



Flux compactifications, gauge algebras and De Sitter

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ABSTRACT

The introduction of (non-)geometric fluxes allows for $\mathcal{N} = 1$ moduli stabilisation in a De Sitter vacuum. The aim of this Letter is to assess to what extent this is true in $\mathcal{N} = 4$ compactifications. First we identify the correct gauge algebra in terms of gauge and (non-)geometric fluxes. We then show that this algebra does not lead to any of the known gaugings with De Sitter solutions. In particular, the gaugings that one obtains from flux compactifications involve non-semi-simple algebras, while the known gaugings with De Sitter solutions consist of direct products of (semi-)simple algebras.

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1. Introduction

Over the years superstring compactifications have been investigated from many different perspectives. The possibility of including various types of fluxes allows for different effective descriptions of four-dimensional physics (see e.g. [1–3]). The effective theories that are in general obtained by flux compactifications in string theory are gauged supergravities. The residual amount of supersymmetries that these four-dimensional theories have depends on the internal manifold chosen for the compactification and on the presence of local sources such as orientifold planes, branes and Kaluza–Klein monopoles. A natural question in this context is whether a vacuum state with positive cosmological constant and spontaneously broken supersymmetry can possibly arise. This would be relevant in order to embed some cosmological features of our four-

dimensional world in string theory. In other words, given a certain $\mathcal{N} = 4$ gauging, it is a highly non-trivial question whether such a gauging has a higher dimensional origin in terms of purely geometric and gauge fluxes. Due to this, the so-called non-geometric fluxes [13,14] (and, relatedly, doubled geometry [15,16]) have been introduced in the literature as flux parameters which are T- and S-dual to the known ones. This basically arises from the concept of mirror symmetry as a way of extending dualities in the presence of fluxes [17]. Using non-geometric fluxes, full stabilisation of all moduli has been achieved in De Sitter vacua in an $\mathcal{N} = 1$ context [18,19].

In the present Letter we first review the gauge algebra of $\mathcal{N} = 4$ gauged supergravity and its formulation in terms of the embedding tensor (Section 2). Secondly, we come to the identification of the correct $\mathcal{N} = 4$ gauge algebras in terms of fluxes (Section 3). In the present Letter we first review the gauge algebra of $\mathcal{N} = 4$ gauged supergravity and its formulation in terms of the embedding tensor (Section 2). Secondly, we come to the identification of the correct $\mathcal{N} = 4$ gauge algebras in terms of fluxes (Section 3). This basically arises from the concept of mirror symmetry as a way of extending dualities in the presence of fluxes [17]. Using non-geometric fluxes, full stabilisation of all moduli has been achieved in De Sitter vacua in an $\mathcal{N} = 1$ context [18,19]. In the present Letter we first review the gauge algebra of $\mathcal{N} = 4$ gauged supergravity and its formulation in terms of the embedding tensor (Section 2). Secondly, we come to the identification of the correct $\mathcal{N} = 4$ gauge algebras in terms of fluxes (Section 3). This basically arises from the concept of mirror symmetry as a way of extending dualities in the presence of fluxes [17]. Using non-geometric fluxes, full stabilisation of all moduli has been achieved in De Sitter vacua in an $\mathcal{N} = 1$ context [18,19]. None of the known gaugings giving rise to De Sitter solutions fall in this class of flux-induced algebras. In the conclusions we suggest a possibility how one could evade this no-go theorem (Section 5).

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In the context of type IIA string theory a number of no-go results [4–10] essentially forbid the existence of De Sitter vacua as long as a limited list of fluxes is considered. Some of these results have been obtained in the case of $SU(3)$ structure manifolds. Further recent works have investigated the link between $\mathcal{N} = 4$ gauged supergravity and string theory background fluxes in the presence of orientifold planes [11,12]. Such an analysis shows that the $\mathcal{N} = 4$ supergravity side allows for much more freedom at the level of deformations of the theory with respect to what is

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2. Gauge algebras in $\mathcal{N} = 4$

Half-maximal $\mathcal{N} = 4$ supergravity corresponds to the low-energy effective description of e.g. ten-dimensional type I string theory on a torus, or of type II string theories on $T^2 \times K3$ or on a torus in the presence of an orientifold plane. The theory consists of a supergravity multiplet and an additional number of vector multiplets, which for our purposes will be six. In this case the theory enjoys a global symmetry

$$SL(2) \times SO(6, 6). \quad (2.1)$$

The doublet representation of $SL(2)$ will be denoted by α , whereas the fundamental representation of $SO(6, 6)$ will be given by M . We will take the corresponding metric to be

$$\eta_{MN} = \begin{pmatrix} & \mathbb{I}_6 \\ \mathbb{I}_6 & \end{pmatrix}, \quad M = (1, \dots, 6, \bar{1}, \dots, \bar{6}), \quad (2.2)$$

i.e. we use light-cone coordinates.

The bosonic fields form representations of this global symmetry group. The scalars form a coset manifold based on (2.1) and hence split up in two parts, of dimensions 2 and 36, respectively. The vectors $A_{M\alpha}$ transform in the fundamental representation of $SO(6, 6)$. Furthermore, a crucial point is that the electric and magnetic parts of the vectors transform as doublets of $SL(2)$.

In the ungauged theory, there are Abelian gauge transformations associated with every gauge vector. In other words, the theory has a $U(1)^{12}$ gauge symmetry, in addition to the global symmetry (2.1). If one wants to include the magnetic part of the vectors as well, one could even say that the theory has a $U(1)^{24}$ gauge symmetry. However, this is only a symmetry of the equations of motion, as the Lagrangian is formulated in terms of the electric gauge potentials only.

The only deformations of this theory are the gaugings of some subgroup of the global symmetry group (2.1). These are parametrised by the components of the so-called embedding tensor [22]. For $\mathcal{N} = 4$ these components consist of $\xi_{\alpha M}$ and $f_{\alpha MNP}$ [23], where the latter is completely anti-symmetric in its $SO(6, 6)$ indices. We will restrict to the case with $\xi_{\alpha M} = 0$, implying that the gauge group is restricted to act within $SO(6, 6)$. In this case the commutation relations read

$$[X^{M\alpha}, X^{N\beta}] = f^{\alpha MN}{}_P X^{P\beta}, \quad (2.3)$$

where $X^{M\alpha}$ is the generator corresponding to the gauge vector $A_{M\alpha}$.

The deformation parameters need to satisfy certain consistency constraints which are called quadratic constraints. One way to derive these is by requiring the embedding tensor components to be invariant under gauge transformation. This results in [23]

$$f_{\alpha R[MN} f_{\beta P]Q}{}^R = 0, \quad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^R = 0. \quad (2.4)$$

The first of these should be thought of as the Jacobi identity leading to closure of the gauge algebra. The other imposes the orthogonality of charges, i.e. ensures that one is not using both the electric and magnetic part of a vector for a gauging, but only a linear combination.

Note that the commutation relation (2.3) in fact is not manifestly anti-symmetric on the right-hand side. This is related to the fact that the 24 generators $X^{M\alpha}$ do not furnish a basis, as there are only twelve physical gauge vectors and hence the total gauge algebra can at most be twelve-dimensional. For that reason there have to be linear relations between the different generators. These are

$$\epsilon_{\alpha\beta} f^{\alpha}{}_{MNR} X^{R\beta} = 0. \quad (2.5)$$

Taken in the adjoint representation this is exactly the second condition of (2.4). Due to this condition, the right-hand side of (2.3) is in fact anti-symmetric in the interchange of the two pairs of indices, as is clear from the left-hand side.

3. (Non-)geometric flux compactifications

Now let us see what gauge algebras can be induced by flux compactifications. The starting point in this discussion are the results of Kaloper and Myers [24]. They found that the dimensional reduction of type I supergravity to four dimensions leads to a non-Abelian gauge algebra if one includes fluxes. In particular, they derived the four-dimensional effect of the following fluxes for the ten-dimensional field content consisting of the metric, a two-form and a dilaton.¹

When reducing the metric from ten to four dimensions, one can generalise ordinary dimensional reduction by replacing the torus with a group manifold [27]. A group manifold is specified by structure constants $\omega_{mn}{}^p$, where the indices run over the dimension of the group manifold. The four-dimensional effect of such so-called geometric fluxes is to convert the gauge group $U(1)^6$, that corresponds to general coordinate transformations on the torus, to a non-Abelian group with commutation relations

$$[Z_m, Z_n] = \omega_{mn}{}^p Z_p, \quad (3.1)$$

where Z_m is the generator corresponding to the internal coordinate transformation $\delta x^m = \lambda^m$.

Due to the presence of the two-form gauge potential in the ten-dimensional theory, the four-dimensional gauge algebra is actually larger. In particular, there is an additional $U(1)^6$ corresponding to internal gauge transformations of the form $\delta B_{mn} = \partial_{[m} \lambda_{n]}$. We will denote these generators by X^p . These commute amongst themselves, but form a representation of the group spanned by (3.1). Furthermore, one can introduce gauge fluxes H_{mnp} for this potential. The total algebra spanned by the six Kaluza–Klein and six gauge generators reads [24]

$$\begin{aligned} [Z_m, Z_n] &= \omega_{mn}{}^p Z_p + H_{mnp} X^p, \\ [Z_m, X^n] &= -\omega_{mp}{}^n X^p, \\ [X^m, X^n] &= 0. \end{aligned} \quad (3.2)$$

Note that the resulting algebra is purely electric. Furthermore, the gauge generators span an ideal of the algebra, and hence the full algebra is non-semi-simple.

In order to make contact with the $SO(6, 6)$ notation of $\mathcal{N} = 4$ supergravity, one needs to split up the $SO(6, 6)$ index $M = (m, \bar{m})$. The twelve doublets of generators then split up according to $X^{M\alpha} = (Z_m{}^\alpha, X^{m\alpha})$. The identification between the embedding tensor and the fluxes is then apparent:

$$f_{+mnp} = H_{mnp}, \quad f_{+mn}{}^p = \omega_{mn}{}^p, \quad (3.3)$$

while the magnetic components vanish.

A natural question is how to generalise this to the case where one includes, in addition to gauge and geometric flux, also the types of non-geometric fluxes introduced by [13]. If one assumes that H and ω are both NS–NS, these will transform into each other under T-duality. Furthermore, these will transform into the non-geometric NS–NS fluxes Q and R under T-duality. The action of

¹ We will only include fluxes for the metric and the two-form. There is a similar possibility for the dilaton, which we will not consider, that leads to gauging with non-vanishing $\xi_{M\alpha}$ [25]. In this paper also the first line of the identification (3.7) was made. Moreover, we will not consider trombone gaugings of the type introduced in [26] for the maximal theory.

T-duality on NS–NS fluxes is to raise and lower the indices of the different types of fluxes:

$$H_{mnp} \leftrightarrow \omega_{mn}{}^p \leftrightarrow Q_m{}^{np} \leftrightarrow R^{mnp}. \quad (3.4)$$

From this, one can derive what the generalisation of the algebra (3.2) is. It can be seen that this reads as [13]

$$\begin{aligned} [Z_m, Z_n] &= \omega_{mn}{}^p Z_p + H_{mnp} X^p, \\ [Z_m, X^n] &= -\omega_{mp}{}^n X^p + Q_m{}^{np} Z_p, \\ [X^m, X^n] &= Q_p{}^{mn} X^p + R^{mnp} Z_p. \end{aligned} \quad (3.5)$$

Note that this algebra, with all types of NS–NS fluxes, is still purely electric.

Subsequently one could reason that in the IIB duality frame with O3-planes one needs to mod out with the \mathbb{Z}_2 symmetry $(-)^{F_L} \Omega I_{4..9}$. Under this symmetry, the only allowed fluxes are H and Q . Therefore the algebra for these fluxes reads

$$\begin{aligned} [Z_m, Z_n] &= H_{mnp} X^p, \\ [Z_m, X^n] &= Q_m{}^{np} Z_p, \\ [X^m, X^n] &= Q_p{}^{mn} X^p. \end{aligned} \quad (3.6)$$

The relation between the embedding tensor and the fluxes can be easily read off from this algebra. Before we give it, let us introduce a slight generalisation by including S-duality related fluxes as well. For the two-form gauge potentials this is very natural, as we know that these form a doublet (H, F) under S-duality. Similarly, it has been conjectured that there is a doublet of non-geometric fluxes (Q, P) as well [14]. Including the two doublets of gauge and non-geometric fluxes, the relation to the embedding tensor is

$$\begin{aligned} f_{+mnp} &= H_{mnp}, & f_{+m}{}^{np} &= Q_m{}^{np}, \\ f_{-mnp} &= F_{mnp}, & f_{-m}{}^{np} &= P_m{}^{np}. \end{aligned} \quad (3.7)$$

The full algebra, including the commutation relations between electric and magnetic generators, then follows trivially from (2.3). Similarly, one can deduce the full set of constraints on the fluxes from (2.4).

Note that the algebra (3.6) in general does not have any non-trivial ideals, and hence is not necessarily non-semi-simple. This form of the algebra has been used in e.g. [18] in their classification of the possible solutions of the corresponding Jacobi identities. Indeed, they encountered simple and semi-simple possibilities. This poses a clear puzzle: we claim to have performed a number of dualities, under which the effective description should transform covariantly, and nevertheless the algebra (3.2) of the starting point clearly differs from (3.6). Indeed, one is necessarily non-semi-simple while the other is not. What has happened? In our opinion, the confusion stems from the identification of the starting point.

The starting point of Kaloper and Myers corresponds to the heterotic string, and therefore contains an NS–NS two-form gauge potential. However, in order to make contact with type II string theories with orientifold planes, e.g. the preferred duality frame of type IIB with O3-planes, one should first perform an S-duality. This takes one to type I string theory, or equivalently type IIB with O9-planes. In this case the two-form is not NS–NS but rather R–R, which will be a crucial distinction when applying T-duality. As mentioned before, in the NS–NS sector T-duality raises and lowers indices. In contrast, in the R–R sector the effect of T-duality is to create or annihilate indices:

$$T_p : \begin{cases} F_{m_1 \dots m_n} \rightarrow F_{m_1 \dots m_n p}, \\ F_{m_1 \dots m_n p} \rightarrow F_{m_1 \dots m_n}. \end{cases} \quad (3.8)$$

In other words, a gauge potential remains a gauge potential but its rank changes.

The correct starting point for our purpose is

$$\begin{aligned} [Z_m, Z_n] &= \omega_{mn}{}^p Z_p + F_{mnp} X^p, \\ [Z_m, X^n] &= -\omega_{mp}{}^n X^p, \\ [X^m, X^n] &= 0, \end{aligned} \quad (3.9)$$

where F_{mnp} is the R–R three-form flux. Upon a six-tuple T-duality to go to the type IIB duality frame with O3-planes, this transforms into

$$\begin{aligned} [Z_m, Z_n] &= 0, \\ [Z_m, X^n] &= Q_m{}^{np} Z_p, \\ [X^m, X^n] &= Q_p{}^{mn} X^p + \tilde{F}^{mnp} Z_p, \end{aligned} \quad (3.10)$$

where $\tilde{F}^{mnp} = \frac{1}{6} \epsilon^{mnpqrs} F_{qrs}$. This fixes the complete electric part of the gauge algebra. The remaining part follows straightforwardly once one has made the identification between the embedding tensor and the fluxes. Again we will give an S-duality covariant set of fluxes, including the gauge doublet (F, H) and the non-geometric doublet (Q, P) . With the algebra (3.10) this identification reads

$$\begin{aligned} f_{+mnp} &= \tilde{F}^{mnp}, & f_{+m}{}^{np} &= Q_m{}^{np}, \\ f_{-mnp} &= \tilde{H}^{mnp}, & f_{-m}{}^{np} &= P_m{}^{np}. \end{aligned} \quad (3.11)$$

The full algebra and corresponding quadratic constraints then follow from (2.3) and (2.4). The latter read

$$\begin{aligned} Q_r{}^{[mn} Q_q{}^{p]r} &= P_r{}^{[mn} P_q{}^{p]r} = 0, \\ P_r{}^{[mn} Q_q{}^{p]r} &= Q_r{}^{mn} P_q{}^{pr} - P_r{}^{mn} Q_q{}^{pr} = 0, \end{aligned} \quad (3.12)$$

involving only non-geometric flux, and

$$\begin{aligned} \tilde{F}r{}^{[mn} Q_r{}^{pq]} &= \tilde{H}r{}^{[mn} P_r{}^{pq]} = 0, \\ \tilde{F}r{}^{[mn} P_r{}^{pq]} &+ Q_r{}^{[mn} \tilde{H}r{}^{pq]} = 0, \end{aligned} \quad (3.13)$$

involving gauge fluxes as well. The fully anti-symmetric parts of the latter set of equations imply the absence of any 7-branes; these would break supersymmetry further to $\mathcal{N} = 1$. The same form of the algebra and quadratic constraints was recently derived in the beautiful work² [28] from a different starting point.

Note the differences between the two algebras³ (3.5) and (3.10). First of all, NS–NS fluxes induce a purely electric gauging in the former algebra [25], while in the latter this involves magnetic generators as well. Moreover, the former can describe a (semi-)simple algebra (see e.g. [29,30,18]), while the latter is always non-semi-simple algebra, as it should. This crucial difference between the two stems from the appearance of the Hodge dualised three-form \tilde{F} , instead of the three-form itself, in (3.10). This qualitative difference can be traced back to the different behaviour of NS–NS and R–R gauge potentials under T-duality.

Finally, the quadratic constraints (3.13) are in general different for the two algebras. For instance, it can be seen from the $SL(2)$ scaling weight that the last equation of (3.13) could never

² Due to different conventions regarding the $SO(6, 6)$ and $SL(6)$ indices, our form of the identification (3.11) does not involve any non-trivial metrics, as in [28]. Moreover, the quadratic constraints given in [28] are not all linearly independent, and hence can be written in a more economic way.

³ Most of the literature that uses (3.5) takes place in an $\mathcal{N} = 1$ context, where the scalar potential is not given in terms of structure constants but rather a superpotential. Therefore our argument does not affect any of the results on $\mathcal{N} = 1$ moduli stabilisation etc.

arise from (3.7). However, in the truncation where one of the two non-geometric fluxes vanishes, e.g. $P = 0$, the quadratic constraints bilinear in the NS–NS fluxes are in fact identical (provided $Q_m^{mn} = 0$). There is still a difference in the constraints bilinear in Q and F : these are much stronger for the first identification (3.7) than those given in (3.13).

4. What about De Sitter?

All the gaugings that are known to give rise to De Sitter solutions in $\mathcal{N} = 4$ gauged supergravity [20,21] are of the form

$$G = G_1 \times G_2 \times \dots, \quad (4.1)$$

i.e. a direct product of a number of gauge factors. This is a solution to the quadratic constraints (2.4) once the Jacobi identities are separately satisfied in the different factors. Moreover, in order to have a De Sitter solution, the gauge group must contain electric and magnetic factors. Finally, the gauge factors have to be specific (semi-)simple groups. In particular, we will focus on the case of two gauge factors. Each factor is of the form $SO(p, q)$ with $p + q = 4$ and embedded in an $SO(3, 3)$ factor. A number of examples of such gaugings with De Sitter solutions was discussed in [20,21]. Moreover, it was shown in [31] that the contracted versions $CSO(p, q, r)$ with $p + q + r = 4$ of such gauge groups do not lead to any solutions with a positive scalar potential. In this section we will assess to what extent one can obtain such gaugings from the flux compactifications considered earlier.

The direct product structure (4.1) leads us to split $SO(6, 6)$ into two $SO(3, 3)$ factors in which to embed G_1 and G_2 respectively. Without loss of generality, we will take the first to be electric and lie in the directions $\{1, 2, 3, \bar{1}, \bar{2}, \bar{3}\}$, while the second is taken magnetic and lies in the complementary directions. We will discuss the embedding of the first factor in some detail; the discussion for the second factor is completely analogous. However, before we discuss $SO(4)$ embeddings in $SO(3, 3) \simeq SL(4)$, we first generalise this to arbitrary N .

In general, the embedding of $SO(N)$ and its analytic continuations into $SL(N)$ can be written in terms of the following generators in the fundamental representation

$$(T_{ij})^k{}_l = 4\delta^k_{[i}M_{j]l}, \quad (4.2)$$

in terms of a symmetric matrix M , that can always be diagonalised by a convenient choice of basis. It is in fact given by the identity in the case of $SO(N)$. These generators labelled by anti-symmetric pairs of indices satisfy the following commutation relations

$$[T_{ij}, T_{kl}] = f_{ij,kl}{}^{mn}T_{mn}, \quad f_{ij,kl}{}^{mn} = 8\delta^{[m}_{[i}M_{j][k}\delta^{n]}_{l]}. \quad (4.3)$$

Analytic continuations of $SO(N)$ correspond to a number of minus signs in the M -matrix. Contractions thereof, denoted by $CSO(p, q, r)$ with $p + q + r = N$ (see e.g. [31]), can be understood in this notation by replacing r non-zero diagonal entries of M with zero entries.

However, the most general form of $CSO(p, q, r)$ structure constants for the special case of $N = 4$ is given in terms of two symmetric matrices rather than one [32], which we will denote by M and \tilde{M} . The generators are then given by

$$(T_{ij})^k{}_l = 4\delta^k_{[i}M_{j]l} - 2\varepsilon_{ijml}\tilde{M}^{mk}, \quad (4.4)$$

giving rise to the following general expression of the structure constants

$$f_{ij,kl}{}^{mn} = 8\delta^{[m}_{[i}M_{j][k}\delta^{n]}_{l]} - \varepsilon_{ijl'j'}\varepsilon_{klk'l'}\tilde{M}^{mni'l'}. \quad (4.5)$$

With such a form we need some extra consistency constraints in terms of M and \tilde{M} , coming from imposing the Jacobi identities. These translate into

$$M_{ij}\tilde{M}^{jk} - \frac{1}{4}\delta_i^k M_{jl}\tilde{M}^{jl} = 0. \quad (4.6)$$

If one still diagonalises M by a convenient basis choice, the Jacobi identity imply \tilde{M} to be diagonal as well. In this case the constraints reduce to

$$M_{11}\tilde{M}^{11} = M_{22}\tilde{M}^{22} = M_{33}\tilde{M}^{33} = M_{44}\tilde{M}^{44}. \quad (4.7)$$

Let us now connect the adjoint representation in terms of $SL(4)$ indices to fundamental $SO(3, 3)$ indices. This relation is given by

$$\{1, 2, 3, \bar{1}, \bar{2}, \bar{3}\} \simeq \{12, 13, 14, 43, 24, 32\}. \quad (4.8)$$

This leads to the following identification between the diagonal components of the two matrices M and \tilde{M} , and the components of the embedding tensor $f_{\alpha MNP}$ in the first $SO(3, 3)$ factor:

$$M = \text{diag}(f_{+123}, f_{+1\bar{2}\bar{3}}, f_{+\bar{1}2\bar{3}}, f_{+\bar{1}\bar{2}3}), \\ \tilde{M} = \text{diag}(f_{+\bar{1}\bar{2}\bar{3}}, f_{+\bar{1}23}, f_{+\bar{1}\bar{2}3}, f_{+\bar{1}2\bar{3}}). \quad (4.9)$$

Other components of the embedding tensor in this $SO(3, 3)$ factor, such as $f_{+\bar{1}\bar{1}2}$ and $f_{+\bar{1}\bar{1}\bar{2}}$, correspond to off-diagonal components of M and \tilde{M} and hence have been set equal to zero.

We have discussed in the previous sections how the embedding tensor can be sourced by different fluxes. In particular, we have discussed the two identifications (3.7) and (3.11). It will be illuminating to illustrate the different consequences of the two identifications in this context. Using the first identification, the matrices are given by

$$M = \text{diag}(H_{123}, Q_1^{23}, Q_2^{31}, Q_3^{12}), \\ \tilde{M} = \text{diag}(0, 0, 0, 0). \quad (4.10)$$

In this case it would therefore be possible to use the different fluxes to generate a simple gauge factor. Given that the discussion in the second, magnetic factor is completely analogous, one could e.g. generate an $SO(4)_{\text{el}} \times SO(4)_{\text{magn}}$ gauge group, which certainly leads to De Sitter solutions. However, we have argued that this is not the correct identification; instead, one should use (3.11). In this case, the matrices read

$$M = \text{diag}(0, Q_1^{23}, Q_2^{31}, Q_3^{12}), \\ \tilde{M} = \text{diag}(F_{456}, 0, 0, 0). \quad (4.11)$$

The crucial point is that in this case the gauge flux does not enter in the M matrix to make it non-singular; instead, it enters in the other matrix \tilde{M} . These singular matrices only lead to non-semi-simple gauge groups. In particular, the matrix M gives rise to $ISO(3)$ and analytic continuations and contractions thereof. Provided the three components $Q_i{}^{jk}$ are non-zero, the additional parameter F_{456} does not modify the gauge group, but only describes different embeddings of it in $SO(3, 3)$. Three of these are inequivalent, corresponding to F_{456} being positive, zero or negative. Exactly the same embeddings of $ISO(3)$ and $ISO(2, 1)$ were considered in⁴ [31], where it was found that such gauge groups do not give rise to scalar potentials with positive extrema.

Indeed, one can infer from the same reasoning that none of the gauge groups discussed in [20,21] follows from a flux compactification with the identification (3.11). The simple bottom line

⁴ The relation to the notation of [31] is $\lambda^2 = (1 - F_{456})/(1 + F_{456})$.

is that all the gauge groups necessarily consist of (semi-)simple gauge factors, while one can only get non-semi-simple factors from flux compactifications.

5. Conclusions

One of the main points of this Letter is to point out the gauge algebra (3.10) that arises from (non-)geometric flux compactifications to $D = \mathcal{N} = 4$. In contrast to (3.5), this algebra is always non-semi-simple due to the presence of an Abelian ideal spanned by the generators Z_m . As a consequence, it is impossible to build any of the gauge groups consisting of simple factors that are known to give rise to De Sitter solutions [20,21].

There is a number of directions in which to extend this work. Amongst them are generalisations of the flux compactifications and gauge algebras discussed in Section 3. For instance, one could include the world-volume excitations of D3-branes to change the number of $\mathcal{N} = 4$ vector multiplets. Similarly, one could consider the truncation to $\mathcal{N} = 1$ supergravity by including O7-planes and D7-branes. Some aspects of these extensions can be found in [28]. Finally, one could consider going beyond the type of flux compactifications discussed here⁵ to account for the missing components of the embedding tensor in (3.11), and in this way build up (semi-)simple gauge algebras.

As for the possibilities of De Sitter, again a number of generalisations are possible. In [20,21] an analysis was made which gauge groups lead to a positive cosmological constant in the origin. Naturally, this could be extended to a larger portion of the moduli space. Indeed, such an analysis was performed in the very recent work [18,19] for $\mathcal{N} = 1$ flux compactifications with $P = 0$. In a clever way all possible Minkowski vacua were determined, and a band of De Sitter vacua was found closeby (in moduli and parameter space). It can be seen that one of their cases,⁶ where the Q -flux spans an $SO(3, 1)$ algebra, allows for an interpretation in terms of $\mathcal{N} = 4$ as well; in this case, all quadratic constraints (3.12) and (3.13) can be fulfilled. Therefore it is possible to obtain De Sitter solutions from $\mathcal{N} = 4$ non-geometric compactifications. A natural question concerns the gauge algebra in this case; in other words, given the fluxes, what algebra does (3.10) correspond to? It appears that it is no longer of the direct product form (4.1) but rather a semi-direct product, where e.g. the electric part of the gauge group has a non-trivial action on the magnetic part. We leave this question for future investigation.

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⁵ One could e.g. investigate the possibility of O3-planes on doubled geometry [15, 16,33] with non-geometric fluxes. In contrast to the heterotic duality frame, where non-geometric fluxes and doubled geometry give rise to the same generalisation of [24], in the O3 duality frame doubled geometry could lead to a doubling of the flux components in (3.11). In such a "doubled non-geometric" set-up all components of the $\mathcal{N} = 4$ embedding tensor could be turned on.

⁶ There is in fact a close link between another of their cases, where the Q -flux spans an $ISO(3)$ algebra, and the unstable De Sitter solution found in [7] from a IIA flux compactification on an $SU(2) \times SU(2)$ group manifold. We have checked explicitly that the fluxes given in the latter paper correspond to the angles $\theta_\xi = 0.38\pi$ and $\theta_\epsilon = 1.47\pi$ in the notation of [19]. Therefore the single example of an unstable De Sitter solution can be understood to lie in a narrow band of such solutions that borders the line of unstable Minkowski solutions.