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Small-x evolution of jet quenching parameter

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ABSTRACT

Concept of transverse deflection probability of a parton that travels through strongly interacting medium, recently introduced by D'Eramo, Liu and Rajagopal, has been used to derive high energy evolution equation for the jet quenching parameter in stochastic multiple scatterings regime. Jet quenching parameter, $\hat{q}(x)$, appears to evolve with x, with an exponent $0.9\bar{\alpha}_s$, which is slightly less than that of $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function.

1. Introduction

The heavy-ion collision programs at CERN's Large Hadron Collider have opened new access for exploration of extreme hot and dense nuclear matter. Precision tomography of the nuclear matter is now becoming feasible as an accurate test of the underlying quantum chromodynamic (QCD) theory. This is instrumental in discovering yet unexplored characteristics of various nuclear effects and collective phenomena that the nuclear matter may possess. One possibility is to explore the QCD scale/energy evolution of various observables in this extreme ambiance. Advancement of study for hard sector observables at the LHC elevated the medium modification of high energy jets as prevailing topic of investigation. In this context study on scale/energy evolution of the jet quenching parameter, attributed as the stopping power of the medium for a certain probe of the medium, is now viable. High energy quarks and gluons passing through the interacting nuclear matter have their transverse momentum distribution broadened due to multiple scatterings with the constituents of the medium. While travelling through the strongly coupled medium the hard parton loses energy as well as its direction of momentum changes. Change in the direction of momentum is referred to as 'transverse momentum broadening' for the travelling parton. In the context of jet-medium interaction the broadening refers to the effect on the jet when the direction of the momenta of an ensemble of partons changes due to the random kicks. Even though there is no apparent change in mean momenta, the spread of the momentum distribution of individual parton within that ensemble broadens.

Evolution of the momentum broadening was first studied by Liou, Mueller and Wu [3] by introducing radiative modification over the leading order momentum broadening effect. The authors showed that average momentum broadening $\langle p_\perp^2 \rangle$ has both double and single logarithmic terms. Both the double logarithmic terms, $\ln^2(L/l_0)$, and single logarithmic terms, $\ln(L/l_0)$ are coming from gluon radiation induced by the medium interactions. Here the length of the nuclear matter is L and l_0 is the size of constituents of the matter. Their estimation showed that the radiative contribution is to be a sizeable correction to the nonradiative leading value of $\langle p_\perp^2 \rangle$. Later an evolution equation has been obtained for the inclusive one-gluon distribution, through the concepts of classical branching process and cascade of partons [4]. This explicitly takes into account the dependence of the observed gluon spectrum upon the energy and the transverse momentum. The explicit transverse momentum dependence of the splitting kernel then enables one to identify large corrections to the jet quenching parameter. Subsequent studies on non-linear evolution lead to prescribe the renormalization of the jet-quenching parameter [5,6].

In this paper, in order to study the energy evolution of jet quenching parameter, we have adopted the idea of transverse deflection probability of a parton, that travels through the nuclear medium. Following a recent work by D'Eramo, Liu and Rajagopal [1] we then relate the momentum broadening to the S-matrix of the nuclear interaction for a dipole. The Balitsky–Kovchegov equation as the evolution equation of the S-matrix is then used to derive high energy evolution equation for the jet quenching parameter in stochastic multiple scatterings regime. The known result of double log enhancement emerges as a special case in the limit when the single scattering is only contributing. Power-counting techniques borrowed from Soft-Collinear-Effective-Theory (SCET) [17–20] have been used to identify the leading contributions in the stochastic multiple scatterings region. For an almost constant $\hat{q}(\omega)$ we recovered the double log result (in the limit $Q_s^2 \to \hat{q}L$) first derived in [3] and subsequent other studies [4–6].

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We have also shown that the double log enhancement gets diluted when we go beyond single scattering limit as previously argued in [3]. Jet quenching parameter $\hat{q}(x)$ is found to evolve with x, slightly weaker than $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function.

2. Probability density for transverse deflection $P(k_\perp)$

The transverse momentum broadening of a parton can be studied by introducing concept of a probability density, denoted in this article as $P(k_\perp)$. It signifies the probabilistic weight for the event where, after travelling a medium of length L, amount of transverse momentum that the parton acquired is $k_\perp$ [1]. The probability density $P(k_\perp)$ is chosen to be normalized in the following way,

$$\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = \frac{1}{4\pi} \int dk_\perp^2 \; P(k_\perp) = 1. \tag{1}$$

Using this probability density one can quickly estimate mean transverse momentum picked up by the hard parton per unit distance travelled,

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{4\pi L} \int dk_\perp^2 \; k_\perp^2 \; P(k_\perp). \tag{2}$$

This defines the jet broadening (or quenching) parameter $\hat{q}$ that may have intrinsic dependence on x and Q of the probe through $P(k_\perp)$.

Its important to make a certain caveat when defining the jet quenching parameter, $\hat{q}$, as done in Eq. (2). In absence of any finite upper bound to the integral, the right-hand side of Eq. (2) could be diverging, e.g., when $P(k_\perp)$ behaves as power law, $P(k_\perp) \sim 1/k_\perp^4$ in the event of lowest order perturbative gluon production at large-$k_\perp$. This gives a logarithmic UV divergence, resulting both average transverse momentum $\langle k_\perp^2 \rangle$ as well as jet quenching parameter, $\hat{q}$, infinite. Although, in the case of multiple scattering with Sudakov like form factor or in the event of multiple stochastic scatterings where $P(k_\perp)$ takes the form of an exponentially damping factor, right hand side of Eq. (2) should be finite even without any apparent upper bound in the integral.

In this paper we will discuss the event of multiple stochastic scatterings, where the probe receives random transverse kicks and $P(k_\perp)$ takes the form of a Gaussian with a variance of $\hat{q}L/2$,

$$P(k_\perp) = \frac{4\pi}{\hat{q}L} \; \exp\left(-\frac{k_\perp^2}{\hat{q}L} \right). \tag{3}$$

In this study we are looking for transverse momentum broadening of the hard parton that has initial light cone momentum $p(p^+, p^-, p_\perp) \sim Q\,(0, 1, 0)$ and enters in a brick of strongly interacting medium of length L. In order to do the power counting, we have introduced the dimensionless small parameter λ. Power corrections in λ to some hard process are generally suppressed in the presence of a hard scale, $Q^2 \gg \Lambda_{QCD}^2$, which act as base for the power corrections. This is a concept borrowed from the soft collinear effective theory (SCET) studies [17–20]. In SCET studies the power counting protocol is as follows, terms that are subleading in two orders of magnitude can be dropped but the terms that are suppressed by one order of magnitude should be kept. While travelling through nuclear medium the hard partons interact repeatedly with the Glauber gluons which scale as $Q\,(\lambda^2, \lambda^2, \lambda)$. After a first few scatterings the hard parton's momentum becomes of order $Q\,(\lambda^2, 1, \lambda)$, though still quite collinear. In this scenario of high energy regime where Glauber gluons are mostly effective, it has been shown in Ref. [1] that $P(k_\perp)$ can be expressed as the Fourier transform in $r_\perp$ of the expectation value of two light-like path-ordered Wilson lines transversely separated by $r_\perp$,

$$P(k_\perp) = \int d^2 r_\perp \, e^{-ik_\perp r_\perp} \, \mathcal{S}(r_\perp), \tag{4}$$

where,

$$\mathcal{S}(r_\perp) \equiv \frac{1}{N_c} \langle \, \mathrm{Tr}\,[\mathcal{W}\,(0, r_\perp)\, \mathcal{W}\,(0, 0)] \, \rangle, \tag{5}$$

with,

$$\mathcal{W}(y^+, y_\perp) \equiv \mathcal{P}\left\{ \exp\left[ig \int_0^{L^-} dy^- A^+\left(y^+, y^-, y_\perp\right) \right] \right\}. \tag{6}$$

Within a dipole picture transverse separation $y_\perp - y'_\perp$ can be taken as the transverse size of the dipole $r_\perp$. The length of the medium is $L = L^-/\sqrt{2}$. In this work we have assumed that $P(k_\perp)$ as the Fourier transform of $\mathcal{S}(r_\perp)$ is well valid while in high energy regime i.e. at small-x and all the evolution characteristics for $P(k_\perp)$ exclusively contained in $\mathcal{S}(r_\perp)$, so that we may write,

$$P(k_\perp, Y) = \int d^2 r_\perp \, e^{-ik_\perp r_\perp} \, \mathcal{S}(r_\perp, Y), \tag{7}$$

and,

$$\frac{\partial P(k_\perp, Y)}{\partial Y} = \int d^2 r_\perp \, e^{-ik_\perp r_\perp} \, \frac{\partial \mathcal{S}(k_\perp, Y)}{\partial Y}, \tag{8}$$

and therefore from Eq. (2),

$$\frac{\partial \hat{q}(Y)}{\partial Y} = \frac{1}{4\pi L} \int dk_\perp^2 \, k_\perp^2 \int d^2 r_\perp \, e^{-ik_\perp r_\perp} \, \frac{\partial \mathcal{S}(r_\perp, Y)}{\partial Y} = \hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[\frac{\partial \mathcal{S}(r_\perp, Y)}{\partial Y} \right]. \tag{9}$$

Where for brevity in notations we have introduced $\hat{\mathcal{F}}$,

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]}[\mathcal{O}] \equiv \frac{1}{4\pi L} \int dk_\perp^2 \, k_\perp^2 \int d^2 r_\perp \, e^{-ik_\perp r_\perp} \, \mathcal{O}. \tag{10}$$

It can also be shown that,

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[r_\perp^2 \exp\left(-\frac{\hat{q}}{4} L r_\perp^2 \right) \right] = -\frac{4}{L}, \tag{11}$$

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]} \left[r_\perp^4 \exp\left(-\frac{\hat{q}}{4} L r_\perp^2 \right) \right] = 0. \tag{12}$$

Now, the non-linear evolution of the S-matrix, $\mathcal{S}(r_\perp = y_\perp - y'_\perp)$, is governed by the Balitsky–Kovchegov equation (BK) [2,7–9], in large-N_c limit as,

$$\frac{\partial \mathcal{S}(y_\perp, y'_\perp; Y)}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2} \int d^2 z_\perp \frac{(y_\perp - y'_\perp)^2}{(y_\perp - z_\perp)^2 (z_\perp - y'_\perp)^2} \left[\mathcal{S}(y_\perp, y'_\perp; Y) - \mathcal{S}(y_\perp, z_\perp; Y) \mathcal{S}(z_\perp, y'_\perp, Y) \right]. \tag{13}$$

Using Eq. (9) and Eq. (13) with $r_\perp = y_\perp - y'_\perp$, together with fact that the medium is transnationally invariant, evolution equation for $\hat{q}(Y)$, can now be written as,

$$\frac{\partial \hat{q}(Y)}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2} \hat{\mathcal{F}}_{[k_\perp, r_\perp]}[\mathcal{M}(r_\perp)], \tag{14}$$

where $\mathcal{M}(r_\perp)$ is an integral over the daughter dipoles' transverse coordinates,

$$\mathcal{M}(r_\perp) = \int d^2 B_\perp \frac{r_\perp^2}{(r_\perp - B_\perp)^2 B_\perp^2} \left[\mathcal{S}(r_\perp, Y) - \mathcal{S}(r_\perp - B_\perp, Y) \mathcal{S}(B_\perp, Y) \right] \tag{15}$$

The Balitsky–Kovchegov (BK) equation results from summing up long-lived soft gluon emissions, off almost onshell hard quark, where the gluon lifetimes usually much longer than the size of the target, which is taken to be a brick of size L here. The integral in Eq. (6) should therefore run from $-\infty$ to $+\infty$, which means that the quark, scattering on the brick, is created at time, $\tau_i = -\infty$, flies through semi-infinite empty space almost onshell while developing the gluon cascade giving the evolution of Eq. (13), scatters on the brick, and flies off until $\tau_f = +\infty$ again developing the cascade [11]. Such a physical picture certainly does not apply in the context of jet quenching in heavy ion collisions where quark jets, are produced in the actual collision (at $\tau = 0$), with a high virtuality Q and quickly losses the virtuality by ordered emission through (medium modified) DGLAP [12–14] evolution and have no time to develop a gluon cascade accordance with Eq. (13) before interacting with the plasma. In the present discussion we therefore presume $\hat{q}$ as momentum broadening long after the quark leaves the brick when it again becomes almost onshell.

In the event when $Y \sim 0$ the jet quenching parameter $\hat{q}_0$ and the saturation momentum Q_{s0} are approximately related as $\hat{q}_0 \sim 4Q_{s0}^2/L$. In the rest of the article we will evaluate Eq. (14) inside or around saturation region which is the region of interest relevant in the context of transverse momentum broadening and energy loss phenomenology in relativistic heavy-ion collision studies.

3. Evolution of $\hat{q}$ in stochastic multiple scatterings regime

Kinematic domains where in-medium interactions can be approximated by stochastic multiple scatterings, $\mathcal{S}(r_\perp)$ appears to be a Gaussian in $r_\perp$ with variance $2/\hat{q}L$ [1],

$$\mathcal{S}[r_\perp] = \exp\left[-\frac{\hat{q}}{4}Lr_\perp^2\right]. \tag{16}$$

Above form of $\mathcal{S}(r_\perp)$ in terms of $\hat{q}L$ and $r_\perp^2$ is limited, if not unique, in the sense that its Fourier transform leaves again a Gaussian form of $P(k_\perp)$ as given in Eq. (3) and also returns $\hat{q}$ once $\hat{\mathcal{F}}$ acts over it,

$$\hat{\mathcal{F}}_{[k_\perp, r_\perp]}\left[\exp\left(-\frac{\hat{q}}{4}Lr_\perp^2\right)\right] = \hat{q}. \tag{17}$$

The imaginary part of the forward scattering amplitude $\mathcal{N}(r_\perp)$ in the Glauber–Gribov–Mueller (GGM) multi-rescattering model [15] also has a similar form,

$$\mathcal{N}(r_\perp) = 1 - \exp\left\{-\frac{\alpha_s \pi^2}{2N_c}T(b_\perp)xG_N\left(x, \frac{1}{r_\perp^2}\right) r_\perp^2\right\}, \tag{18}$$

where imaginary part of the forward scattering amplitude $\mathcal{N}$ is related with S-matrix element, $\mathcal{S}(r_\perp)$ as,

$$\mathcal{N}(r_\perp) = 1 - \mathcal{S}(r_\perp). \tag{19}$$

As long as one is not deep inside the saturation region Eq. (18) provides the correct form of saturation scale with the identification [10],

$$Q_s^2(b_\perp) = \frac{\alpha_s \pi^2}{2N_c}T(b_\perp)xG_N\left(x, \frac{1}{r_\perp^2}\right) \sim \frac{1}{4}\hat{q}L. \tag{20}$$

In order to calculate $\hat{q}$ at strong coupling via AdS/CFT correspondence, form of $\mathcal{S}$ as given in Eq. (16) was even taken to define nonperturbative, quantum field theoretic definition of jet quenching parameter: $\hat{q}$ is the coefficient of $Lr_\perp^2/4$ in $\log\mathcal{S}$ for small $r_\perp$ [16]. In this paper we have assumed form of all $\mathcal{S}$ in Eq. (15) as Gaussian in dipoles' transverse size, with a variance of $2/\hat{q}L$, retains while it goes through high-energy evolution, with the caveat that deep inside the saturation region this may not true. We therefore take S-matrices in Eq. (15) as,

$$\mathcal{S}(r_\perp, Y) = \exp\left[-\frac{\hat{q}(Y)}{4}Lr_\perp^2\right] \tag{21}$$

$$\mathcal{S}(r_\perp - B_\perp, Y) = \exp\left[-\frac{\hat{q}(Y)}{4}L(r_\perp - B_\perp)^2\right] \tag{22}$$

$$\mathcal{S}(B_\perp, Y) = \exp\left[-\frac{\hat{q}(Y)}{4}LB_\perp^2\right]. \tag{23}$$

Its important to recall that in this region of interest just inside the saturation region (and multiple stochastic scatterings works as well) S-matrices are small, $\mathcal{S} \ll 1$ and transverse width of daughter dipoles are large $B_\perp, B_\perp - r_\perp \geq 1/Q_s$. Once we replace Eq. (21)–(23) in Eq. (15), $\mathcal{M}(r_\perp)$ becomes,

$$\mathcal{M}(r_\perp) = \exp\left(-\frac{\hat{q}(Y)}{4}Lr_\perp^2\right)\int d^2B_\perp \frac{r_\perp^2}{(r_\perp - B_\perp)^2 B_\perp^2} \left[1 - e^{-\frac{1}{2}\hat{q}(Y)L(B_\perp - r_\perp)B_\perp}\right]. \tag{24}$$

However, the Gaussian assumptions of Eq. (21)–Eq. (23) may not correct at low-x both at small and large $r_\perp$. Small-x evolution leads to anomalous dimension modifying the power of $r_\perp$ from $r_\perp^2$. This means that $\hat{q}$ in Eq. (21)–Eq. (23) is not a function of Y, but more appropriately is the initial condition of $\hat{q}(Y)$ at $Y = 0$. Therefore, Eq. (24) should be understood as one step of BK evolution, where daughter dipoles interact through Glauber gluons are approximated by Gaussians in Eq. (21)–Eq. (23). The subsequent evolution equations should then be understood as a fairly crude approximation of the actual evolution.

Now, around the saturation line, the dipole size $r_\perp \sim 1/Q_{s0}$, and $B_\perp, (B_\perp - r_\perp) \gtrsim 1/Q_s(Y)$ (however they are still well below $1/\Lambda_{QCD}$). We will further assume that transverse size of the daughter dipole ($\sim$ inverse of the gluon's transverse momentum) is much larger than the transverse size of the parent dipole i.e. $B_\perp, (B_\perp - r_\perp) \gg r_\perp$. With appropriate limits in the integral, Eq. (24) then can be written as,

$$\mathcal{M}(r_\perp) = \exp\left(-\frac{\hat{q}(Y)}{4}Lr_\perp^2\right) \pi\, r_\perp^2 \int_{1/Q_s^2(Y)}^{1/\Lambda_{QCD}^2} \frac{dB_\perp^2}{B_\perp^4}\left[1 - e^{-\frac{1}{2}\hat{q}(Y)LB_\perp^2}\right]. \tag{25}$$

Integral in Eq. (25) is so converging that the upper limit may be taken at infinity instead of $1/\Lambda_{QCD}^2$, leaving the integral I_{dip} as,

$$I_{dip} \equiv \int_{1/Q_s^2(Y)}^{\infty} \frac{dB_\perp^2}{B_\perp^4}\left[1 - e^{-\frac{1}{2}\hat{q}(Y)LB_\perp^2}\right] = Q_s^2(Y)\left\{1 - \exp\left(-\frac{1}{2}\frac{\hat{q}(Y)L}{Q_s^2(Y)}\right) - \frac{1}{2}\frac{\hat{q}(Y)L}{Q_s^2(Y)}\mathrm{Er}\left(-\frac{1}{2}\frac{\hat{q}L}{Q_s^2(Y)}\right)\right\}, \tag{26}$$

where 'Er' is the Exponential Integral function. Using Eq. (26) one can now derive the evolution equation of the jet quenching parameter in stochastic multiple scatterings regime just around saturation line as,

$$\frac{\partial\hat{q}(Y)}{\partial Y} = \frac{\alpha_s N_c}{\pi}\,\frac{2Q_s^2(Y)}{L}\left[1-\exp\left(-\frac{\hat{q}(Y)L}{2Q_s^2(Y)}\right) -\frac{\hat{q}(Y)L}{2Q_s^2(Y)}\,\mathrm{Ei}\left(-\frac{\hat{q}(Y)L}{2Q_s^2(Y)}\right)\right]. \tag{27}$$

Eq. (27) can be written in terms of incomplete gamma function $\Gamma(n,a)$,

$$\begin{aligned}\frac{\partial\ln\hat{q}(Y)}{\partial Y} &= \bar{\alpha}_s\left[\frac{1}{2\mathcal{E}}\left(1-e^{-2\mathcal{E}}\right)+\Gamma(0,2\mathcal{E})\right]\\ &= \bar{\alpha}_s\left[\ln 2+\gamma_E-\ln\frac{1}{\mathcal{E}}+\frac{1}{2\mathcal{E}}\left(1-e^{-2\mathcal{E}}\right)+\sum_{k=1}^{\infty}\frac{(-\mathcal{E})^k}{k(k!)}\right]\end{aligned} \tag{28}$$

where,

$$\mathcal{E} = \frac{1}{4}\frac{\hat{q}(Y)L}{Q_s^2(Y)}, \tag{29}$$

with $\bar{\alpha}_s = \alpha_s N_c/\pi$. Around $\mathcal{E}\sim 1$, solution of Eq. (28) can be approximately estimated as,

$$\hat{q}(x) \propto \left(\frac{1}{x}\right)^{0.9\bar{\alpha}_s}. \tag{30}$$

Jet quenching parameter $\hat{q}(x)$ therefore evolves with x, with an exponent 0.9 $\bar{\alpha}_s$, which is just slightly less than that of $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function.

3.1. *Special case: double log enhancement*

For smaller dipole with $r_\perp \ll 1/Q_s$ the imaginary part of forward scattering amplitude $\mathcal{N}$ is small, and saturation and unitarity bound effects are not that very important. Principle of color transparency ensures that the forward scattering amplitude $\mathcal{N}$ approaches to zero (S-matrix, $\mathcal{S}$, goes to one). This allows us to take that $\mathcal{N}\ll 1$ for small dipole size even when small-x evolution is included. For $\mathcal{N}\ll 1$ we can linearise Eq. (24) as,

$$\mathcal{M}(r_\perp) = \frac{\hat{q}(Y)}{2}L\exp\left(-\frac{\hat{q}(Y)}{4}Lr_\perp^2\right)\int d^2B_\perp\frac{r_\perp^2}{(r_\perp-B_\perp)^2B_\perp^2}\left[(B_\perp-r_\perp)B_\perp\right]. \tag{31}$$

In the limit of $r_\perp \ll B_\perp$, Eq. (31) can be further simplified,

$$\mathcal{M}(r_\perp) = \pi\frac{\hat{q}(Y)}{2}Lr_\perp^2\exp\left(-\frac{\hat{q}(Y)}{4}Lr_\perp^2\right)\int\frac{dB_\perp^2}{B_\perp^4}B_\perp^2. \tag{32}$$

In a dipole system when a single real gluon emits, the gluon can be characterized by its energy, ω, and transverse size, $B_\perp$, that it makes with the (anti)-quark legs of the parent dipole having transverse size $r_\perp$. When $B_\perp \gg r_\perp$ the probability of emitting a gluon with given energy ω goes as $r_\perp^2 dB_\perp^2/B_\perp^4$. Despite being non logarithmic and converging, the integral becomes logarithmically diverging once the single scattering in included which gives an additional $B_\perp^2$ in the numerator [3]. This is evident from Eq. (32) which together with the logarithm coming from energy ω integral gives double logarithmic contribution to the modification in transverse momentum broadening and so as for the jet quenching parameter. Substituting Eq. (17) and Eq. (32) in Eq. (14) gives the Y evolution of jet quenching parameter,

$$\frac{\partial\hat{q}(Y)}{\partial Y} = \frac{\alpha_s N_c}{\pi}\int\frac{dB_\perp^2}{B_\perp^2}. \tag{33}$$

Integral Eq. (33) can be compared with Eq. (26) of [3] and Eq. (4.35) of [5]. This double log enhancement, first derived in [3] and subsequent later studies [4–6], need to be supplemented with appropriate kinematic limits of the integral both for $B_\perp$ and ω.

In order to fix the kinematic limits for the energy w and $B_\perp$ here we follow the arguments made in [3] to isolate the region of double log enhancement. Two conditions have to be satisfied in order to achieve the double log enhancement. Firstly, the inverse transverse size of the daughter dipole $1/B_\perp^2$ ($\sim$ transverse momentum of the emitted gluon) needs to be just below saturation momentum Q_s but sufficient large that the multiple scattering is not important and one can linearise Eq. (15) with confidence. Secondly lifetime of the fluctuations ($\sim$ formation time of the emitted gluon) $\tau\sim\omega B_\perp^2$ has to be greater than nucleon size l_0, and less than the length of the medium L. Inverse transverse size of the daughter dipole is however well above the saturation scale Q_s which ensures $r_\perp^2 \ll B_\perp^2$.

$$r_\perp^2 \ll \frac{1}{Q_s^2} \le B_\perp^2 \le \frac{1}{\hat{q}l_0} \ll \frac{1}{\Lambda_{QCD}^2} \tag{34}$$

$$l_0 \le \omega B_\perp^2 \le L \tag{35}$$

Eq. (35), together with the condition that $B_\perp$ have to be sufficiently small so that one can linearise Eq. (15), $\hat{q}LB_\perp^2 \le 1$, implies,

$$\frac{l_0}{B_\perp^2} \le \omega \le \frac{1}{\hat{q}B_\perp^4}. \tag{36}$$

As $\partial/\partial Y \equiv \omega\partial/\partial\omega$ double logarithmic enhancement of jet quenching parameter in high energy would be,

$$\begin{aligned}\Delta\hat{q} &= \frac{\alpha_s N_c}{\pi}\int_{l_0/B_\perp^2}^{1/\hat{q}(\omega)B_\perp^4}\frac{d\omega}{\omega}\,\hat{q}(\omega)\int_{1/Q_s^2}^{1/\hat{q}(\omega)l_0}\frac{dB_\perp^2}{B_\perp^2}\\ &= \frac{\bar{\alpha}_s}{2}\,\hat{q}(0)\,\log^2\frac{Q_s^2}{\hat{q}l_0},\end{aligned} \tag{37}$$

where ω integration has been performed before the $B_\perp$ integration. For an almost constant $\hat{q}(\omega)$ we recovered in Eq. (37) the double log result (in the limit $Q_s^2\to\hat{q}L$) first derived in [3] and subsequent other studies [4–6]. This double log enhancement however diluted by multiple scattering effects [3] as evident from Eq. (28).

4. Summary and outlook

There are ongoing phenomenological efforts to extract values for the jet transport parameter $\hat{q}$ at various central heavy-ion collisions done at various energies for prevailing energy loss models. Here parameters for the medium properties are constrained by experimental data on the nuclear modification factor R_{AA} [21]. Following a recent work by D'Eramo, Liu and Rajagopal [1] we have introduced the concept of transverse deflection probability of a parton, that travels through strongly interacting medium, and derived high energy evolution equation for the jet quenching parameter in stochastic multiple scatterings regime, which is the region of interest in the context of jet quenching phenomenology of the heavy-ion collider experiments. The Balitsky–Kovchegov (BK) equation, as the evolution equation of the S-matrix, is used to derive high energy evolution equation for the jet quenching parameter. We have shown that $\hat{q}(x)$ evolves with small x, with an exponent $\sim 0.9\ \bar{\alpha}_s$, which is just slightly less than that of $x\mathcal{G}(x)$ where $\mathcal{G}(x)$ is the gluon distribution function. The known result of double log enhancement emerges as a special case in the limit when the single scattering is only contributing.

In this article Eq. (7)–Eq. (10) constitute a complete set of equations needed to determine the energy dependence of $\hat{q}$. To date there is no complete exact analytical solution of BK equation. Although there are several approximate analytical solutions, along with some numerical solutions. One can therefore use the approximate analytical solutions, e.g., solution outside the saturation region which exhibits extended geometric scaling or solution inside the saturation region, the Levin–Tuchin solution [22], that exhibits complete geometric scaling [23] to calculate the energy evolution of jet quenching parameter. As an obvious consequence of the scaling, one could end up finding energy-dependence is proportional to saturation momentum, $Q_s(Y)$, i.e., $\hat{q}(Y) \sim Q_s^2(Y)$. The result could therefore modify at deep inside the saturation regions and also in the event when the LPM effect modifies the spectrum strongly. Both the issues which may farther squeeze the evolution of jet quenching parameter, will be explore in a future effort.

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