Available online at www.sciencedirect.com

ScienceDirect

Computers and Mathematics with Applications 56 (2008) 1236–1244

An International Journal
computers & mathematics with applications

www.elsevier.com/locate/camwa

Finite element solution for flow of a third grade fluid past a horizontal porous plate with partial slip

M. Sajid[a,*], R. Mahmood[a], T. Hayat[b]

[a] *Theoretical Plasma Physics Division, PINSTECH, P.O. Nilore, Islamabad 44000, Pakistan*
[b] *Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan*

Received 23 October 2007; received in revised form 18 January 2008; accepted 6 February 2008

Abstract

The steady flow of a third grade fluid past a horizontal porous plate with partial slip is investigated. The arising non-linear problem is solved numerically using a finite element method. The results of no-slip and slip conditions are presented graphically and the effects of fluid parameters, suction velocity and slip parameter have been discussed. Comparison of the present analysis is made with the existing results.

Keywords: Third grade fluid; Porous plate; Slip effects; Suction velocity; Numerical solution

1. Introduction

The equations which govern the flows of viscous fluids are the Navier–Stokes equations. In nature, there are many fluids which do not obey the Newtonian law of viscosity and the Navier–Stokes equations are inadequate for such fluids. These fluids are termed as the non-Newtonian fluids. Considerable interest has been shown in the non-Newtonian fluids during the past few decades in view of their potential applications in industry and technology. Due to this fact several investigators [1–16] are engaged in studying the flows of such fluids. Nevertheless, the resulting problems in the study of non-Newtonian fluids pose a challenge to applied mathematicians, physicists, modelers and computer scientists. In general these stem from the fact that the viscoelasticity of the fluid introduces some extra non-linear terms in the momentum equations which make the task of obtaining the accurate solutions a difficult one. The constitutive equations of even the simplest of viscoelastic fluids namely the second grade fluid are such that the momentum equations give rise to boundary value problems in which the order of the differential equations is greater than the number of available boundary conditions. Hence the adherence boundary conditions are insufficient for a unique solution. A detail discussion on this issue can be found in the Reference [17–21].

In all the above mentioned studies, no attention has been focused on exploring the slip effects on the flows of non-Newtonian fluids. Although the no-slip condition is widely used for flows involving non-Newtonian fluids it is

* Corresponding author. Tel.: +92 51 9290231.
E-mail address: sajidqau2002@yahoo.com (M. Sajid).

0898-1221/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2008.02.025

inadequate in certain cases namely the mechanics of thin films, problems involving multiple interfaces and the flow of rarefied fluids etc. Furthermore, it is known now that a large class of polymeric materials slip or stick slip on solid boundaries. There is a sudden increase in the throughput at a critical pressure gradient when polymer melts flow under the application of a pressure gradient. This mechanism is known as "spurt". Experimentalists usually associate "spurt" with slip at the wall. The detailed review relevant to the slip condition has been provided in the studies [22–24].

The purpose of the present attempt is to analyze the slip effects on the steady flow of a third grade fluid past a porous plate. The slip condition depends on the shear stress only. Numerical solution of the arising problem is given by employing a finite element method. This is an efficient method and is widely used in finding the solutions in different fields ranging from solid mechanics to fluid mechanics. It has been successfully applied to all engineering disciplines; but civil, mechanical and aerospace engineers are the most frequent users of this method. The present analysis provides an application of this method to a strong non-linear problem. The paper is organized as follows:

Section 2 includes the problem formulation. The necessary steps involved in finite element method are presented in Section 3. Section 4 consists of numerical results and discussion. Section 5 synthesis the concluding remarks.

2. Formulation of the problem

Consider the steady flow of a third grade fluid past a porous plate. The x-axis is taken parallel to the plate and the y-axis normal to it, the velocity field depends only on y. The flow past a porous plate with suction or injection and with uniform stream at infinity has a two-dimensional structure. The governing equations of the third grade fluid consists of the incompressibility condition

$$\text{div}\,\mathbf{V} = 0 \tag{1}$$

and the momentum equation

$$\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{1}{\rho}\text{div}\,\mathbf{T}, \tag{2}$$

where ρ is the fluid density, $\mathbf{V}$ is the velocity and $\mathbf{T}$ the Cauchy stress tensor. Under the consideration of flow, it follows from Eq. (1) that for a uniformly porous plate

$$u = u(y), \qquad v = -V_0, \tag{3}$$

in which $V_0 > 0$ is the suction velocity and $V_0 < 0$ corresponds to the injection velocity.

The Cauchy stress tensor in a third grade fluid is [25]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \mathbf{S}, \tag{4}$$

$$\mathbf{S} = \beta_1\mathbf{A}_3 + \beta_2\left(\mathbf{A}_2\mathbf{A}_1 + \mathbf{A}_1\mathbf{A}_2\right) + \beta_3\left(\text{tr}\,\mathbf{A}_1^2\right)\mathbf{A}_1, \tag{5}$$

in which p is the hydrostatic pressure, $\mathbf{I}$ is the identity tensor, μ is the coefficient of viscosity and α_i $(i = 1, 2)$, β_i $(i = 1$ to $3)$ are material constants. The kinematical tensors $\mathbf{A}_i$ $(i = 1, 2, 3)$ are the Rivlin–Ericksen tensors given by [26]

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^{\top}, \tag{6}$$

$$\mathbf{A}_n = \frac{\mathrm{d}\mathbf{A}_{n-1}}{\mathrm{d}t} + \mathbf{A}_{n-1}\mathbf{L} + \mathbf{L}^{\top}\mathbf{A}_{n-1}, \quad n = 2, 3, \ldots, \tag{7}$$

$$\mathbf{L} = \nabla \boldsymbol{V}, \tag{8}$$

where ∇ is the gradient operator and $\mathrm{d}/\mathrm{d}t$ is the material time derivative defined by

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (\mathbf{V}\cdot\nabla), \tag{9}$$

in which $\partial/\partial t$ is the partial time derivative. A detail thermodynamic analysis of the model, represented by Eqs. (4) and (5), is given in [27], where it is shown that if all the motions of the fluid are to be compatible with thermodynamics in the sense that these motions satisfies the Clausius–Duhem inequality and if it is assumed that the specific Helmholtz free energy is a minimum when the fluid is locally at rest, then

$$\mu \geq 0, \ \alpha_1 \geq 0, \quad |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \quad \beta_1 = \beta_2 = 0, \ \beta_3 \geq 0. \tag{10}$$

In present analysis we assume that the fluid is thermodynamically compatible and therefore Eqs. (4) and (5) reduces to

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3\left(\mathrm{tr}\,\mathbf{A}_1^2\right)\mathbf{A}_1. \tag{11}$$

The non-zero components of the stress for the problem under consideration are

$$T_{xx} = -p + \alpha_2\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2, \tag{12}$$

$$T_{xy} = \mu\frac{\mathrm{d}u}{\mathrm{d}y} - \alpha_1 V_0\frac{\mathrm{d}^2u}{\mathrm{d}y^2} + 2\beta_3\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^3, \tag{13}$$

$$T_{yy} = -p + (2\alpha_1 + \alpha_2)\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2, \tag{14}$$

$$T_{zz} = -p, \tag{15}$$

$$T_{xz} = T_{zy} = 0, \tag{16}$$

and

$$T_{xy} = T_{yx}, \qquad T_{xz} = T_{zx}, \qquad T_{yz} = T_{zy}. \tag{17}$$

The momentum equation takes the form

$$-\rho V_0\frac{\mathrm{d}u}{\mathrm{d}y} = \mu\frac{\mathrm{d}^2u}{\mathrm{d}y^2} - \alpha_1 V_0\frac{\mathrm{d}^3u}{\mathrm{d}y^3} + 6\beta_3\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2\frac{\mathrm{d}^2u}{\mathrm{d}y^2} - \frac{\partial\widehat{p}}{\partial x}, \tag{18}$$

where the modified pressure

$$\widehat{p} = -T_{yy}. \tag{19}$$

Here, the second and third components of Eq. (9) give $\partial\widehat{p}/\partial y = \partial\widehat{p}/\partial z = 0$ and we have, for the uniformity of the free stream at infinity, $\partial\widehat{p}/\partial x = 0$ [21] and the governing equation (18) thus becomes

$$-\rho V_0\frac{\mathrm{d}u}{\mathrm{d}y} = \mu\frac{\mathrm{d}^2u}{\mathrm{d}y^2} - \alpha_1 V_0\frac{\mathrm{d}^3u}{\mathrm{d}y^3} + 6\beta_3\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2\frac{\mathrm{d}^2u}{\mathrm{d}y^2}. \tag{20}$$

The relevant boundary conditions on u are [21]

$$u(0) = \frac{\kappa}{\mu}T_{xy}\big|_{y=0}, \tag{21}$$

$$u(y) \to U_0, \quad \frac{\mathrm{d}u}{\mathrm{d}y} \to 0, \quad \frac{\mathrm{d}^2u}{\mathrm{d}y^2} \to 0 \quad \text{as } y \to \infty, \tag{22}$$

where U_0 is the mainstream velocity.

In order to carry out the non-dimensional analysis, we define the following variables

$$\bar{u} = \frac{u}{U_0}, \qquad \bar{y} = \frac{U_0 y}{\upsilon}, \qquad \bar{V}_0 = \frac{V_0}{U_0}, \qquad \bar{\alpha} = \frac{\alpha_1 U_0^2}{\rho\upsilon^2},$$
$$\bar{\beta} = \frac{\beta_3 U_0^4}{\rho\upsilon^3}, \qquad \bar{\gamma} = \frac{\kappa U_0}{\nu}. \tag{23}$$

and thus the boundary value problem becomes

$$\frac{d^2u}{dy^2} + V_0\frac{du}{dy} - \alpha_1 V_0\frac{d^3u}{dy^3} + 6\beta\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2} = 0, \tag{24}$$

$$u = \gamma\left[\frac{du}{dy} - \alpha_1 V_0\frac{d^2u}{dy^2} + 2\beta\left(\frac{du}{dy}\right)^3\right] \quad \text{at } y = 0, \tag{25}$$

$$u \to 1, \quad \frac{du}{dy} \to 0, \quad \frac{d^2u}{dy^2} \to 0 \quad \text{as } y \to \infty. \tag{26}$$

For simplicity we omitted the bars of the non-dimensional quantities. Integrating Eq. (24) once and using the conditions (26) we get

$$\frac{du}{dy} + V_0 u - \alpha_1 V_0\frac{d^2u}{dy^2} + 2\beta\left(\frac{du}{dy}\right)^3 - V_0 = 0. \tag{27}$$

At $y = 0$ above equation reads as

$$\left[\frac{du}{dy} - \alpha_1 V_0\frac{d^2u}{dy^2} + 2\beta\left(\frac{du}{dy}\right)^3\right]_{y=0} = V_0\left[1 - u(0)\right]. \tag{28}$$

Comparison of Eq. (25) and (28) yields

$$u(0) = \frac{\gamma V_0}{1 + \gamma V_0}. \tag{29}$$

3. Finite element method

We use the finite element method for the solution of non-linear differential equation given by Eq. (27) subject to the boundary conditions given in Eqs. (26) and (29). In the finite element method, a weak form or variational form for a given boundary value problem is first constructed. A weak form is a weighted integral statement of a differential equation in which the differentiation is distributed among the dependent variable and the weight function or test function. It also includes the natural boundary conditions of the problem. The weak formulation has two desirable characteristics. First, it requires weaker continuity of the dependent variable by distributing the differentiation between the solution and the weight function w (due to its weaker requirement of continuity, it has been given the name weak form). Second, the natural boundary conditions of the problem are included in the weak form and the solution is required to satisfy only the essential boundary conditions of the problem. Whenever the classical solution exists it coincides with the weak solution of the problem. The finite element method then follows an orderly step by step process.

1. *Discretization of domain*

The first step is to divide/discretize the continuous physical model or domain into a finite number of smaller elements/sub-domains. The domain for the boundary value problem is viewed as an assemblage of these sub-domains usually known as the finite element mesh/grid. A variety of element shapes may be used, and, with care different element shapes may be employed in the same solution region. If we partition the domain Ω into a finite number E of elements $\Omega_1, \Omega_2, \ldots, \Omega_E$, then these elements should be non overlapping and cover the domain Ω in the sense that,

$$\Omega_e \cap \Omega_f = \phi \quad \text{for } e \neq f \quad \text{and} \quad \bigcup_{e=1}^{E} \bar{\Omega}_e = \bar{\Omega}. \tag{30}$$

The number and the type of elements to be used in a given problem are matters of mathematical or engineering judgement. Since these elements can be put together in a variety of ways, they can be used to represent very complex domain shapes. The mesh consists of line segments in one dimension, in two dimensions it may consist of triangles or quadrilaterals and in three dimensions it may consist of tetrahedra or hexahedra. All these are known as finite elements or simply elements.

2. *Selection of approximation function*

In this step nodes (nodal points) are assigned to each element, these are normally the points at which the elements are connected (though an element may have a few interior nodes depending on the finite element solution procedure followed). Approximation functions or interpolation functions are then chosen to represent the variation of the dependent/field variable in terms of the assumed approximation functions over the element. The approximating functions are defined in terms of linear combinations of algebraic polynomials called basis functions and the values of the field variables at node points. The nodal values of the field variable and the basis functions for the elements completely define the behavior of the field variable within the elements. For the finite element representation of a problem the nodal values of the field variable become unknowns to be determined. A finite element approximate solution U is of the type,

$$U = \sum_{i=1}^{N} u_i \psi_i, \tag{31}$$

where u_i are solution values at the node points to be determined and ψ_i are chosen approximating functions and N is the total number of nodes. The choice of algebraic polynomials as a basis function has two reasons. First the interpolation theory of numerical analysis can be used to develop the approximate functions systematically over an element. Second, numerical evaluation of integrals of algebraic polynomials is easy. The degree of the polynomial chosen depends on the number of nodes assigned to the element, the nature and number of unknowns at each node and certain continuity requirements imposed at the nodes and along the element boundaries. For example, in two dimensions on triangles the field variables may be approximated by linear polynomials $p = a_1 + a_2x + a_3y$, with three nodes at the vertices of the triangle, or by quadratic polynomials $p = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2$, with six nodes, three at the vertices and three at the mid points of the triangle edges. Basis functions ψ_i have the following properties.

i. The functions ψ_i are bounded and continuous, that is, $\psi_i \in C(\bar{\Omega})$.
ii. The total number of basis functions is equal to the number of nodes present in the mesh and each function ψ_i is nonzero only on those elements that are connected to node i: $\psi_i(\underline{x})|_{\Omega_e} \equiv 0$ if $i \notin \bar{\Omega}_e$, where $\underline{x}$ is any interior point of $\bar{\Omega}_e$.
iii. The basis function ψ_i is equal to 1 at node i, and equal to zero at the other nodes, $\psi_i(y_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$, where y_j is the position of jth node.

3. *Obtaining element properties*

Once the elements and their interpolation functions have been selected we are ready to determine the matrix equations expressing the properties of the individual elements. Instead of solving the problem for the entire domain in one step, attention is mainly devoted to the formulation of the properties of the constituent elements. A standard element is selected from the mesh and then the finite element formulation is constructed for this element.

4. *Assembling the system equations*

Results are recombined to represent the whole domain/mesh by assembling the element properties to obtain the system equations. To find the properties of the overall system modeled by the mesh of elements we must interface all the element properties.

5. *Imposing the boundary conditions*

At this stage we impose known nodal boundary values of the dependent variable i.e. system equations must be modified to account for the boundary conditions of the problem.

6. *Solving the system equations*

The assembly process yields a large set of simultaneous algebraic equations. After imposing the essential and natural boundary conditions the problem is thus reduced to one of solving the set of simultaneous equations where the number of equations is equal to the number of nodes at which the solution is required. In matrix form this set of equations can be written as

$$K\underline{u} = \underline{f}, \tag{32}$$

where the matrix K is known as the stiffness matrix, $\underline{f}$ is known as the load vector and $\underline{u}$ is the solution vector.

Since $\psi_i = 0$ for all elements that do not have node i as a node, it follows that this property of basis functions will result in the matrix K having a sparse structure or, with an appropriate ordering, a banded structure in which all nonzero entries are clustered around the main diagonal. There are two main approaches to solving this set of simultaneous equations: direct methods or iterative methods, and there are many variants of each method.

3.1. Implementation of finite element method

The variational form associated with Eqs. (26), (27) and (29) is given by

$$\int_{\Omega} \phi \left(\frac{\mathrm{d}u}{\mathrm{d}y} + uV_0 - \alpha V_0 \frac{\mathrm{d}^2 u}{\mathrm{d}y^2} + 2\beta \left(\frac{\mathrm{d}u}{\mathrm{d}y} \right)^3 - V_0 \right) \mathrm{d}y = 0, \tag{33}$$

where Ω is the domain of the problem, ϕ is arbitrary test functions which can be considered as the variation of u. We have chosen a linear basis function for the solution of this equation which uses a two node line element. We assume the finite element solution over this element is of the form,

$$\mathbf{u} = \sum_{i=1}^{2} u_i \psi_i. \tag{34}$$

The Galerkin weighted residual method is applied for the solution of original equation. In the Galerkin approach the weight function for an element is chosen as

$$\phi = \psi_i \quad (i = 1, 2), \tag{35}$$

where ψ_i are the basis functions for a typical element (y_i, y_{i+1}) and they are defined as

$$\psi_1 = \frac{y_{i+1} - y}{y_{i+1} - y_i}, \qquad \psi_2 = \frac{y - y_i}{y_{i+1} - y_i}; \quad y_i \le y \le y_{i+1}. \tag{36}$$

As Eq. (33) is non linear in velocity the finite element formulation of problem which can be obtained by putting Eqs. (34) and (35) in Eq. (33) will also generate a nonlinear set of algebraic equations. This nonlinear set of algebraic equations can be written in matrix form as

$$\mathbf{F}(\mathbf{u}) = \begin{bmatrix} f_1(\mathbf{u}) \\ f_2(\mathbf{u}) \\ . \\ . \\ f_n(\mathbf{u}) \end{bmatrix} = \mathbf{0}, \tag{37}$$

where n is the total number of unknown solution values and $f_i(\mathbf{u})$ are obtained by putting in the value of $\mathbf{u}$ from Eq. (34) for each element in Eq. (33).

The domain of the problem is divided into a set of 200 elements of equal length. We know the velocity values at the boundary of the domain therefore after incorporating these boundary conditions we obtain a set of 199 simultaneous non linear algebraic equations having 199 unknowns. For the solution of this set of equations we used Newton's iterative method with an initial guess provided to it as

$$u(y) = -(1 - u(0))\mathrm{e}^{-yVo}. \tag{38}$$

Newton's iterative method is generally implemented as a two step procedure. First a vector $\mathbf{z}$ is found which will satisfy

$$J(\mathbf{u}^{(k)})\mathbf{z} = -\mathbf{F}(\mathbf{u}^{(k)}), \tag{39}$$

where $J(\mathbf{u}^{(k)})$ is the Jacobian of $\mathbf{F}(\mathbf{u}^{(k)})$. After this has been carried out the new approximation $\mathbf{u}^{(k+1)}$ can be obtained by

$$\mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \mathbf{z}. \tag{40}$$

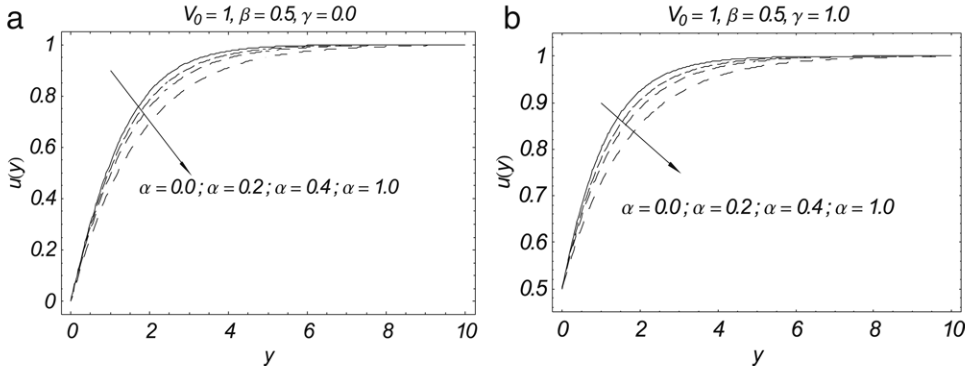

Fig. 1. Influence of second grade parameter α on u (a) no-slip boundary condition (b) partial slip boundary condition.

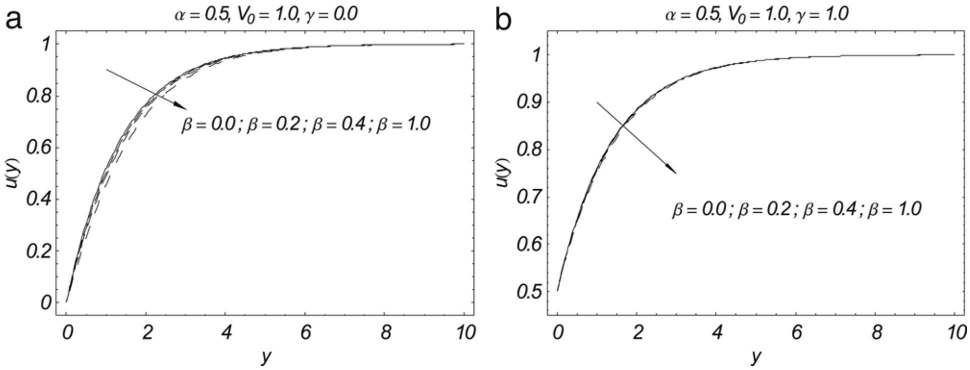

Fig. 2. Influence of third grade parameter β on u (a) no-slip boundary condition (b) partial slip boundary condition.

Newton's method is expected to give quadratic convergence, provided that a sufficiently good starting guess is given.

4. Results and discussion

The graphs for the function $u(y)$ are drawn against y for different values of the parameters α, β, γ and V_0 in both the cases by taking no-slip and partial slip at the boundary. In Fig. 1 the velocity is plotted for different values of the fluid parameter α. It is observed that by increasing α velocity decreases and boundary layer thickness decreases for both no-slip and partial slip cases. It is seen that the partial slip parameter does not effect the behavior of the second grade parameter much, it only increases the starting velocity from 0 to 0.5 as it is obvious from the boundary condition (29). The effects of the third grade parameter on the velocity is shown in Fig. 2. The influence of β on velocity is similar to that of α, but in this case introduction of partial slip reduces the decrease in velocity. The effect of suction velocity V_0 on the velocity u is displayed in Fig. 3. These Figs. show that velocity increases and boundary layer thickness decreases by an increase in the suction velocity. Hence one can use the suction mechanism to control the boundary layer thickness. The influence of slip parameter γ on velocity is presented in Fig. 4. Fig. 4 shows that the velocity increases and boundary layer thickness decreases by an increase in the slip parameter. The introduced slip at the boundary enhances the effects of suction at the plate. In Fig. 5 a comparison of the exact solution obtained in reference [21] for the second grade fluid is presented and it is found that our numerical solution is in an excellent agreement with that of the exact solution.

5. Concluding remarks

In this paper, we found a numerical solution using the finite element method corresponding to the flow of a third grade fluid over a porous plate with partial slip. The results are shown graphically and the influence of the pertinent

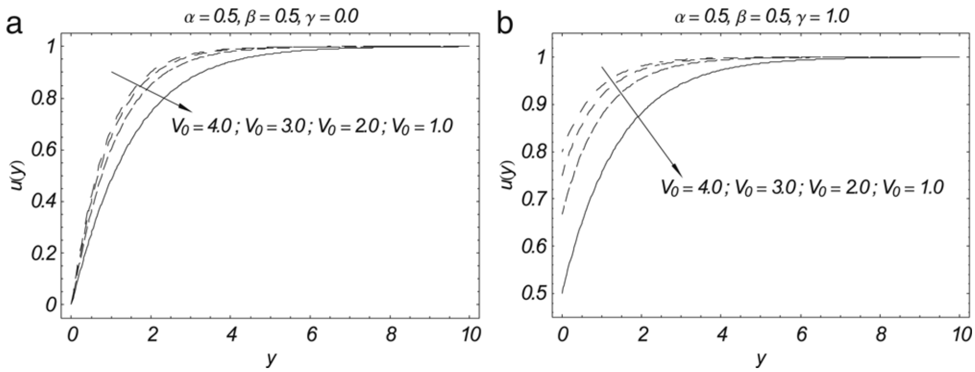

Fig. 3. Influence of suction velocity V_0 on u (a) no-slip boundary condition (b) partial slip boundary condition.

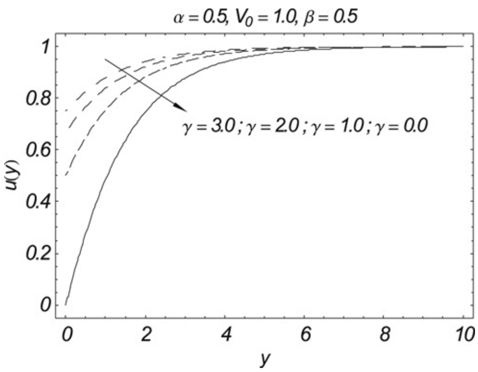

Fig. 4. Influence of slip parameter γ on u.

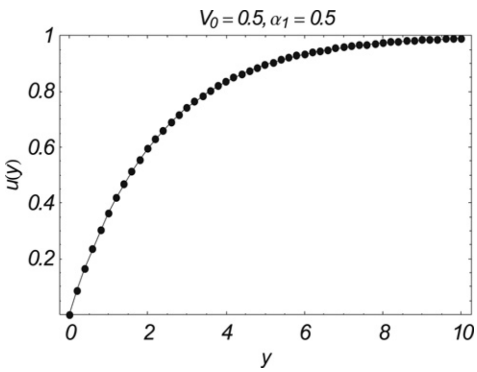

Fig. 5. Comparison between the exact solution [21] (bullets) with the present numerical solution (solid line) for a second grade fluid.

parameters of interest is discussed. It is observed that the suction velocity V_0 and slip parameter γ can be used to control the boundary layer thickness. Finally a comparison is given between exact and numerical solutions for the second grade fluid.

References

[1] K.R. Rajagopal, A note on unsteady unidirectional flows of a non-Newtonian fluid, Int. J. Non-linear Mech. 17 (1982) 369–373.
[2] K.R Rajagopal, On the creeping flow of second order fluid, J. Non-Newtonian Fluid Mech. 15 (1984) 239–246.
[3] K.R. Rajagopal, Longitudinal and torsional oscillations of a rod in a non-Newtonian fluid, Acta Mech. 49 (1983) 281–285.
[4] A.M. Siddiqui, T. Hayat, S. Asghar, Periodic flows of a non-Newtonian fluid between two parallel plates, Int. J. Non-linear Mech. 34 (1999) 895–899.

[5] P.N. Kaloni, A.M. Siddiqui, A note on the flow of a viscoelastic fluid between eccentric disks, J. Non-Newtonian Fluid Mech. 26 (1987) 125–133.
[6] T. Hayat, S. Asghar, A.M. Siddiqui, Periodic unsteady flows of a non-Newtonian fluid, Acta Mech. 131 (1998) 169–175.
[7] T. Hayat, S. Asghar, A.M. Siddiqui, On the moment of a plane disk in a non-Newtonian fluid, Acta Mech. 136 (1999) 125–131.
[8] T. Hayat, S. Nadeem, S. Asghar, A.M. Siddiqui, Fluctuating flow of a third grade fluid on a porous plate in a rotating medium, Int. J. Non-linear Mech. 36 (2001) 901–916.
[9] T. Hayat, S. Asghar, A.M. Siddiqui, Some nonsteady flows of a non-Newtonian fluid, Int. J. Engng. Sci. 38 (2000) 337–346.
[10] C. Fetecau, C. Fetecau, A new exact solution for the flow of a Maxwell fluid past an infinite plate, Int. J. Non-linear Mech. 38 (2003) 423–427.
[11] C. Fetecau, C. Fetecau, Decay of a potential vortex in an Oldroyd-B fluid, Int. J. Non-linear Mech. 43 (2005) 340–351.
[12] F. Franchi, B. Straughan, Convection, stability and uniqueness for a fluid of third grade, Int. J. Non-linear Mech. 23 (1988) 377–384.
[13] F. Franchi, B. Straughan, Stability and nonexistence results in the generalized theory of a fluid of second grade, J. Math. Anal. Appl. 180 (1993) 122–137.
[14] B. Straughan, Energy stability in the Benard problem for a fluid of second grade, ZAMP 34 (1983) 502–509.
[15] B. Straughan, The backward in time problem for a fluid of third grade, Ric. Mat. 36 (1987) 289–293.
[16] B. Straughan, Explosive Instabilities in Mechanics, Springer, 1998.
[17] K.R. Rajagopal, On the boundary conditions for fluids of the differential type, in: A. Sequira (Ed.), Navier–Stokes Equation and Related Non-linear Problems, Plenum Press, New York, 1995, pp. 273–278.
[18] K.R. Rajagopal, Boundedness and uniqueness of fluids of the differential type, Acta Cienc. Indica 18 (1982) 1–11.
[19] K.R. Rajagopal, A.Z. Szeri, W. Troy, An existence theorem for the flow of a non-Newtonian fluid past an infinite porous plate, Int. J. Non-linear Mech. 21 (1986) 279–289.
[20] K.R. Rajagopal, P.N. Kaloni, Some remarks on boundary conditions for fluids of differential type, in: G.A.C. Graham, S.K. Malik (Eds.), Continuum Mechanics and Applications, Hemishpere, New York, 1989, pp. 935–942.
[21] K.R. Rajagopal, A.S. Gupta, An exact solution for the flow of a non-Newtonian fluid past an infinite plate, Meccanica 19 (1984) 158–160.
[22] I.J. Rao, K.R. Rajagopal, The effect of the slip condition on the flow of fluids in a channel, Acta Mech. 135 (1999) 113–126.
[23] C.L. Roux, Existence and uniqueness of the flow of second grade fluids with slip boundary conditions, Arch. Ration. Mech. Anal. 148 (1999) 309–356.
[24] T. Hayat, M. Khan, M. Ayub, The effect of the slip condition on flows of an Oldroyd 6-constant fluid, J. Comp. Appl. Math. 202 (2007) 402–413.
[25] C. Truesdell, W. Noll, The Non-linear Field Theories of Mechanics, 2nd edition, Springer, 1992.
[26] R.S. Rivlin, J.L. Ericksen, Stress deformation relation for isotropic materials, J. Ration. Mech. Anal. 4 (1995) 323–425.
[27] R.L. Fosdick, K.R. Rajagopal, Thermodynamics and stability of fluids of third grade, Proc. R. Soc. Lond. Ser. A 339 (1980) 351–377.