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CP violation from scatterings with gauge bosons in leptogenesis

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ABSTRACT

We present an explicit computation of the CP asymmetry in scattering processes involving the heavy right-handed neutrinos of the type I seesaw framework and the Standard Model gauge bosons. Compared to CP violation in two-body decays and in scatterings with top quarks there are new contributions at one loop in the form of new type of vertex corrections as well as of box diagrams. We show that their presence implies that, unlike the CP asymmetry in scatterings with top quarks, the CP asymmetry in scatterings with gauge bosons is different from the two-body decay asymmetry even for hierarchical right-handed neutrinos. This also holds for the L-conserving CP asymmetry in scatterings with $U(1)_Y$ gauge bosons. Quantitatively however, the effects are not very different in size from those of scatterings with top quarks: the CP asymmetry per scattering is $\mathcal{O}(0.5\text{–}2)\times$ the CP asymmetry per decay in all the relevant temperature range for leptogenesis and the generated baryon asymmetry can only be sizeably affected in leptogenesis models that are sensible to the $T \gtrsim M$ epoch.

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1,2]. In the standard framework the tiny neutrino masses are generated via the (type I) seesaw mechanism [3–6] and since the new singlet neutral leptons (N_i) have heavy lepton number (L) violating Majorana masses they can produce dynamically a lepton asymmetry through out of equilibrium processes. Eventually, this lepton asymmetry is partially converted into a baryon asymmetry due to fast sphaleron processes.

In this framework the CP asymmetry can be generated in the N_i decays [7], i.e. $N_i \to l_j H$ [8] as well as in scattering processes of the N_i with particles in the plasma. In perturbation theory, to lowest order, the CP asymmetry in any of these processes arises from the interference between the tree level and one-loop amplitudes. The CP violating asymmetry in scatterings with top quarks has been considered in Refs. [9–12]. In Refs. [9–11] it was argued that an approximate equality between the scattering and decay asymmetries should hold for hierarchical right-handed neutrinos (RHNs). Ref. [12] presented an explicit computation of the CP asymmetry in top quarks scatterings (TQS) and directly showed that this was correct. It is important to note that there is a one to one correspondence between one-loop diagrams in decays and TQS. Then, as discussed in Ref. [11], the "factorization" of a common CP asymmetry can be easily understood in terms of an effective field theory in which all but the lightest of the RHNs have been integrated out and their effect appears in a dim-5 operator $(H\ell)(H\ell)$. In this approximation only "bubble-like" diagrams contribute to the one-loop amplitudes of both decays and TQS.

As for the CP asymmetry in scatterings with gauge bosons, its effect was estimated in Ref. [12] under the assumption that it also factorizes in terms of the decay CP asymmetry. However, the same argument that leads to the understanding of the equality between the CP asymmetries in decay and TQS does not hold for gauge boson scatterings (GBS) due to the presence of additional contributions to the amplitude at one loop (which in the effective field theory approximation contain three rather than two particles in the loop). With this motivation in this note we carry out an explicit calculation of the CP asymmetry in GBS.[1]

For a given heavy neutrino species (called here N_i) there are three different types of GBS, namely

$$N_i A \to \ell_j H, \qquad N_i \bar{\ell}_j \to H A, \qquad \bar{\ell}_j A \to N_i H, \tag{1}$$

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[1] In principle one must consider all processes at a given order in the coupling constants. In particular the three body decays $N_i \to \ell_j H A$ should be included together with the GBS. However the three body decay rate is phase space suppressed, thus the processes $N_i \to \ell_j H A$ and their CP asymmetries can be safely neglected (see e.g. the appendix of [12]).

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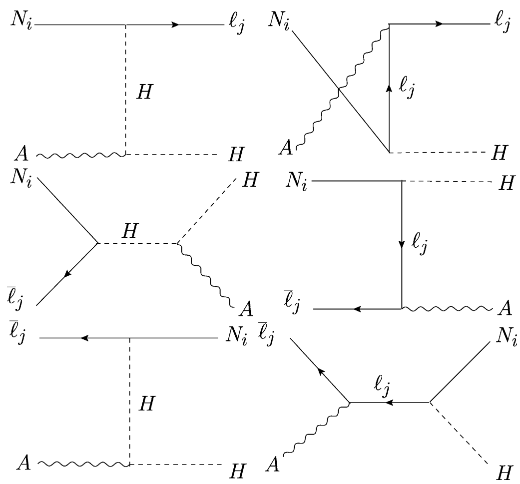

Fig. 1. Tree diagrams for the three scattering processes in (1).

and each one has an associated CP conjugate process. Here $A = W_a$ or B_Y for SU(2)$_L$ and U(1)$_Y$ gauge bosons, respectively. The Lagrangian for the Yukawa and gauge interactions relevant for the computation of these processes reads

$$\mathcal{L}_{Y+A} = -Y_{N_{ij}} \bar{\ell}_j P_R N_i \tilde{H} - \frac{ig_2}{2} (\partial_\mu H)^\dagger W_a^\mu \sigma_a H - \frac{ig_1}{2} (\partial_\mu H)^\dagger B_Y^\mu H + \text{h.c.} + \sum_j \left[\frac{g_2}{2} \bar{\ell}_j \gamma_\mu W_a^\mu \sigma_a P_L \ell_j - \frac{g_1}{2} \bar{\ell}_j \gamma_\mu B_Y^\mu P_L \ell_j \right], \tag{2}$$

where i, j are generation indices of RHNs and lepton doublets $\ell_j^T = (\nu_j, \ell_j^-)$ respectively, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, and $\tilde{H} = i\tau_2 H^*$ with $H^T = (h^+, h^0)$ being the Higgs doublet.

We denote by $\gamma_B^A \equiv \gamma(A \to B)$ the thermally averaged density rate for a state A to go into the state B (summed over initial and final spin and gauge degrees of freedom), and by $\Delta\gamma_B^A \equiv \gamma_B^A - \gamma_{\bar{B}}^{\bar{A}}$ the CP difference between the rates for particle and antiparticle processes. Neglecting the thermal motion with respect to the plasma, the relevant thermal average rates for $12 \to 34$ processes can be written as

$$\gamma_{34}^{12} = \frac{T}{64\pi^4} \int_{s_-}^{\infty} ds \sqrt{s}\, \hat{s}_{34}^{12}(s) K_1\left(\frac{\sqrt{s}}{T}\right) = \frac{T}{2^9\pi^5} \int_{s_-}^{\infty} ds \int_{t_-}^{t_+} dt \frac{|M_{34}^{12}|^2}{\sqrt{s}} K_1\left(\frac{\sqrt{s}}{T}\right),$$

where $\hat{s}_{34}^{12}(s)$ is the reduced cross section, $K_1(x)$ is the modified Bessel function of second kind of order 1, and T is the temperature. We have also introduced the Mandelstam variables in terms of the momenta p_i of the four particles in the process, $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$, with $p_i^2 = m_i^2$. Thus for each process $s_- = \text{Max}[(m_1 + m_2)^2, (m_3 + m_4)^2]$ and

$$t_\mp = \frac{(m_1^2 - m_2^2 - m_3^2 + m_4^2)^2}{4s} - \left[\sqrt{\frac{(s + m_1^2 - m_2^2)^2}{4s} - m_1^2} \pm \sqrt{\frac{(s + m_3^2 - m_4^2)^2}{4s} - m_3^2} \right]^2.$$

For the sake of simplicity in our evaluation of the CP asymmetries we neglect the thermal mass of the gauge bosons, and we include the thermal masses of leptons m_ℓ and the Higgs boson m_H only in the propagators (but not in the loops), in order to regularize the infrared divergences that appear when these states are exchanged in a t- or u-channel.

In this approximation the tree level amplitudes for the three processes in (1) are related by crossing symmetry and are given by

$$\left|M(N_i A \to \ell_j H)\right|^2 = \left|M(N_i A \to \bar{\ell}_j \bar{H})\right|^2 = 2\mathcal{G}_A |Y_{N_{ij}}|^2 I_0(s,t,u), \tag{3}$$

$$\left|M(N_i \bar{\ell}_j \to HA)\right|^2 = \left|M(N_i \ell_j \to \bar{H} A)\right|^2 = -2\mathcal{G}_A |Y_{N_{ij}}|^2 I_0(u,s,t), \tag{4}$$

$$\left|M(\bar{\ell}_j A \to N_i H)\right|^2 = \left|M(\ell_j A \to N_i \bar{H})\right|^2 = 2\mathcal{G}_A |Y_{N_{ij}}|^2 I_0(u,t,s), \tag{5}$$

where $\mathcal{G}_A = 3g_2^2/2$ or $g_1^2/2$ for SU(2)$_L$ and U(1)$_Y$ GBS, respectively.

$I_0(s,t,u) = 2D_{tH} D_{u\ell} s M_i^2 + D_{u\ell}^2 u (M_i^2 - s)$, and D_{XP} comes from the propagator of particle P in the X-channel, $D_{XP} = (X - m_P^2)^{-1}$.

The CP asymmetry in GBS arises from the interference between the tree-level and one-loop amplitudes in Figs. 1 and 2, respectively. In Fig. 2 we show as a "waving" line labeled C_1, C_1' or C_2 the possible branch cuts in which the particles in the propagators can be on-shell and therefore give a contribution to the imaginary part of the corresponding amplitude. As seen in Fig. 2 besides the N_i self-energy (wave) corrections (diagrams (a) and (b)), and the vertex corrections (diagrams (c) and (d)), which are also present at one loop in two-body N_i decays and in TQS, there are new type of vertex diagrams (labeled (e) and (f)), as well as contributions from boxes (diagrams (g) and (h)). Furthermore for process $\bar{\ell}_j A \to N_i H$ additional imaginary parts appear from the last two diagrams 3.(i) and 3.(j).

For each process the sum of amplitudes of wave diagrams (a) and (b) is gauge invariant. The sum of amplitudes for vertex diagrams (e) and (f) is also gauge invariant, while the amplitudes of vertices (c), (d) and boxes (g), (h) are not separately gauge invariant but the sum of the four amplitudes is. Finally for process $\bar{\ell}_j A \to N_i H$ the sum of amplitudes 3.(i) and 3.(j) is gauge invariant.

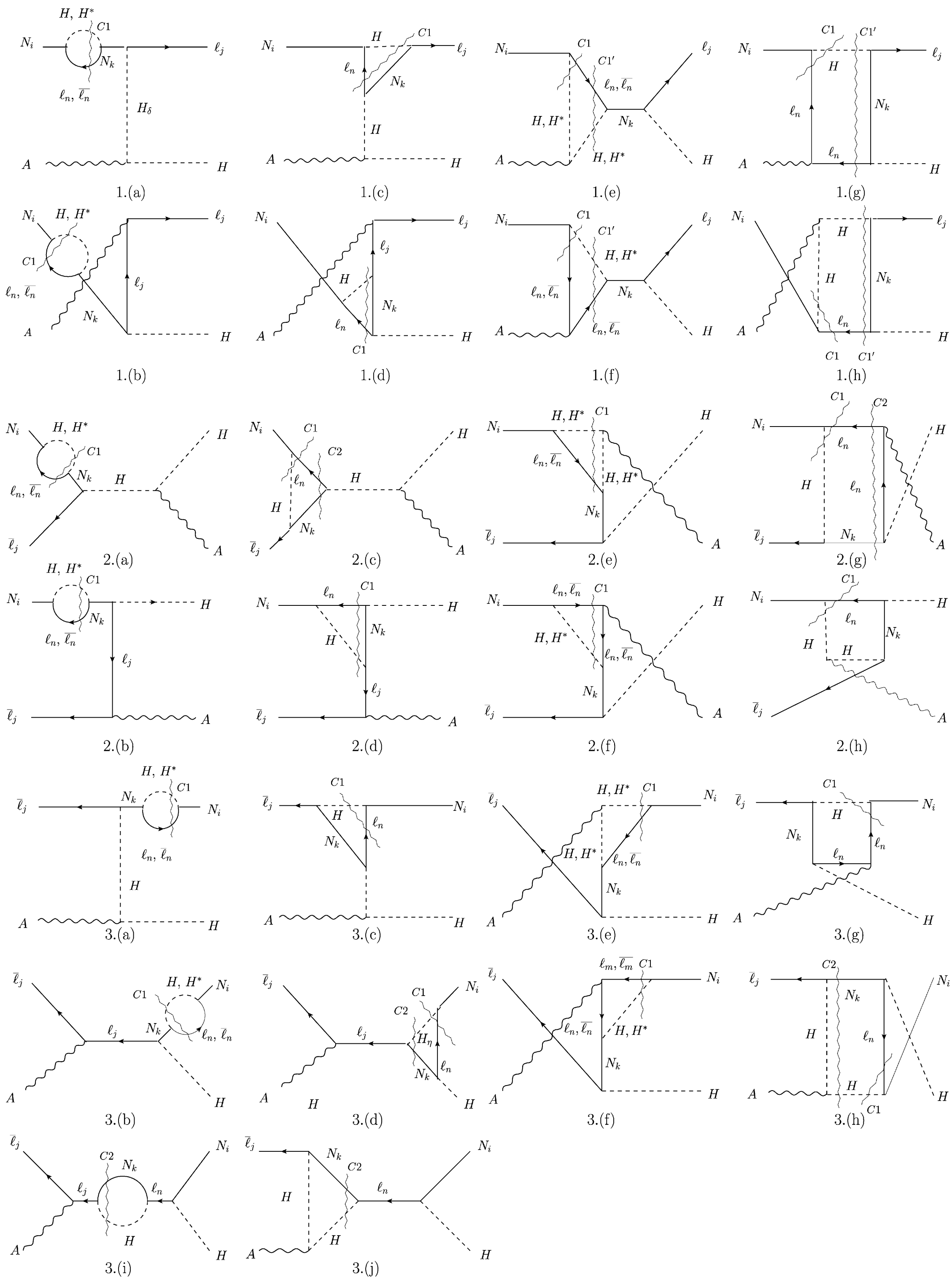

Fig. 2. One-loop diagrams contributing to the CP asymmetry for the scattering processes in (1). We mark with a waving line the possible branch cuts which contribute to the imaginary part of the corresponding amplitude. Additional diagrams which do not contribute to the CP asymmetry include those similar to 3.(i) and 3.(j) for the scatterings $N_i\bar{\ell}_j \to HA$ and $N_iA \to \ell_jH$ for which no branch cut is kinematically allowed, and the corresponding ones with $\ell \leftrightarrow H$ whose coupling with respect to the tree amplitude one is relatively real.

Thus generically the CP asymmetry from any of the three processes in (1) can be written as

$$\epsilon_{34}^{12} \equiv \frac{\gamma_{34}^{12} - \gamma_{\bar{3}\bar{4}}^{\bar{1}\bar{2}}}{\sum_j(\gamma_{34}^{12} + \gamma_{\bar{3}\bar{4}}^{\bar{1}\bar{2}})} \equiv \epsilon_{34}^{12(w)} + \epsilon_{34}^{12(v1)} + \epsilon_{34}^{12(v2)} + \epsilon_{34}^{12(b)} + \Delta\epsilon_{N_iH}^{\bar{\ell}_jA}, \tag{6}$$

where $\epsilon_B^{A(w,v1,v2,b)}$ is generated by the interference between the tree-level amplitudes and the self-energy amplitudes (a) and (b), the vertex amplitudes (c) and (d), the vertex amplitudes (e) and (f), and the box amplitudes (g) and (h), respectively. We denote by $\Delta\epsilon_{N_iH}^{\bar{\ell}_jA}$ the extra asymmetry for the process $\bar{\ell}_jA \to N_iH$ from the diagrams 3.(i) and 3.(j). Notice that each of the diagrams (a), (b), (e) and (f) represents two amplitudes depending on whether the internal lepton line is a lepton or an anti-lepton, which leads respectively to total L-conserving or L-violating contributions.

It is straightforward to show that for any of the processes in (1) we have

$$\epsilon_{\ell_jH}^{N_iA(w)} = \epsilon_{HA}^{N_i\bar{\ell}_j(w)} = \epsilon_{N_iH}^{\bar{\ell}_jA\,(w)} = \epsilon_{\ell_jH}^{N_i\quad(w)} = \frac{\sum_{k\neq i}\left[C_{jk}\frac{\sqrt{a_k}}{a_k-1} + \tilde{C}_{jk}\frac{1}{a_k-1}\right]}{8\pi\left(Y_NY_N^\dagger\right)_{ii}},$$

$$a_k = \frac{M_k^2}{M_i^2}, \qquad C_{jk} = -\operatorname{Im}\left[Y_{N_{ij}}^* Y_{N_{kj}}\left(Y_NY_N^\dagger\right)_{ki}\right], \qquad \tilde{C}_{jk} = -\operatorname{Im}\left[Y_{N_{ij}}^* Y_{N_{kj}}\left(Y_NY_N^\dagger\right)_{ik}\right]. \tag{7}$$

In the above, $\epsilon_{\ell_jH}^{N_i\quad(w)}$ is the contribution to the CP asymmetry in $N_i \to \ell_jH$ from the N_i self-energy diagrams [8] and the first (second) term arises from the L-violating (L-conserving) diagrams.

The contributions to the CP asymmetries from the vertex (labeled with superscripts $(v1), (v2)$), box (superscript (b)), and the extra piece from diagrams 3.(i) and 3.(j) (superscript (Δ)) read[2,3]

$$\begin{aligned}
&\left[\left|M(12\to 34)\right|^2 - \left|M(\bar{1}\bar{2}\to\bar{3}\bar{4})\right|^2\right]^{(v1),(v2),(b),(\Delta)} \\
&\quad= -\sum_{k\neq i}\left\{\frac{\mathcal{G}_AC_{jk}M_kM_i}{2\pi}\left[I_{34}^{12(v1),(b)}(s,t,u) + I'^{12(b)}_{34}(s,t,u) + \theta\left(s-M_k^2\right)J_{34}^{12(v1),(b)}(s,t,u)\right]\right. \\
&\qquad + \frac{\mathcal{F}_AC_{jk}M_kM_i}{2\pi}\left[K_{34}^{12(v2)}(s,t,u) + K'^{12(v2)}_{34}(s,t,u)\right] \\
&\qquad \left. + \frac{\tilde{C}_{jk}M_i^2}{2\pi}\left[\mathcal{F}_A\left(\tilde{K}_{34}^{12(v2)}(s,t,u) + \tilde{K}'^{12(v2)}_{34}(s,t,u)\right) + \mathcal{G}_A\theta\left(s-M_k^2\right)J_{34}^{12(\Delta)}(s,t,u)\right]\right\},
\end{aligned} \tag{8}$$

where $I_{34}^{12(v1),(b)}(s,t,u) \equiv I_{34}^{12(v1)}(s,t,u) + I_{34}^{12(b)}(s,t,u)$ comprises the contribution from the C_1 cuts of the vertex diagrams (c) and (d) (given by $I_{34}^{12(v1)}(s,t,u)$), and box diagrams (g) and (h) (given by $I_{34}^{12(b)}(s,t,u)$). $I'^{12(b)}_{34}(s,t,u)$ contains the contribution from the C_1' cuts present only in the box diagrams of $N_iA \to \ell_jH$. $J_{34}^{12(v1),(b)}(s,t,u) \equiv J_{34}^{12(v1)}(s,t,u) + J_{34}^{12(b)}(s,t,u)$ has the contribution from the C_2 cuts, which are kinematically allowed when $s > M_k^2$ and they are possible only in the processes $N_i\bar{\ell}_j \to HA$ and $\bar{\ell}_jA \to N_iH$. All these contributions are L-violating and they are present for both $SU(2)_L$ and $U(1)_Y$ GBS. $J_{34}^{12(\Delta)}(s,t,u)$ has the – L-conserving – contribution from the C_2 cuts in the diagrams 3.(i) and 3.(j) present only for $\bar{\ell}_jA \to N_iH$.

$K_{34}^{12(v2)}(s,t,u)$ ($\tilde{K}_{34}^{12(v2)}(s,t,u)$) contains the contribution from the C_1 cuts of the L-violating (L-conserving) vertex diagrams in graphs (e) and (f). Similarly, $K'^{12(v2)}_{34}(s,t,u)$ ($\tilde{K}'^{12(v2)}_{34}(s,t,u)$) involves the contribution from the C_1' cuts of the L-violating (L-conserving) diagrams in the graphs (e) and (f) of process $N_iA \to \ell_jH$. Notice that the diagrams (e) and (f) correspond to amplitudes where the gauge boson is coupled to the one-loop self energy $N_i - N_k$. Since the RHNs are $SU(2)_L$ singlets, the sum over the $SU(2)_L$ degrees of freedom cancel in each of these graphs. This, however, is not the case for the $U(1)_Y$ gauge boson. Correspondingly the coupling factors $\mathcal{F}_A = 0$ for $A = W_a$ while $\mathcal{F}_A = \mathcal{G}_A = g_1^2/2$ for $A = B_Y$.

The C_1 cuts are kinematically allowed for all processes and their corresponding contributions to the amplitudes are related by crossing symmetry. For the amplitudes of diagrams (c) and (d) this reads $I_{\ell_jH}^{N_iA(v1)}(s,t,u) = I^{(v1)}(s,t,u)$, $I_{HA}^{N_i\bar{\ell}_j(v1)}(s,t,u) = -I^{(v1)}(u,s,t)$, and $I_{N_iH}^{\bar{\ell}_jA\,(v1)}(s,t,u) = I^{(v1)}(u,t,s)$, with

$$\begin{aligned}
I^{(v1)}(s,t,u) = {} & D_{u\ell}^2M_i^2\left[\frac{\tilde{s}\tilde{u}^2}{(1-\tilde{u})^2}\left(-1 + \frac{a_k}{1-\tilde{u}}L_2(\tilde{u})\right) + \frac{\tilde{t}\tilde{u}}{(1-\tilde{u})^2}\left(1 - \frac{a_k+1-\tilde{u}}{1-\tilde{u}}L_2(\tilde{u})\right)\right] \\
& + D_{tH}D_{u\ell}M_i^2\left[\frac{\tilde{s}}{1-\tilde{t}}\left(1 - \frac{a_k+1-\tilde{t}}{1-\tilde{t}}L_2(\tilde{t})\right) + \frac{\tilde{s}}{1-\tilde{u}}\left(1 - \frac{a_k+1-\tilde{u}}{1-\tilde{u}}L_2(\tilde{u})\right)\right].
\end{aligned} \tag{9}$$

The C_2 contributions to the vertex asymmetry from diagrams (c) and (d) are

$$\begin{aligned}
J_{HA}^{N_i\bar{\ell}_j(v1)}(s,t,u) &= D_{sH}D_{t\ell}M_i^2\frac{\tilde{u}}{1-\tilde{s}}\left[1 - \frac{a_k}{\tilde{s}} - \frac{a_k+1-\tilde{s}}{1-\tilde{s}}L_4(\tilde{s})\right], \\
J_{N_iH}^{\bar{\ell}_jA\,(v1)}(s,t,u) &= D_{tH}D_{s\ell}M_i^2\frac{\tilde{u}}{1-\tilde{s}}\left[a_k - \tilde{s} + \frac{a_k+1-\tilde{s}}{1-\tilde{s}}L_4(\tilde{s})\right] + D_{s\ell}^2M_i^2\frac{\tilde{s}}{(1-\tilde{s})^2}\left[(\tilde{t}-\tilde{u})(a_k-\tilde{s}) + \frac{\tilde{t}(a_k+1-\tilde{s}) - a_k\tilde{s}\tilde{u}}{1-\tilde{s}}L_4(\tilde{s})\right],
\end{aligned} \tag{10}$$

[2] When evaluating $\epsilon_{34}^{12(v1)}$ and $\epsilon_{34}^{12(b)}$ one must specify a gauge since, as mentioned above, the one-loop vertex $(v1)$ and box amplitudes are not separately gauge-invariant. In what follows we will work in the unitary gauge.

[3] Since the resulting expressions are somewhat cumbersome, to double check our results we have computed the asymmetries both by explicit evaluation of the corresponding loop integrals as well as by using the Cutkosky rules that give directly the absorptive part of the Feynman diagrams.

and $J^{N_iA(v1)}_{\ell_j H}(s,t,u)=0$. In Eqs. (9)–(10) and the following we define $\tilde{s}=\frac{s}{M_i^2}$, $\tilde{t}=\frac{t}{M_i^2}$, $\tilde{u}=\frac{u}{M_i^2}$, and

$$\begin{aligned}
&L_1(x)\equiv \ln\frac{a_k+x}{x}, \qquad L_2(x)\equiv \ln\frac{a_k+1-x}{a_k}, \qquad L_4(x)\equiv \ln\frac{x(a_k+1-x)}{a_k},\\
&L_3(x,y)\equiv \ln\frac{xy+a_k(1-x)}{a_k(1-x)}, \qquad L_5(x,y)\equiv \ln\frac{xy+a_k(1-x)}{a_k}.
\end{aligned}\tag{11}$$

For the C_1 contributions to the asymmetry from diagrams (e) and (f) the crossing symmetry reads $K^{N_iA(v2)}_{\ell_j H}(s,t,u)=K^{(v2)}(s,t,u)$, $K^{N_i\bar{\ell}_j(v2)}_{HA}(s,t,u)=-K^{(v2)}(u,s,t)$, $K^{\bar{\ell}_jA\,(v2)}_{N_iH}(s,t,u)=K^{(v2)}(u,t,s)$, and equivalently for the $\tilde{K}$ functions, with

$$\begin{aligned}
K^{(v2)}(s,t,u)&=\frac{2\tilde{s}\{D_{tH}\tilde{t}[(\tilde{t}-2\tilde{u})+\tilde{u}(1-\tilde{s})]-D_{u\ell}\tilde{u}[3\tilde{t}+\tilde{u}(1-\tilde{s})]\}}{(\tilde{s}-a_k)(\tilde{t}+\tilde{u})^3},\\
\tilde{K}^{(v2)}(s,t,u)&=\frac{2\tilde{s}\tilde{u}\{D_{tH}\tilde{t}[3-2(\tilde{t}+\tilde{u})]+D_{u\ell}[(2\tilde{t}-\tilde{u})-\tilde{t}^2+\tilde{u}^2]\}}{(\tilde{s}-a_k)(\tilde{t}+\tilde{u})^3}.
\end{aligned}\tag{12}$$

The contribution from the C_1' cuts in diagrams (e) and (f) (which are only present for $N_iA\to\ell_jH$) is

$$K'^{N_iA(v2)}_{\ell_j H}(s,t,u)=-K^{(v2)}(s,t,u), \qquad \tilde{K}'^{N_iA(v2)}_{\ell_j H}(s,t,u)=-\tilde{K}^{(v2)}(s,t,u),\tag{13}$$

and hence the contributions from vertices (e) and (f) to $N_iA\to\ell_jH$ cancel exactly.

For the box diagrams $I^{N_iA(b)}_{\ell_j H}(s,t,u)=I^{(b)}(s,t,u)\equiv I_{b1}(s,t,u)+I_{b2}(s,t,u)$, $I^{N_i\bar{\ell}_j(b)}_{HA}(s,t,u)=-I^{(b)}(u,s,t)$, and $I^{\bar{\ell}_jA\,(b)}_{N_iH}(s,t,u)=I^{(b)}(u,t,s)$, with

$$\begin{aligned}
I_{b1}(s,t,u)&=D_{tH}\left\{\frac{\tilde{s}\tilde{u}}{(1-\tilde{s})^2}-\frac{\tilde{s}(a_k+1-\tilde{u})}{\tilde{u}(1-\tilde{t})}L_2(\tilde{t})+\left[\frac{a_k+\tilde{s}}{\tilde{u}}+\frac{a_k(a_k+1-\tilde{t})}{\tilde{s}\tilde{t}+a_k(1-\tilde{s})}\right]L_3(\tilde{s},\tilde{t})\right\}\\
&\quad+D_{u\ell}\left[-1-\frac{\tilde{s}\tilde{t}}{(1-\tilde{s})^2}+\frac{a_k+\tilde{s}}{1-\tilde{t}}L_2(\tilde{t})+\frac{(a_k+\tilde{s})(a_k+1-\tilde{t})}{\tilde{s}\tilde{t}+a_k(1-\tilde{s})}L_3(\tilde{s},\tilde{t})\right],
\end{aligned}\tag{14}$$

$$\begin{aligned}
I_{b2}(s,t,u)&=D_{tH}\left\{\frac{\tilde{s}\tilde{t}}{(1-\tilde{s})^2}+\frac{a_k+1-\tilde{u}}{\tilde{t}}\left[-\frac{\tilde{s}L_2(\tilde{u})}{1-\tilde{u}}+\frac{(1-\tilde{s})(a_k+\tilde{s})+a_k\tilde{t}}{\tilde{s}\tilde{u}+a_k(1-\tilde{s})}L_3(\tilde{s},\tilde{u})\right]\right\}\\
&\quad+D_{u\ell}\left\{\frac{\tilde{s}\tilde{t}}{(1-\tilde{s})^2}-\frac{\tilde{u}}{\tilde{t}(1-\tilde{s})}-\frac{(a_k+1-\tilde{u})[\tilde{s}\tilde{t}-\tilde{u}(1-\tilde{u})]}{\tilde{t}^2(1-\tilde{u})}L_2(\tilde{u})\right.\\
&\quad\left.-\left[\frac{1-3\tilde{s}-2a_k}{\tilde{t}}+\frac{(1-\tilde{s})(a_k+\tilde{s})}{\tilde{t}^2}+\frac{a_k-\tilde{s}}{\tilde{s}}-\frac{a_k(a_k+\tilde{s})}{\tilde{s}^2\tilde{u}+a_k\tilde{s}(1-\tilde{s})}\right]L_3(\tilde{s},\tilde{u})\right\}.
\end{aligned}\tag{15}$$

The corresponding contributions from C_1' cuts read $I'^{N_i\bar{\ell}_j(b)}_{HA}(s,t,u)=I'^{\bar{\ell}_jA(b)}_{N_iH}(s,t,u)=0$ and $I'^{N_iA(b)}_{\ell_j H}(s,t,u)=I'^{(b)}(s,t,u)\equiv I'_{b1}(s,t,u)+I'_{b2}(s,t,u)$, with

$$\begin{aligned}
I'_{b1}(s,t,u)&=D_{tH}\left\{-\frac{\tilde{s}\tilde{u}}{(1-\tilde{s})^2}+\frac{a_k+\tilde{s}}{\tilde{u}}L_1(\tilde{s})-\left[\frac{a_k+\tilde{s}}{\tilde{u}}+\frac{a_k(a_k+1-\tilde{t})}{\tilde{s}\tilde{t}+a_k(1-\tilde{s})}\right]L_3(\tilde{s},\tilde{t})\right\}\\
&\quad+D_{u\ell}\left[1+\frac{\tilde{s}\tilde{t}}{(1-\tilde{s})^2}-\frac{a_k+\tilde{s}}{\tilde{s}}L_1(\tilde{s})-\frac{(a_k+\tilde{s})(a_k+1-\tilde{t})}{\tilde{s}\tilde{t}+a_k(1-\tilde{s})}L_3(\tilde{s},\tilde{t})\right],
\end{aligned}\tag{16}$$

$$\begin{aligned}
I'_{b2}(s,t,u)&=D_{tH}\left\{-\frac{\tilde{s}\tilde{t}}{(1-\tilde{s})^2}+\frac{a_k+1-\tilde{u}}{\tilde{t}}\left[L_1(\tilde{s})-\frac{(1-\tilde{s})(a_k+\tilde{s})+a_k\tilde{t}}{[\tilde{s}\tilde{u}+a_k(1-\tilde{s})]}L_3(\tilde{s},\tilde{u})\right]\right\}\\
&\quad+D_{u\ell}\left\{\frac{\tilde{s}}{\tilde{t}}-\frac{1-\tilde{s}+\tilde{s}\tilde{t}}{(1-\tilde{s})^2}+\left[\frac{(1-\tilde{u})^2(a_k+\tilde{s})}{\tilde{s}\tilde{t}^2}+\frac{1-\tilde{u}}{\tilde{t}}-\frac{a_k+1-\tilde{u}}{\tilde{t}^2}\right]L_1(\tilde{s})\right.\\
&\quad\left.+\left[\frac{1-3\tilde{s}-2a_k}{\tilde{t}}+\frac{(1-\tilde{s})(a_k+\tilde{s})}{\tilde{t}^2}+\frac{a_k-\tilde{s}}{\tilde{s}}-\frac{a_k(a_k+\tilde{s})}{\tilde{s}^2\tilde{u}+a_k\tilde{s}(1-\tilde{s})}\right]L_3(\tilde{s},\tilde{u})\right\}.
\end{aligned}\tag{17}$$

From Eqs. (14)–(17) we see that for the scattering process $N_iA\to\ell_jH$ the contributions from C_1 and C_1' partially cancel each other.[4]

[4] By definition $I^{12(b)}_{34}$, $I'^{12(b)}_{34}$, and $J^{12(b)}_{34}$ must be real. It can be easily verified that in their expressions the imaginary contributions from negative arguments in the logarithms of Eq. (11) always cancel out.

The C_2 contributions to the box asymmetry are $J_{\ell_j H}^{N_i A(b)}(s,t,u) = 0$ and

$$J_{HA}^{N_i\bar{\ell}_j(b)}(s,t,u) = D_{sH}\left\{\frac{\tilde{u}(a_k+1-\tilde{s})}{\tilde{t}(\tilde{s}-1)}L_4(\tilde{s}) + \left[\frac{a_k+\tilde{u}}{\tilde{t}} + \frac{a_k(a_k+1-\tilde{s})}{\tilde{s}\tilde{u}+a_k(1-\tilde{u})}\right]L_5(\tilde{u},\tilde{s})\right\} + D_{t\ell}\left\{\frac{(1-\tilde{t})(a_k-\tilde{s})}{\tilde{s}^2} - \frac{(a_k+\tilde{u})}{\tilde{s}-1}L_4(\tilde{s}) + \frac{(a_k+1-\tilde{s})(a_k+\tilde{u})}{\tilde{s}\tilde{u}+a_k(1-\tilde{u})}L_5(\tilde{u},\tilde{s})\right\}, \tag{18}$$

$$J_{N_i H}^{\bar{\ell}_j A\,(b)}(s,t,u) = D_{s\ell}\left\{\frac{\tilde{s}-a_k}{\tilde{t}} - \frac{(a_k-\tilde{s})(a_k+\tilde{u})}{\tilde{s}\tilde{u}+a_k(1-\tilde{u})}L_5(\tilde{u},\tilde{s}) + \frac{a_k+1-\tilde{s}}{\tilde{t}^2}\left[\frac{\tilde{t}\tilde{u}-\tilde{s}(1-\tilde{s})}{1-\tilde{s}}L_4(\tilde{s}) + (\tilde{s}-\tilde{t})L_5(\tilde{u},\tilde{s})\right]\right\} + D_{tH}\left\{\frac{\tilde{u}(a_k+1-\tilde{s})}{\tilde{t}(1-\tilde{s})}L_4(\tilde{s}) - \frac{(a_k+1-\tilde{s})[(a_k+\tilde{u})(1-\tilde{u})+a_k\tilde{t}]}{\tilde{t}[\tilde{s}\tilde{u}+a_k(1-\tilde{u})]}L_4(\tilde{s})\right\}. \tag{19}$$

Finally the C_2 contributions from diagrams 3.(i) and 3.(j) read

$$J_{N_i H}^{\bar{\ell}_j A\,(\Delta)}(s,t,u) = -D_{s\ell}\frac{M_i^2(\tilde{s}-a_k)}{2\tilde{s}}\left[\frac{\tilde{s}-a_k}{\tilde{s}} + D_{s\ell}M_i^2(\tilde{s}+a_k)\right]\left[D_{tH}\tilde{u} + D_{s\ell}\tilde{s}(1-\tilde{u})\right]. \tag{20}$$

The ratios between the L-violating asymmetries in GBS and the L-violating asymmetry in decays are shown in Fig. 3 as a function of T/M_1 (left panels) and of M_2/M_1 (right panels). For simplicity we have considered the virtual effects of only one neutrino species $N_k = N_2$ different from $N_i = N_1$. It is apparent that for $N_1\bar{\ell}_j \to HA$ and $\bar{\ell}_j A \to N_1 H$ this ratio deviates from unity even for hierarchical neutrinos.[5] In the relevant temperature range for leptogenesis $0.1 \lesssim T/M_1 \lesssim 10$ the deviation can be of several tens of percent, e.g. 50% for $\bar{\ell}_j B_Y \to N_i H$ at $T = M_1$. Conversely for $N_1 A \to \ell_j H$ the ratio tends to one for hierarchical neutrinos, hence the asymmetry can be well approximated by the decay one if $M_2/M_1 \gg 10$, but corrections appear at high temperatures for milder hierarchies.

These results can be easily understood analytically by expanding to lowest order in M_i/M_k the expressions in Eqs. (9), (12), and (14)–(17) which gives

$$I^{(v1)}(s,t,u) = -\frac{1}{2}\frac{I_0(s,t,u)}{M_k^2}, \tag{21}$$

$$I^{(b)}(s,t,u) = \frac{M_i^2}{M_k^2}\frac{\tilde{s}\{D_{u\ell}\tilde{u}[3\tilde{t}+\tilde{u}(1-\tilde{s})] - D_{tH}\tilde{t}[(\tilde{t}-2\tilde{u})+\tilde{u}(1-\tilde{s})]\}}{(\tilde{t}+\tilde{u})^3}, \tag{22}$$

$$I'^{(b)}(s,t,u) = -I^{(b)}(s,t,u), \tag{23}$$

$$K^{(v2)}(s,t,u) = 2I^{(b)}(s,t,u) = -K'^{(v2)}(s,t,u), \tag{24}$$

$$\tilde{K}^{(v2)}(s,t,u) = -\frac{M_i^2}{M_k^2}\frac{2\tilde{s}\tilde{u}\{D_{tH}\tilde{t}[3-2(\tilde{t}+\tilde{u})] + D_{u\ell}[(2\tilde{t}-\tilde{u}) - \tilde{t}^2 + \tilde{u}^2]\}}{(\tilde{t}+\tilde{u})^3}. \tag{25}$$

Including these results in Eq. (8) and using Eqs. (3)–(5) one finds that

$$\epsilon_{\ell_j H}^{N_i A(v1)} = \epsilon_{HA}^{N_i\bar{\ell}_j(v1)} = \epsilon_{N_i H}^{\bar{\ell}_j A\,(v1)} = \frac{\sum_{k\neq i} C_{jk}}{16\pi (Y_N Y_N^\dagger)_{ii}}\frac{M_i}{M_k} = \epsilon_{\ell_j H}^{N_i\;(v)}. \tag{26}$$

In the last equality we have used that the vertex CP asymmetry from the decay $N_i \to \ell_j H$

$$\epsilon_{\ell_j H}^{N_i\;(v)} = -\frac{\sum_{k\neq i} C_{jk}}{8\pi (Y_N Y_N^\dagger)_{ii}}\sqrt{a_k}\left[1 - (1+a_k)\ln\frac{1+a_k}{a_k}\right] \simeq \frac{\sum_{k\neq i} C_{jk}\frac{M_i}{M_k}}{16\pi (Y_N Y_N^\dagger)_{ii}}, \tag{27}$$

where the last approximation holds for $M_k/M_i \gg 1$. So, as in the case of TQS, both wave and vertex contributions to the CP asymmetry in scatterings with $SU(2)_L$ gauge bosons are equal to the corresponding contributions to the decay CP asymmetry in the hierarchical limit ($M_k/M_i \gg 1$).

For the process $N_i A \to \ell_j H$ Eqs. (22)–(24) imply that in the hierarchical limit the CP symmetries from vertices (e) and (f) with $U(1)_Y$ gauge bosons, and from boxes with either $SU(2)_L$ or $U(1)_Y$ gauge bosons vanish,

$$\epsilon_{\ell_j H}^{N_i W_a(b)} = \epsilon_{\ell_j H}^{N_i B_Y(b)} = \epsilon_{\ell_j H}^{N_i B_Y(v2)} = 0. \tag{28}$$

However for the other two processes we find that the L-violating contributions to the CP asymmetries satisfy

$$\epsilon_{HW_a}^{N_i\bar{\ell}_j(b)} \neq 0, \qquad \epsilon_{N_i H}^{\bar{\ell}_j W_a(b)} \neq 0, \qquad \epsilon_{HB_Y}^{N_i\bar{\ell}_j(b)} = \frac{1}{2}\epsilon_{HB_Y}^{N_i\bar{\ell}_j(v2)} \neq 0, \qquad \epsilon_{N_i H}^{\bar{\ell}_j B_Y(b)} = \frac{1}{2}\epsilon_{N_i H}^{\bar{\ell}_j B_Y(v2)} \neq 0. \tag{29}$$

[5] Unlike the CP asymmetry in decays, the CP asymmetry in scatterings depends on the temperature, thus when comparing both types of asymmetries at a given T, the RHNs are considered to be hierarchical enough if $M_{k\neq 1} \gg M_1, T$.

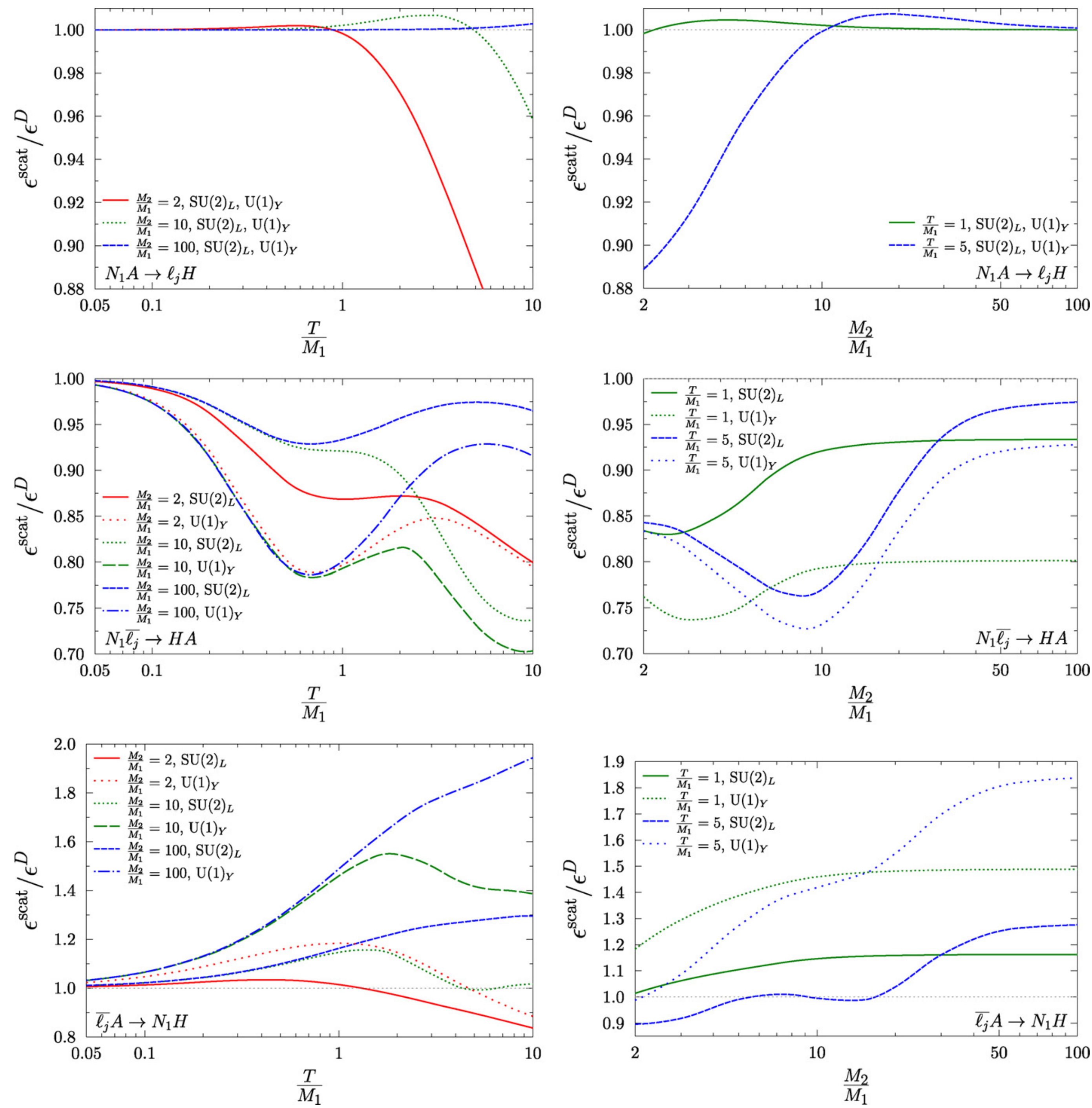

Fig. 3. Ratio between the L-violating asymmetry in scatterings $\epsilon^{scat} \equiv \epsilon_{34}^{12}$ for the different processes as labeled in the figure and the corresponding quantity for decays $\epsilon^D \equiv \epsilon_{\ell_j H}^{N_i}$. To illustrate we have used $m_\ell = 0.18T$ and $m_H = 0.34T$. The left panels show the ratio as a function of T/M_1 for three different hierarchies $M_2/M_1 = 2, 10, 100$. The right panels show the ratio as a function of M_2/M_1 for two values of $T/M_1 = 1, 5$.

We note that the vanishing of $\epsilon_{\ell_j H}^{N_i A(b)}$ and $\epsilon_{\ell_j H}^{N_i B_Y(v2)}$ at leading order in M_i/M_k is due to the presence of the additional C_1' cuts, which, as mentioned above, results in a partial or total cancellation of the contributions from the C_1 cuts so that only terms of higher order in M_i/M_k remain.

We have mentioned before that for each process the sum of amplitudes of wave diagrams is independently gauge invariant, as it is the sum of the amplitudes for the vertex ($v2$) diagrams, but the amplitudes of vertex ($v1$) and box diagrams are not separately gauge invariant. However at the leading order in M_i/M_k, the box and vertex ($v1$) diagrams are independently gauge invariant, thus the results in Eqs. (26), (28), and (29) are also gauge independent.

Summarizing, we find that even for very hierarchical heavy RHNs

$$\epsilon_{\ell_j H}^{N_i A} = \epsilon_{\ell j H}^{N_i} \neq \epsilon_{HA}^{N_i \bar{\ell}_j} \neq \epsilon_{N_i H}^{\bar{\ell}_j A}, \tag{30}$$

where $\epsilon_{\ell jH}^{N_i} = \epsilon_{\ell jH}^{N_i\ (w)} + \epsilon_{\ell jH}^{N_i\ (w)}$. Hence, we conclude that the CP asymmetry in the scattering $N_i A \to \ell_j H$ factorizes in the hierarchical limit. However, this is not the case for the scatterings $N_i \bar{\ell}_j \to HA$ and $\bar{\ell}_j A \to N_i H$.

Also, as mentioned above, in the processes $N_i \bar{\ell}_j \to H B_Y$ and $\bar{\ell}_j B_Y \to N_i H$ there are new L-conserving vertex contributions to the CP asymmetry. They imply that in the hierarchical limit the L-conserving CP asymmetry in these scatterings is not equal to the L-conserving CP asymmetry in decays. Note that although the L-conserving asymmetries are always suppressed by a higher power of M_i/M_k compared to the L-violating ones, they are the only source of CP violation in models with conservation of L [13–15].

Finally, a few comments about the impact of our results for the determination of the baryon asymmetry are in order. Although the CP asymmetry in GBS has more interesting features compared to the one in TQS, we have found that still the CP asymmetry per scattering is (0.5–2) $\times$ the CP asymmetry per decay. Therefore the generation of CP asymmetry through GBS is relevant at high temperatures ($T \gtrsim M_i$) when the scattering rates are larger than the decay rates. Hence it can have effects in models for leptogenesis that are sensible to the high temperature regime. This includes standard type I leptogenesis in the weak washout regime and low energy models in which the sphaleron processes freeze out before the RHNs have decayed [14,16]. Quantitatively including the GBS can result into an increase of the final baryon asymmetry by more than a factor $\sim$2 for standard type I leptogenesis in the weak washout regime for zero initial abundances of the heavy neutrinos, i.e., when they are produced only by the CP-violating Yukawa interactions. For the low energy models, the effects can be larger than even one order of magnitude if the sphalerons freeze out when the scattering rates are much larger than the decay rates.

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