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Corrigendum

Corrigendum to "Half-linear equations and characteristic properties of the principal solution" [J. Differential Equations 208 (2005) 494–507]☆

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The limit and integral characterization of principal solutions of the half-linear equation

$$(a(t)\Phi(x'))' + b(t)\Phi(x) = 0, \tag{1}$$

where the functions a, b are positive and continuous for $t \geqslant 0$ and $\Phi(u) = |u|^{p-2}u$, $p > 1$, are considered and the following two cases:

$$(i_1)\ \ J_a + J_b < \infty, \quad (i_2)\ \ J_a + J_b = \infty$$

are separately studied, where

$$J_a := \int_0^\infty \Phi^*(1/a(t))\,dt, \quad J_b := \int_0^\infty b(t)\,dt$$

and Φ^* is the inverse of the map Φ.

We employed a result by Došlý–Elbert [5, Theorem 3.3]; see also [6, Theorem 4.2.8]). However, this result fails to hold in some particular cases, as we have discovered

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by using a similar argument that is given in [3, Theorem 4]. For this reason, some additional assumptions, which we state below, are needed.

More precisely, the Došlý–Elbert result [5, Theorem 3.3] reads for (1) as follows.

Theorem (*Došlý and Elbert [5, Theorem 3.3]*). *Let* (1) *be nonoscillatory. Assume* $J_a = \infty$. *Then* u *is principal solution of* (1) *if and only if*

$$I = \int^{\infty} \frac{u'(t)}{a(t)\Phi(u'(t))u^2(t)}\, dt = \infty.$$

The following example illustrates that this theorem is not correct when $1 < p < 2$.

Example. Consider the equation

$$(\Phi(x'))' + e^{-t}\Phi(x) = 0 \tag{2}$$

with $1 < p < 2$. This equation is nonoscillatory and has both bounded and unbounded solutions, as it follows, for instance, from [7, Theorems 4.1, 4.2] (see also [2, Theorems 3.3, 4.1], [8, Theorem 4.3]). Taking into account that if u is a principal solution of (2), then the ratio u/x is eventually either positive decreasing or negative increasing for any other solution x of (2) such that $x \neq \lambda u$, $\lambda \in \mathbb{R}/\{0\}$, necessarily any principal solution of (2) is bounded as $t \to \infty$. Without loss of generality, let u be an eventually positive and bounded solution of (2): clearly, u' is positive decreasing for large t and so $\lim_{t\to\infty} u'(t) = 0$. Integrating (2) for large t we have

$$u'(t) = \Phi^* \left(\int_t^{\infty} e^{-s}\Phi(u(s))\, ds \right).$$

Since u is bounded, there exist two positive constants $h_1 < h_2$ such that

$$h_1 e^{-t/(p-1)} < u'(t) < h_2 e^{-t/(p-1)}.$$

Then for any bounded solution of (2) we have

$$I \sim \int^{\infty} \left(u'(t)\right)^{2-p}\, dt < \infty,$$

which gives a contradiction with Theorem 3.3 in [5].

The above theorem [5, Theorem 3.3] fails by virtue of an incorrect application of [5, Lemma 2.4] when $1 < p < 2$ [4]. A sufficient condition, under which such a result remains valid, is $p \geqslant 2$, i.e.,

$$J_a = \infty, \quad p \geqslant 2.$$

Consequently, results hold under the additional condition H_1 or

$$J_b = \infty, \quad 1 < p \leqslant 2.$$

Summarizing, it is sufficient that (Introduction, p. 497, line 5 from the top) the sentence

$$\textit{In this paper, the case } b(t) > 0 \textit{ is examined} \ldots$$

be replaced by

$$\textit{In this paper, the case } b(t) > 0 \textit{ is examined under the assumption}$$

$$J_a = \infty, \;\; p \geqslant 2 \quad \text{or} \quad J_b = \infty, \;\; 1 < p \leqslant 2 \quad \text{or} \quad J_a + J_b < \infty.$$

Observe also that, in Example 4, the application of Theorem 3, for the above reason, is not convenient.

Let us note that the remaining cases

$$J_a = \infty, \;\; 1 < p < 2 \quad \text{or} \quad J_b = \infty, \;\; p > 2,$$

are considered in the forthcoming paper [1], together with an exhaustive analysis of the general situation and comparisons between various cases.

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