. (1996) who proposed practical formula for tension force in a slender cable considering both cable sag and flexure. However there is significant difference between the values of the practical formula and experimental results for the first vibration mode of large sag cables.

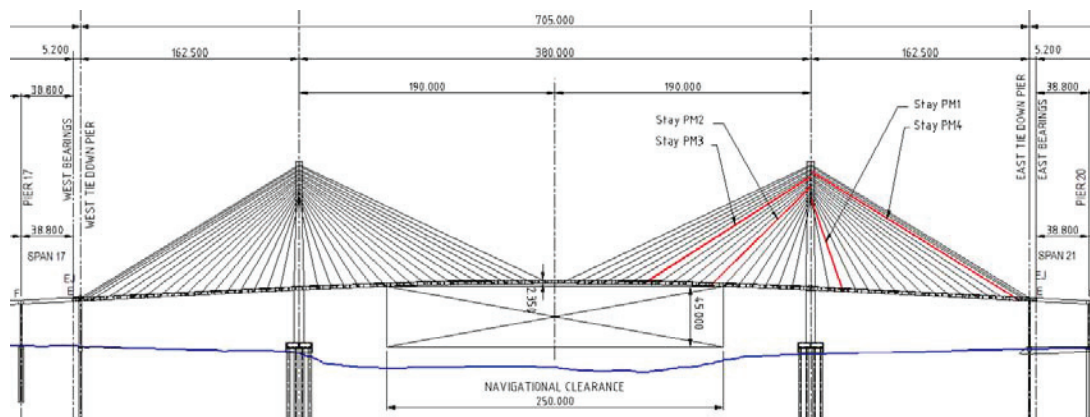


Figure 1. Phú Mỹ Bridge in Ho Chi Minh City

In a different approach, this study attempts to derive asymptotic forms of the wave number equation of a general cable, from which the cable tension can be estimated in a simpler and straightforward procedure using measured natural frequencies. The developed procedure can be applied for a wide range of flexural rigidity as well as sag-to-span ratio in cable, through two dimensionless parameters consistently defined in Irvine and Caughey (1974), and in recent studies of the first author (Hoang and Fujino 2007, Fujino and Hoang 2008). Finally it is verified by realistic data collected from sensors in the SHM scheme of Phú Mỹ Bridge.

2. GOVERNING EQUATIONS OF A GENERAL CABLE

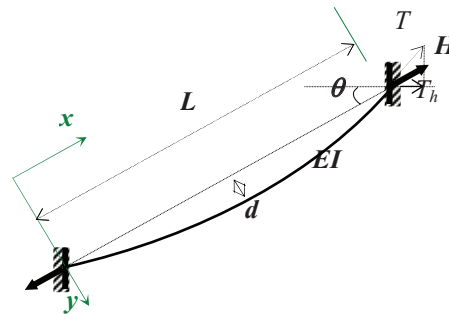


Figure 2. A model of an inclined cable

Consider an inclined cable under tension force T as shown in Figure 2. A coordinate system is defined with the x -axis along the cable chord and the y -axis in the perpendicular direction. The cable has a mass per unit length m , a chord length L , a finite flexural rigidity EI and is inclined at an angle θ to the horizontal ($0 \leq \theta < \pi/2$). The dynamic equation of the cable in-plane motion $v(x, t)$ (in y -direction) is expressed as (Fujino and Hoang 2008, Zui et al. 1996)

$$H \frac{\partial^2 v}{\partial x^2} - m \frac{\partial^2 v}{\partial t^2} + h \frac{d^2 y}{dx^2} - EI \frac{\partial^4 v}{\partial x^4} = 0 \quad (1)$$

where $H = T_h / \cos \theta$ is the chord tension, and T_h is the horizontal component of the cable tension. Eq. (1) is established assuming that the cable tension T is sufficiently large so that the static profile of the cable can be accurately described by the parabola (Irvine 1981)

$$y = 4d \frac{x}{L} \left(1 - \frac{x}{L} \right) \quad (2)$$

Here d is the sag at mid-span, $d = mgL^2 \cos \theta / (8H)$, in which g denotes the gravitational constant. In Eq. (1), $h(t)$ is the additional tension in the cable caused by the motion, which is obtained from the elastic and geometric compatibility of the cable element (Irvine and Caughey 1974)

$$\frac{hL_e}{EA} = \frac{8d}{L^2} \int_0^L v(x, t) dx \tag{3}$$

in which EA is the axial rigidity of the cable and $L_e = L[1 + 8(d/L)^2]$. Eq. (1) corresponds to the most general case of a cable where both the sag and flexure in the cable are taken into account. The equation has been studied by the first author in investigating the damping effect of a stay cable with a damper (Fujino and Hoang 2008). Two important parameters are here noted (i) the sag parameter λ^2 , defined by Irvine and Caughey (1974),

$$\lambda^2 = \left(\frac{8d}{L}\right)^2 \frac{L}{HL_e/EA} \tag{4}$$

and (ii) the flexural rigidity parameter ε (Hoang and Fujino 2007),

$$\varepsilon = EI/HL^2 \tag{5}$$

Considering the free vibration of the cable with (complex) natural frequency ω . Denotes a dimensionless parameter, i.e., wave number $\beta = \omega\sqrt{m/H}$, the following characteristic equation for β can be derived (Fujino and Hoang 2008):

$$f_1(f_2 + \lambda^2 f_3) = 0 \tag{6}$$

where

$$f_1 = 8\gamma_a\gamma_b(\gamma_a^2 + \gamma_b^2) [\gamma_a \sin(\frac{1}{2}\gamma_b L) \cosh(\frac{1}{2}\gamma_a L) - \gamma_b \cos(\frac{1}{2}\gamma_b L) \sinh(\frac{1}{2}\gamma_a L)] \tag{6a}$$

$$f_2 = \gamma_a\gamma_b \beta^2 L^3 [\gamma_a \cos(\frac{1}{2}\gamma_b L) \sinh(\frac{1}{2}\gamma_a L) + \gamma_b \sin(\frac{1}{2}\gamma_b L) \cosh(\frac{1}{2}\gamma_a L)] \tag{6b}$$

$$f_3 = -\gamma_a\gamma_b^2 L \sin(\frac{1}{2}\gamma_b L) \cosh(\frac{1}{2}\gamma_a L) - \gamma_a^2\gamma_b L \cos(\frac{1}{2}\gamma_b L) \sinh(\frac{1}{2}\gamma_a L) + 2(\gamma_a^2 + \gamma_b^2) \sin(\frac{1}{2}\gamma_b L) \sinh(\frac{1}{2}\gamma_a L) \tag{6c}$$

$$\gamma_{a,b}^2 = \frac{1}{2\varepsilon L^2} (\sqrt{1 + 4\varepsilon \beta^2 L^2} \pm 1) \tag{6d}$$

It is noted that in case of an anti-symmetric vibration mode, the sag will generate no additional cable tension. As a result there exist different solutions of Eq. (6) for symmetric and anti-symmetric modes of the cable. For the anti-symmetric modes (mode index $n = 2, 4, \dots$), the wave number is determined by equating the first term f_1 in Eq. (6) to zero which yields

$$\tan(\frac{1}{2}\gamma_b L) = \frac{\gamma_b}{\gamma_a} \tanh(\frac{1}{2}\gamma_a L) \quad \text{for } n = 2, 4, \dots \tag{7}$$

For symmetric modes ($n = 1, 3, \dots$), equating the remaining bracket to zero, after rearranging, gives

$$\frac{\sqrt{\varepsilon}L(\gamma_a^2 + \gamma_b^2)\sin(\frac{1}{2}\gamma_b L)\sinh(\frac{1}{2}\gamma_a L)}{\gamma_a \cos(\frac{1}{2}\gamma_b L)\sinh(\frac{1}{2}\gamma_a L) + \gamma_b \sin(\frac{1}{2}\gamma_b L)\cosh(\frac{1}{2}\gamma_a L)} = \frac{\beta L}{2} - \frac{4}{\lambda^2} \left(\frac{\beta L}{2}\right)^3 \quad \text{for } n = 1, 3, \dots \quad (8)$$

Eq. (6) and consequently Eqs. (7) and (8) are transcendental equations and can be solved for the wave number β using iterative numerical methods, such as Newton-Raphson, starting with an appropriate value. It is seen that this equation covers a broad range of traditional problems in which either the sag parameter λ^2 or the flexural rigidity parameter ε is separately included. For example, the wave number of a taut flexural cable ($\lambda^2 = 0$) is determined by equating the product of $f_1 \times f_2$ to zero, which results in the same cable characteristic equation derived by Zui et al. (1996). When $\lambda^2 \neq 0$, in the limit of zero flexural rigidity $\varepsilon \rightarrow 0$, after rearranging like terms in the bracket and equating it to zero, the well-known wave number equation for a (non-flexural) sag cable, established by Irvine and Caughey (1974), is obtained

3. ESTIMATION OF CABLE TENSION

The cable tension can be obtained by its relation with the wave number. With transcendental equations like Eqs. (7) and (8), an expression for the wave number β_{0n} of an individual mode n , and thus the cable tension, cannot be explicitly obtained. However by considering proper simplifying assumptions of a small flexural rigidity parameter, the cable tension can be estimated in a simpler and straightforward way. When ε is small so that $4\varepsilon\beta_{0n}^2 L^2 \ll 1$, from Eq. (6d), the following approximations can be made

$$\gamma_a \cong 1/\sqrt{\varepsilon L^2}, \quad \gamma_b \cong \beta_{0n}, \quad \gamma_a^2 + \gamma_b^2 \cong \gamma_a^2 - \gamma_b^2 = 1/(\varepsilon L^2) \quad (9a)$$

$$\sinh \gamma_a L \cong \cosh \gamma_a L \gg 1 \quad (9b)$$

Introducing these approximations into Eqs. (7&8), and rearrange the resulting expressions, yielding

$$\tan(\frac{1}{2}\beta_{0n}L) = \beta_{\varepsilon n}, \quad n = 2, 4, \dots \quad (10)$$

$$\tan(\frac{1}{2}\beta_{0n}L) = \frac{\beta_{\lambda n}}{1 - \beta_{\varepsilon n} \beta_{\lambda n}}, \quad n = 1, 3, \dots \quad (11)$$

where $\beta_{\varepsilon n} = \sqrt{\varepsilon} \beta_{0n} L$ and $\beta_{\lambda n} = \frac{1}{2} \beta_{0n} L - (4/\lambda^2)(\frac{1}{2} \beta_{0n} L)^3$

Eqs. (10) and (11) are logical extensions of the wave number equation for even and odd vibration modes of a sag cable given by Irvine and Caughey (1974), to considering the cable flexural rigidity.

Note that in long-span cable-stayed bridges the value of λ^2 is normally less than 3 (Tabatabai and Mehrabi 2000), while a range of $2.5 \times 10^{-6} - 10^{-4}$ is common for ε (Hoang and Fujino 2007). The graphs of variation of β_{0n} of the first six vibration modes ($n = 1 - 6$) for a typical value $\varepsilon = 10^{-4}$ can be visualized in Figure 3. In the figure the wave number of sag cable by Irvine and Caughey equation ($\varepsilon = 0$) is plotted by thin lines for reference. It is seen that the cable flexural rigidity causes a variation in the wave number which is rather slight for first low modes but then significant for higher modes. The variation is quite steady with respect to sag parameters for both even and odd vibrations modes. The accuracy of the asymptotic equations (10) and (11) can be verified by comparing its result to the exact solution of Eqs. (7) and (8) in Figure 3.

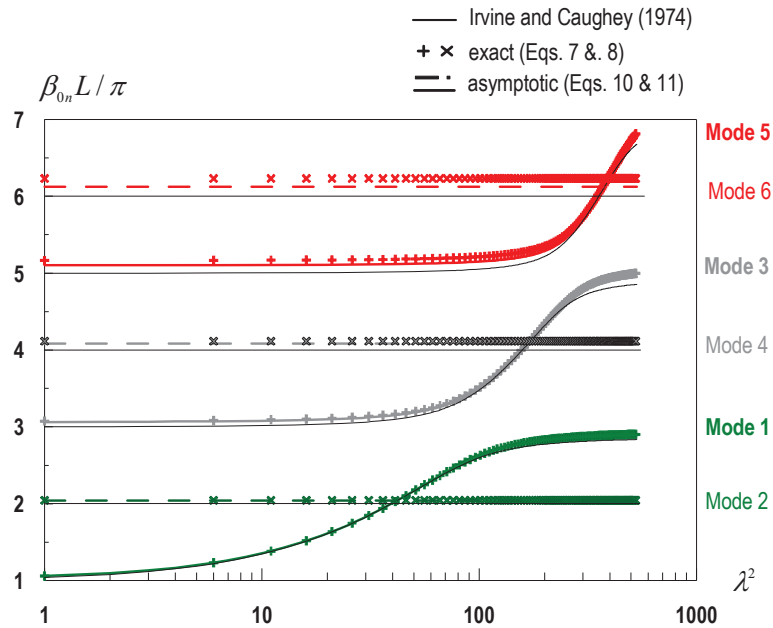


Figure 3. Variation of wave number of the first six vibration modes for $\varepsilon = 10^{-4}$

Hence using simple equations (10) and (11), the wave number or natural frequencies of a general sag and flexural cable can be readily calculated, given the cable tension and properties. By this way, a curve relating the cable tension and its natural frequency can be built, from which cable tension can be estimated from measured natural frequencies. Regarding the vibration modes n to be considered in the estimation, it depends on the level of sag in the cable. For short cable with small sag parameter, the first mode should be the most crucial, while the higher modes would give easier and more accurate estimate for long cable (Zui et al. 1996). In general, Eq. (12) should be satisfied for any value of n .

4. CASE STUDY OF PHU MY BRIDGE

The accuracy of the above procedure for cable tension estimation can be verified through realistic data. In this study, the cable properties and measurement data collected from a cable-stayed bridge in Vietnam are employed. It is Phú Mỹ Bridge which crosses SaiGon River to form part of a new ring road around Ho Chi Minh City - an important transport link from the southern Mekong delta region to the Central and Northern parts of Vietnam. The main span of the bridge is 380 m, with 16-m deck width, 90-m pylon height. The cable system here includes 144 cables in multi-fan type. These cables are of Freyssinet monostrand which consists of a group of parallel individually protected type S15 strands. The length of cables varies from 60 to 202 m. The properties of typical 4 cables selected for this study, namely PM1 – PM4 as in Figure 1, are given in **Table 1**. In evaluating the inertial moment of a parallel-strand stay cable, the slip occurring between the individual strands should be considered via void ratio (Gimsing 1997), which may be as low as 20% that of a homogeneous section. Using the data in this table, the curves relating the tension and natural frequencies of cables PM1 – PM4 can be made as shown in Figure 5 for first four fundamental vibration modes.

Table 1. Cable properties

Cable No.	No. of strand	Outer Dia.	Cable weight	Sectional Area	Void Ratio	Inertia	Length	Inclination angle
		mm	kg/m	mm ²		m ⁴	m	
PM1	27	160	31.86	4050.00	0.260	8.363E-06	68.470	68.01
PM2	35	180	41.30	5250.00	0.266	1.372E-05	101.586	46.02
PM3	45	200	53.10	6750.00	0.277	2.178E-05	145.224	33.11
PM4	51	200	60.18	7650.00	0.314	2.468E-05	179.361	28.37

In August 2009, a field test has been conducted for stay cables of Phú Mỹ Bridge in normal weather conditions (Figure 4): temperature varied from 32°C to 35°C, and wind speed varied from 2.0 to 11.20 m/s. The cables were excited by human power until significant motion then released, and free vibration decay was recorded. The purpose of the test is to estimate the cable tension through natural vibration frequency. These data are to be used as a reference state for a structural health monitoring scheme of the Bridge later on. For the 4 cables considered in this study, the measured natural frequencies are presented in Table 2. Marking these measured frequencies on the corresponding curves in Figure 5, the tension which fits all considered modes can be recognized and its value is noted in Table 2. These estimated values agree well with the references load cells readings provided by the contractor, and also with the values calculated using formulae (35) in Zui et al. (1996), given in the last column of Table 2.

Table 2. Measured natural frequency and measured tension force in cable

Cable No.	f_1	f_2	f_3	f_4	Estimated H	H by Zui et al.
	Hz	Hz	Hz	Hz	KN	KN
PM1	2.070	4.067	6.101	8.188	2480	2426
PM2	1.297	2.694	3.972	5.251	2900	2746
PM3	0.994	1.999	2.966	3.999	4400	4292
PM4	0.818	1.576	2.373	3.151	4800	5057

5. CONCLUSIONS

The combined effects of the sag and flexural rigidity of the cable on its tension have been analytically studied. By introducing proper simplifying approximations, asymptotic forms for the wave number equation of an inclined cable considering sag and flexural rigidity have been explicitly obtained and their accuracy is confirmed. It renders a practically applicable procedure to estimate cable tension using measured natural frequencies of the cable. The developed procedure has been verified by realistic data of Phú Mỹ Bridge in Vietnam.



Figure 4. Measurement of cable vibration at Phú Mỹ Bridge

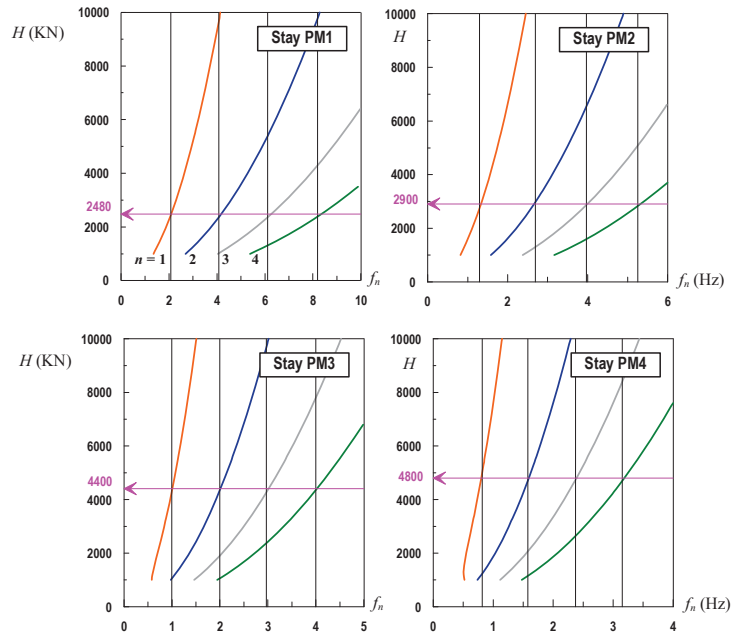


Figure 5. Curves relating tension and natural frequencies for stay cables of Phú Mỹ Bridge

REFERENCES

- [1] Fujino, Y., and Hoang, N. (2008). “Design formulas for damping of a stay cable with a damper.” *Journal of Structural Engineering*, ASCE, 134(2), 269-278.
- [2] Gimsing, J. N. (1997). *Cable Supported Bridges*, Second Ed., John Wiley & Sons, Chichester.
- [3] Hoang, N., and Fujino, Y. (2007). “Analytical study on bending effects in a stay cable with a damper.” *Journal of Engineering Mechanics*, ASCE, 133(11), 1241-1246.
- [4] Irvine, H.M., and Caughey, T.K. (1974). “The linear theory of free vibrations of a suspended cable.” *Proceedings of the Royal Society London, Series A*, 341, 299–315.
- [5] Irvine, H.M. (1981). *Cable Structures*. Dover, New York.
- [6] Zui, H., Shinke, T., and Namita, Y. (1996). “Practical formulas for estimation of cable tension by vibration method.” *Journal of Structural Engineering*, ASCE, 122(6), 651-656.
- [7] Vietnamese Ministry of Transportation (2009) - Design of Phú Mỹ Bridge Documents.