Scale-invariance as the origin of dark radiation?

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A R T I C L E   I N F O

A B S T R A C T

Recent cosmological data favor R^2-inflation and some amount of non-standard dark radiation in the Universe. We show that a framework of high energy scale invariance can explain these data. The spontaneous breaking of this symmetry provides gravity with the Planck mass and particle physics with the electroweak scale. We found that the corresponding massless Nambu–Goldstone bosons – dilatons – are produced at reheating by the inflaton decay right at the amount needed to explain primordial abundances of light chemical elements and anisotropy of the cosmic microwave background. Then we extended the discussion on the interplay with Higgs-inflation and on general class of inflationary models where dilatons are allowed and may form the dark radiation. As a result we put a lower limit on the reheating temperature in a general scale invariant model of inflation.

1. Introduction

Particle physics teaches us that in a renormalizable theory at high energy only dimensionless couplings are relevant. Thus, the Standard Model (SM) becomes scale-invariant at classical level in this limit. Though quantum corrections generally violate scale invariance, one can speculate that at high energy the model is indeed modified to be scale-invariant, which provided the argument by Bardeen [1] can solve the naturalness problem in the SM Higgs sector (suffered from the quadratically divergent quantum corrections to the Higgs boson mass squared, see e.g. [2]). Then spontaneous breaking of the scale invariance provides low energy particle physics with the only (at the tree level) dimensionful parameter of the SM, that is the value of the electroweak scale $v = 246$ GeV.[1]

The same logic may be applied to gravity. Then at high energy the classically scale invariant gravity action[2] contains both scalar curvature R and dilaton X,

$$S_0 = \int d^4x \sqrt{-g}\frac{1}{2}\big[\beta R^2 + (\partial_\mu X)^2 - \xi X^2 R\big], \tag{1}$$

with dimensionless real parameters $\beta, \xi > 0$. Once the scale invariance breaks, dilaton X gains non-zero vacuum expectation value and the last term in (1) yields the Einstein–Hilbert low-energy action. Dilaton remains massless in perturbation theory, so the scale invariance may be maintained at the quantum level, see e.g. [4]. As the Nambu–Goldstone boson, dilaton couples to other fields via derivative thus avoiding bounds on a fifth force.

Remarkably, with R^2-term in gravity action (1), the early Universe exhibits inflationary stage of expansion suggested by Starobinsky [6]. At this stage the Universe becomes flat, homogeneous and isotropic as we know it today. Also, quantum fluctuations of the responsible for inflation scalar degree of freedom in (1) (inflaton, which is also called *scalaron* in this particular model) transform to the adiabatic perturbations of matter with almost scale-invariant power spectrum. These perturbations are believed to be seeds of large scale structures in the present Universe and they are responsible for the anisotropy of the cosmic microwave background (CMB). With β normalized to the amplitude of CMB anisotropy $\delta T/T \sim 10^{-4}$ and ξX^2 fixed by the Planck mass value in order to produce the usual gravity, the action (1) has no free parameters. Therefore, the inflationary dynamics is completely determined. Interestingly, recent analyses of cosmological data [7,8]

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[1] The dark energy may be understood as either an effective cosmological constant emerging after spontaneous breaking of scale invariance or a special dynamics of dilaton field, see e.g. [5].

[2] Quadratic terms in the Riemann and Ricci tensors generally give rise to ghost-like and other instabilities and are omitted hereafter. Since the physics responsible for violation of the scale invariance is also beyond the scope of this paper, we omit the dilaton potential in Eq. (1) and disregard its impact on the early time cosmology. Note that the absence (smallness) of a scale-invariant quartic term X^4 in (1)

may be related to vanishing (tiny) cosmological constant at later stages of the Universe expansion [3].

favor this prediction over those of many other models of inflation driven by a single scalar field.

One may treat these results as a hint of scale invariance at high energy. Yet the theory we consider apart from the Starobinsky model contains also massless dilaton coupled to gravity through the last term in (1). In this Letter we show that this term is also responsible for the scalaron decays into dilatons at post-inflationary reheating. In the late Universe the massless dilatons affect the Universe expansion. Surprisingly, the relic amount of produced massless dilatons is precisely what we need to explain the additional (to active neutrinos) dark radiation component[3] suggested by the recent analyses of CMB anisotropy data [10,7,8,11,12], and favored by the observation of primordial abundance of light chemical elements [13]. We consider this finding as possibly one more hint of scale invariance at high energy.

To complete the study we then discuss the SM Higgs boson sector in the model following Refs. [14–16] and outline the regions of the model parameter space where the SM Higgs contributes to the inflationary dynamics. Finally, we investigate the dilaton production in a general scale invariant model of a single field inflation and set a lower limit on the reheating temperature from avoiding the dilaton overproduction.

2. Dilaton–scalaron inflation

We start from considering the scale invariant extension of the Starobinsky model with action (1). Following [17] we introduce new scalar fields Λ and $\mathcal{R}$ and find the equivalent form of action (1):

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}\left(\beta\mathcal{R}^2 + (\partial_\mu X)^2 - \xi X^2 \mathcal{R}\right) - \Lambda\mathcal{R} + \Lambda R\right]. \quad (2)$$

Integrating out auxiliary field $\mathcal{R}$ (solving the corresponding equation of motions for $\mathcal{R}$) we obtain

$$S = \int d^4x \sqrt{-g}\left[\Lambda R + \frac{1}{2}(\partial_\mu X)^2 - \frac{1}{2\beta}\left(\Lambda + \frac{1}{2}\xi X^2\right)^2\right]. \quad (3)$$

Going to the Einstein frame through the conformal transformation $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega^2 = -2\Lambda/M_P^2$, and omitting tildes thereafter (all quantities below are evaluated with metric $\tilde{g}_{\mu\nu}$) we arrive at

$$S = \int d^4x \sqrt{-g}\left[-\frac{M_P^2}{2}R + \frac{6M_P^2}{2\omega^2}\left[(\partial_\mu\omega)^2 + (\partial_\mu X)^2\right] - \frac{M_P^4}{8\beta}\left(1 - \frac{6\xi X^2}{\omega^2}\right)^2\right], \quad (4)$$

here $\omega = \sqrt{6}M_P\Omega$, and the reduced Planck mass M_P is defined through the Newtonian gravitational constant G_N as $1/M_P^2 = 8\pi G_N$. After changing the variables $\omega = r\sin\theta$, $X = r\cos\theta$ the kinetic term K and potential term V become

$$K = \frac{6M_P^2}{2\sin^2\theta}\left((\partial_\mu\log r)^2 + (\partial_\mu\theta)^2\right),$$
$$V = \frac{M_P^4}{8\beta}\left(1 - 6\xi\cot^2\theta\right)^2, \quad (5)$$

or, casting them in terms of new variables

$$\rho = \sqrt{6}M_P\log\frac{r}{M_P}, \qquad f - f_0 = \sqrt{6}M_P\log\tan\frac{\theta}{2}, \quad (6)$$
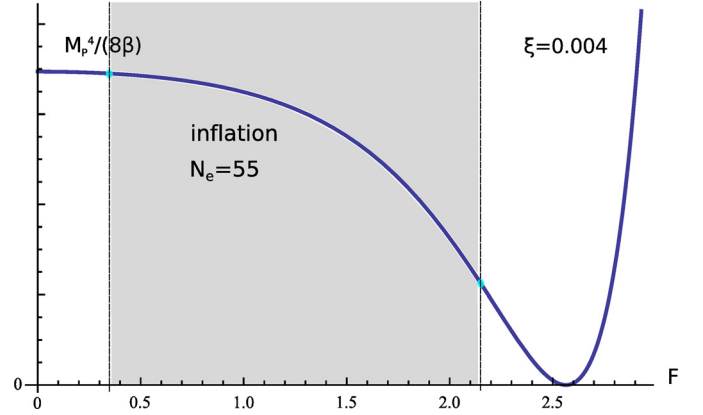

Fig. 1. Inflationary potential: field $F \equiv (f_0 - f)/(M_P\sqrt{6})$ slowly moves from close to hilltop $f \simeq f_0$ towards the minimum at $f = 0$. The potential is symmetric under reflection $F \to -F$.

we find

$$K = \frac{1}{2}(\partial_\mu\rho)^2\cosh^2\left(\frac{f_0 - f}{\sqrt{6}M_P}\right) + \frac{1}{2}(\partial_\mu f)^2,$$
$$V = \frac{M_P^4}{8\beta}\left(1 - 6\xi\sinh^2\left(\frac{f_0 - f}{\sqrt{6}M_P}\right)\right)^2. \quad (7)$$

Both kinetic and potential parts (7) are invariant under reflection $f \to 2f_0 - f$. Choosing one of two minima of V to be at $f = 0$ implies that integration constant f_0 obeys

$$\sinh^2\left(\frac{f_0}{\sqrt{6}M_P}\right) = \frac{1}{6\xi}. \quad (8)$$

The inflation may occur at values $0 < f < f_0$ (or in a mirror interval $f_0 < f < 2f_0$, that we ignore in what follows), see Fig. 1. The potential is similar to one considered in [14], so for the tilt of scalar perturbations seeded by inflaton fluctuations one has

$$n_s \simeq 1 - 8\xi\coth(4\xi N_e), \quad (9)$$

where N_e is the number of e-foldings remained till the end of inflation from the moment when perturbations of the CMB-experiments pivot scale $k/a_0 = 0.002$ Mpc^{-1} exit horizon. To have $N_e \approx 55$ e-folds [18] (since the reheating temperature is about 3.1×10^9 GeV [19] provided scalaron decays to the Higgs bosons) and fit into the favored by cosmological analyses interval $n_s = 0.9603 \pm 0.0073$ [7], we need

$$\xi < 0.004, \quad (10)$$

hence $f_0 > 6.28 \times M_P$. In order to obtain the right value of scalar perturbation amplitude $\Delta \approx 5 \times 10^{-5}$ we should choose the parameter β (weakly depending on ξ) to be in the range $(2 - 0.8) \times 10^9$.

3. Reheating and dilaton production

After inflation the energy is confined in homogeneous oscillating around minimum of the scalaron potential. Scalaron coupling to other fields provides oscillation decay. It reheats the Universe when the Hubble parameter becomes comparable to the inflation decay rate. It is well-known that scalaron couples to any *conformally non-invariant* part of the lagrangian, see discussion in [19, 20]. Within the SM the most relevant is coupling to the Higgs field. Scalaron decay rate to the Higgs bosons is the same as in case of the usual Starobinsky model, and for a more general variant with

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[3] Particular models with massless (Nambu–Goldstone) bosons were considered in literature to address the dark radiation problem, see e.g. [9].

the Higgs non-minimally coupled to gravity through the lagrangian term $-\xi' R H^\dagger H$ one obtains [20,21]

$$\Gamma_H = \left(\frac{1}{6\beta}\right)^{3/2} \frac{4 M_P}{192\pi} (1 + 6\xi')^2. \tag{11}$$

Generally, scalaron decays preferably into model scalars, as their kinetic terms are non-conformal. The kinetic term in (7) yields (after canonical normalization $\rho\sqrt{1 + \xi/6} \to \rho$) for the scalaron decay rate to dilatons ρ

$$\Gamma_\rho = \left(\frac{1}{6\beta}\right)^{3/2} \frac{M_P}{192\pi}. \tag{12}$$

We see from Eqs. (11), (12) that $\Gamma_\rho / \Gamma_H = 1/(4(1 + 6\xi')^2)$ giving the same ratio $\rho_\rho / \rho_H = 1/(4(1 + 6\xi')^2)$ at reheating. Produced at reheating dilatons never equilibrate in the Universe and other mechanisms of their production (e.g. nonperturbative as discussed in [15] or in scattering of SM particles) are inefficient. Dilatons contribute to the energy density and pressure of primordial plasma and hence change the Universe expansion rate. In particular, the existence of the dilaton rises the effective number of additional to the SM relativistic degrees of freedom at Big Bang Nucleosynthesis [15]:

$$\Delta N_{eff} \simeq 2.85 \frac{\rho_\rho}{\rho_H} = \frac{0.71}{(1 + 6\xi')^2}. \tag{13}$$

The last 9th WMAP release (more exactly, combined WMAP + eCMB + BAO + H$_0$ data) gives

$$N_{eff} = 3.84 \pm 0.40, \tag{14}$$

when helium abundance is fixed [8]. The first result by Planck Collaboration [7] gives $N_{eff} = 3.36 \pm 0.34$ in agreement with the SM prediction $N_{eff} = 3.046$. However, when independent data on direct measurements of the present Hubble parameter are included into fit (which may cure the anomaly at small multipoles $l \sim 15$–30) as was done in the WMAP result (14), the estimate becomes [7] (see also [10])

$$N_{eff} = 3.62 \pm 0.25. \tag{15}$$

Hence $\Delta N_{eff} \simeq 1$ is still allowed, which is generally consistent with $\xi' \lesssim 1$ (when Higgs field dynamics does not change inflation, see details in Section 4). Moreover, one finds that for minimally coupled Higgs, $\xi' = 0$, the predicted amount of dark radiation (13) is exactly what we need to explain observations (14), (15).

In the conformal case $\xi' = -1/6$ or close to it, the Universe reheats by the anomalous inflaton decay to gauge fields [21]. The decay rate due to the conformal anomaly is

$$\Gamma_{gauge} = \frac{\Sigma b_i^2 \alpha_i^2 N_i}{4\pi^2} \left(\frac{1}{6\beta}\right)^{3/2} \frac{M_P}{192\pi}. \tag{16}$$

Here b_i, α_i, N_i are coefficient in β-function, gauge coupling constant and the number of colors correspondingly for the SM gauge fields. Numerically $\Gamma_{gauge} \sim \Gamma_\rho / 130$ which means that actually all inflatons decay to dilatons. So the case of conformal or close to conformal Higgs is forbidden.

4. Scalaron inflation or Higgs inflation?

To justify our study of the non-minimally coupled to gravity Higgs in the context of R^2-dilaton inflation we need to understand when the nonminimal coupling ξ' starts to change the inflationary dynamics. Consider the scale invariant action for the gravity, dilaton X and Higgs field h in the unitary gauge,

$$S_0 = \int d^4 x \sqrt{-g} \left[\frac{1}{2}\left(\beta R^2 + (\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2\right) - \frac{\lambda}{4}(h^2 - \alpha^2 X^2)^2 \right]. \tag{17}$$

The dilaton vacuum expectation value (vev) $\langle X \rangle$ defines the reduced Planck mass (cf. Eqs. (2) and (4)) as $M_P = \sqrt{\xi}\langle X \rangle$, and the last term in (17) defines the SM Higgs field vev as $v = \alpha\langle X \rangle$. Hence the upper limit on ξ (10) implies $\alpha < 10^{-17}$. For this study we suppose that at inflationary scale $\lambda > 0$, which is consistent with recent analyses [22] when uncertainties are accounted for.

Applying the same technique as in Section 2 to action (17) we obtain the Einstein frame lagrangian

$$L = -\frac{M_P^2}{2} R + \frac{6 M_P^2}{2\omega^2}\left((\partial_\mu \omega)^2 + (\partial_\mu X)^2 + (\partial_\mu h)^2\right) - V, \tag{18}$$

$$V = \frac{9\lambda M_P^4}{\omega^4}(h^2 - \alpha^2 X^2)^2 + \frac{M_P^4}{8\beta}\left(1 - 6\xi\frac{X^2}{\omega^2} - 6\xi'\frac{h^2}{\omega^2}\right)^2. \tag{19}$$

The appropriate change of variables in this case looks as:

$$\omega = r\sin\theta, \qquad X = r\cos\theta\cos\Phi, \qquad h = r\cos\theta\sin\Phi. \tag{20}$$

So we come to the lagrangian (see Eq. (6))

$$L = \frac{1}{2}(\partial_\mu \rho)^2 \cosh^2 F + \frac{1}{2}(\partial_\mu \phi)^2 \sinh^2 F + \frac{1}{2}(\partial_\mu f)^2 - V, \tag{21}$$

$$V = \frac{M_P^4}{8\beta}\left[1 - 6(\xi\cos^2\Phi + \xi'\sin^2\Phi)\sinh^2 F\right]^2 + 9\lambda M_P^4\left[(1 + \alpha^2)\sin^2\Phi - \alpha^2\right]^2\sinh^4 F, \tag{22}$$

where we used the following notations: $(f_0 - f)/(\sqrt{6} M_P) \equiv F$ and $\phi \equiv \sqrt{6} M_P \Phi$. Since α is expected to be tiny, its impact on inflationary dynamics is negligible and we set $\alpha = 0$ hereafter.

If ξ' is small enough the situation is similar to that considered in Section 2. Namely, the field f takes superplanckian values and drives slow roll inflation while the 'Higgs' ϕ takes small (subplanckian) values, see the left plot in Fig. 2. But if $\xi' > \xi$ then $\phi = 0$ is a maximum of the potential (in ϕ-direction) for $f > 0$, see the right plot in Fig. 2, so the mentioned inflationary trajectory becomes unstable. The stable trajectory lies in the valley described by the condition

$$\frac{\partial V}{\partial \Phi} = 0 \tag{23}$$

implying for the given case

$$\sin^2\Phi = \frac{\xi' - \xi}{2\beta\lambda + (\xi' - \xi)^2}\frac{1 - 6\xi\sinh^2 F}{6\sinh^2 F}. \tag{24}$$

The inflation along this valley exactly reproduces Higgs–dilaton inflation [14,23] for $\xi'^2 \gg \beta\lambda$, see the right plot in Fig. 2. In the general case (any ξ'^2 and $\beta\lambda$ but with $\xi' \gg \xi$) one observes that the kinetic term of the field f remains close to canonical when inflaton is far from its minimum ($(1 - 6\xi\sinh^2 F) \sim 1$):

$$(\partial f)^2 + \sinh^2 F(\partial\phi)^2$$
$$= (\partial f)^2\left(1 + \frac{(\xi' - \xi)\cosh^2 F}{[1 - 6\xi\sinh^2 F][12\beta\lambda\sinh^2 F + (\xi' - \xi)(6\xi'\sinh^2 F - 1)]}\right) \tag{25}$$

and the effective potential along the valley (23) is

$$V(F) = \frac{\lambda M_P^4}{4}\frac{1}{2\beta\lambda + (\xi' - \xi)^2}\left(1 - 6\xi\sinh^2 F\right)^2. \tag{26}$$
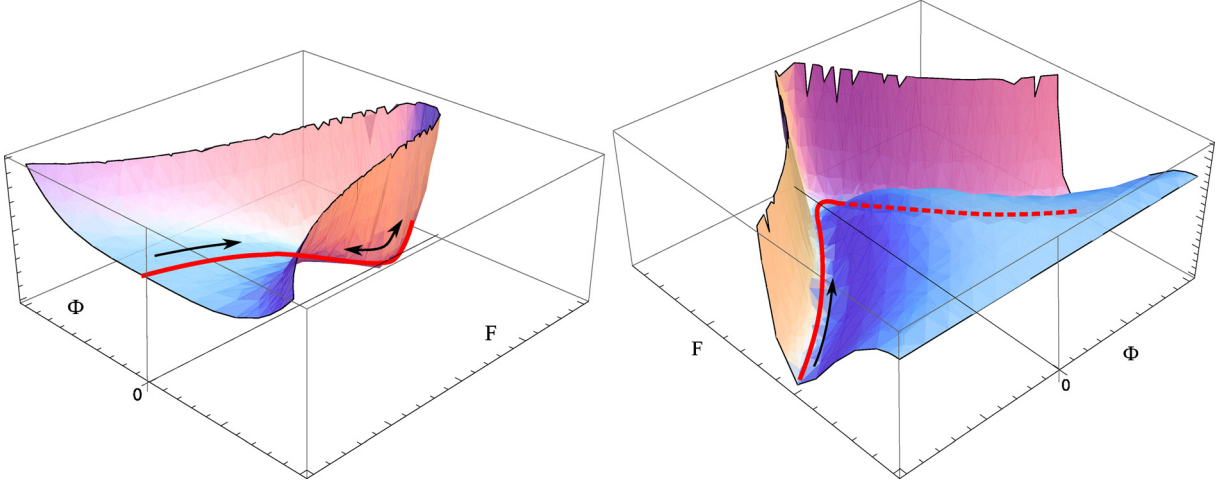
Fig. 2. Competition of the SM Higgs field ϕ and scalaron f at inflationary stage. *Left plot:* $\xi' < \xi$; scalaron drives inflation and later reheats the Universe (like in R^2-dilaton inflation considered in Section 2). *Right plot:* $\xi' > \xi$; mostly the Higgs field drives inflation (like in Higgs-driven inflation [23]).
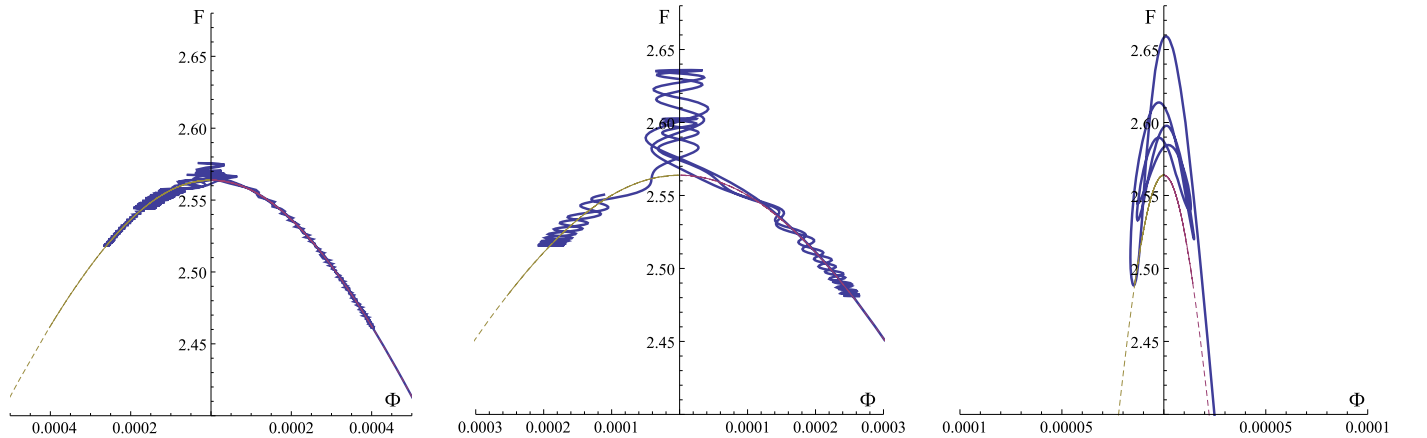

Fig. 3. Post-inflationary trajectories in (Φ, F) space. The dashed line corresponds to the valley (23). *Left plot:* $\xi'^2 \gg 2\beta\lambda$; both inflation and reheating as in the Higgs-inflation [23]. *Right plot:* $\xi'^2 \ll 2\beta\lambda$; Higgs-like inflation in the valley (23) with subsequent reheating due to scalaron decays. *Middle plot:* The intermediate case when $\xi'^2 \approx 2\beta\lambda$; after inflation energy converts to both degrees of freedom.

With potential (26) the amplitude of scalar perturbations is determined by both β and ξ', hence the latter may be chosen to be not as large as needed in the original Higgs–dilaton inflation [14], provided the appropriate value of β.

In all these cases the corresponding valleys attract the inflaton trajectories: starting from general in the context of chaotic inflation initial conditions the inflaton field approaches the attractor and then slowly rolls along the valley. At the latter stage the last 50–60 e-foldings happen. Then following [25] one checks that neither non-gaussian nor isocurvature perturbations are produced to be relevant in cosmology.

To proceed with discussion of the postinflationary stage, one observes that in all these cases, $f = 0$, $\phi = 0$ is an absolute minimum of the potential (22), the inflation drives the fields towards the origin. The expansion near this vacuum reads ($\tilde{\phi} \equiv \phi/\sqrt{6\xi}$ is canonically normalized)

$$V \approx \left(\frac{\sqrt{1+6\xi}}{\sqrt{12\beta}} M_P f + \frac{\xi'-\xi}{\sqrt{8\beta}} \tilde{\phi}^2 \right)^2 + \frac{\lambda}{4} \tilde{\phi}^4. \qquad (27)$$

At low energy field f is superheavy so it decouples from the low energy dynamic and can be integrated out leaving the SM Higgs potential.

The two cases mentioned above differ by direction of the inflaton oscillations after inflation. 'Higgs'-like inflation with $\xi' \gg \xi$ and

$\xi'^2 \gg 2\beta\lambda$ ends by oscillation in ϕ-direction which corresponds to the ordinary Higgs field, see the trajectories on the left plot in Fig. 3. It rapidly decays to SM particles reheating the Universe [15,24]. The dilaton production is negligible due to high reheating temperature [15]. If $\xi'^2 \ll 2\beta\lambda$ (and $\xi > \xi'$) the energy converts mostly to oscillations of the field f, see the right plot in Fig. 3. When $\xi' < \xi$ both inflation and oscillations take place only in f-direction, see the left plot in Fig. 2, with couplings suppressed by the Planck mass (which is similar to the Starobinsky model [6]) and the reheating is delayed [19]. The relevant for inflation regions of model parameter space are outlined in Fig. 4. Note that change in the reheating temperature implies (small) change in the number of e-foldings which determines the values of cosmological parameters (spectral indices, etc.) in an inflationary model.

5. Bounds on scale-invariant inflation

In this section we extend our study on a general inflationary model with scale invariance. Indeed, since the massless dilaton exists in all possible models with spontaneously broken scale invariance there arises the question whether dilaton production at reheating is high enough to give a noticeable contribution to ΔN_{eff}. Consider the scale-invariant lagrangian for the dilaton X and inflaton ϕ with a scale invariant potential:
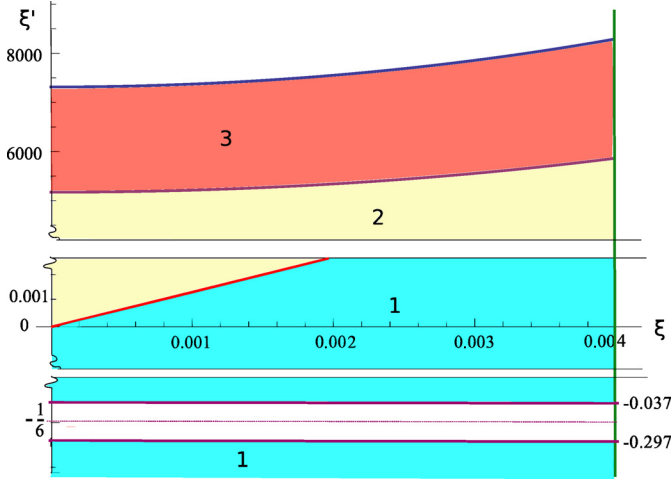
Fig. 4. Shaded regions in (ξ, ξ') plane are allowed from successful inflation and reheating. Region labeled '1' ($\xi' < \xi$) refers to simple scalaron inflation ended by oscillations of the field f and reheating described in Sections 2 and 3. The region near $\xi' = -1/6$ is forbidden because of dilaton overproduction. Domain '2' corresponds to inflation along valley (23) ended by oscillations dominantly in f-direction and the reheating like in previous case. Domain '3' is for the Higgs-like inflation when subsequent oscillations take place in ϕ-direction inside the valley leading to the reheating like in the Higgs-inflation case [24]. In all these cases parameter β is defined by curvature perturbation amplitude $\Delta \simeq 5 \times 10^{-5}$, the e-folding number is $N_e = 55$ and $\lambda = 0.01$.

$$L = -\frac{1}{2}\xi X^2 R + \frac{1}{2}(\partial_\mu X)^2 + \frac{1}{2}(\partial_\mu \phi)^2 - X^4 V\left(\frac{\phi}{X}\right). \quad (28)$$

After conformal transformation $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$ with $\Omega^2 = \xi X^2 / M_P^2$ and redefinition of fields

$$X = \frac{r \sin \theta}{\sqrt{1 + 6\xi}}, \qquad \phi = r \cos \theta \quad (29)$$

we obtain kinetic term K in the form

$$2K = M_P^2 \zeta^2 \left[\frac{(\partial r)^2}{r^2 \sin^2 \theta} + \frac{(\partial \theta)^2}{\sin^2 \theta}\right], \quad \zeta = \sqrt{\frac{1 + 6\xi}{\xi}}. \quad (30)$$

Canonically normalizing the field θ and defining $\rho = M_P \zeta \log r$ we obtain:

$$L = \frac{1}{2}(\partial f)^2 + \frac{1}{2}(\partial \rho)^2 \cosh^2 \tilde{f} - \frac{M_P^4}{\xi^2} V(\sqrt{1 + 6\xi} \sinh \tilde{f}). \quad (31)$$

Here $\tilde{f} \equiv f / \zeta M_P$ and $\sin \theta \equiv 1/\cosh \tilde{f}$. Note that if we start from the Higgs–dilaton-like renormalizable potential $\lambda_0(\phi^2 - \alpha^2 X^2)^2$ in the Jordan frame we arrive at a potential with an exponentially flat plateau. It predicts close to the case of the Starobinsky model values of tilt n_s and tensor-to-scalar ratio r and is strongly supported by the Planck data [7].

We see that the inflaton field f couples to the massless dilaton ρ through its non-canonical kinetic term. So the inflaton can decay to dilatons after inflation and produce the dark radiation. Whether the dilaton production is negligible or not, depends on the function V in (31). Namely, if $V(y)$ has a minimum at $y = 0$ then inflaton oscillates around the origin and the inflaton coupling to dilaton is suppressed by $1/M_P^2$ and hence negligible. But if the minimum of potential is at some nonzero $f = f_0$ the suppression factor is only $1/M_P$. Expanding around the minimum ($f = f_0 + \delta f$) we obtain in this case the interaction term

$$L_{int} = \frac{\text{th} \, \tilde{f}_0}{\zeta M_P}(\partial \rho)^2 \delta f, \quad (32)$$

which corresponds to the decay width of inflaton to dilatons

$$\Gamma_\rho = \frac{m^3 \, \text{th}^2 \, \tilde{f}_0}{32\pi \zeta^2 M_P^2}, \quad (33)$$

where m is inflaton mass. Requiring not to overproduce dilatons constrains the mechanism responsible for the Universe reheating after inflation. Namely, at reheating the dilaton production rate must be sufficiently low as compared to the Universe expansion rate. This sets a lower limit on the reheating temperature:

$$T_{reh} > \frac{1.87}{\sqrt{\Delta N_{max}}} g_*^{-1/4} \sqrt{\Gamma_\rho M_P}, \quad (34)$$

where $\Delta N_{max} = N_{eff} - 3.04$ is the maximal still allowed amount of non-standard dark radiation, a rough estimate from (14), (15) is $\Delta N_{max} \simeq 1$.

Note in passing that gravity interaction and scale invariance in action (28) supplemented with all scale-invariant terms suggest two natural reheating mechanisms: decay to the SM Higgs bosons and anomalous decay to the SM gauge bosons (due to the conformal anomaly). Similarly to Section 3 we have $\Gamma_H / \Gamma_\rho = 4(1 + 6\xi')^2$. For the conformal or nearly conformal Higgs the decays into SM gauge fields dominate, so

$$\Gamma_{gauge} = \Sigma b_i^2 \alpha_i^2 N_i \frac{m^3 \, \text{th}^2 \, \tilde{f}_0}{128\pi^3 \zeta^2 M_P^2}, \quad (35)$$

adopting the same notations as in Section 3. This case is unacceptable, since exactly as it was obtained in Section 3, $\Gamma_{gauge} \sim \Gamma_\rho/130$, which means that mostly all inflatons decay to dilatons grossly violating (14). The model becomes viable after introducing a reheating mechanism more efficient than the conformal anomaly.

6. Conclusions

We investigated the possibility that the probably observed additional dark radiation has an origin associated with the scale invariance. Namely, the additional relativistic degree of freedom may be massless dilaton: the Nambu–Goldstone boson of spontaneously broken scale invariance. Dilaton exists in all possible scale invariant models, but its production in the early Universe and hence its relic abundance is model-dependent. For example, in the Higgs–dilaton model of inflation [14] the dilaton gives negligible impact to the effective number of relativistic degrees of freedom [15].

We examined a natural scale invariant extension of the Starobinsky inflationary model and found that the dilaton production in this case may be significant and explain the observed additional dark radiation. Also we studied the inflation and reheating taking into consideration two fields: scalaron and Higgs in order to distinguish the parameter space of Higgs-like inflation with negligible dilaton production and R^2-like inflation giving a possibility to provide observable amount of dark radiation. Finally, we investigated a minimal scale-invariant extension of a single field inflation and presented general conditions when the dilaton is produced in the amount compatible with the recent observations.

For major part of parameter space the inflation is driven by one field only. The slow roll valley is an attractor, and when either dilaton or scalaron drives inflation, the orthogonal to the inflation trajectory direction in the field space has large mass and the valley is deep enough, similar to what one has in case of Higgs–dilaton inflation [14]. For generic chaotic inflation initial conditions the field starts roll towards larger curvature of the potential and after brief damped oscillations proceeds rolling inside the valley. The latest stage of effectively single field slow roll inflation includes the last

50–60 e-foldings of inflation that we can observe. Thus one naturally expects neither non-gaussianity nor isocurvature perturbations at a noticeable amount [25,26]. However, in specific regions of parameter space (where both dilaton and scalaron actively participate in observable inflationary dynamics) some non-standard perturbations may be produced. Since both scalaron and Higgs decays into the SM particles, the isocurvature perturbations turn into adiabatic, which may change the amplitude of the spectrum. Nevertheless, they may be of some interest in model extensions, where i.e. the dark matter particles or baryon (lepton) asymmetry are produced by scalaron or Higgs field at the reheating stage (see e.g. [19,20,27]). Dilaton is massless and its isocurvature modes would resemble those of neutrinos. Likewise the Universe may reheat by joint work of gravity and SM interactions, which somewhat changes the spectral indices. Numerical calculation of these effects and of the sensitivity to the initial (preinflationary) state we leave for future study.

Acknowledgements

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