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Fundamental Problems of Modeling the Fracture Processes in Concrete II: Size Effect and Selection of the Solution Approach

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Abstract

Theory of concrete fracture, despite all the efforts of numerous researchers, still did not provide the clear answer to the problem of modeling the fracture processes of concrete. Three well known theories are at hand: fracture mechanics, plasticity theory and mechanics of continuous damages. The fundamental assumptions, those theories are based on, do not completely correspond to the nature of concrete. They all are confronted with numerous problems, out of which the four are fundamental: damages micromechanics, damages localization, size effects and the dilemma when to apply the phenomenological and when the micromechanical approach to considering this problem. In this paper are considered the last two of those problems: size effect (scaling laws) and decision making what would be the best way in solving problems of modeling the fracture processes in concrete and concrete structures.

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Keywords: concrete fracture; modeling; quasi brittle material; microechanics; damage localization; size effect

1. Introduction

Problems of understanding, explaining and solving modeling of the fracture processes in concrete and concrete structures are extensively studied for decades. However, despite the voluminous research conducted, to what

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testifies numerous published scientific papers, these and dissertations [1-21], behavior of concrete is still not well understood and explained.

Concrete belongs into a group of the so-called quasi-brittle materials and its complex behavior on the macro level is determined by its heterogeneous structure, which can be observed at the nano, micro or macro level. For modeling the complex processes in fracture of concrete, three branches of sciences can be applied: theory of plasticity, fracture mechanics and continuum damage mechanics. Regardless of which of the three theories is applied, there are several main issues the researchers are confronted with, out of which the most important are: micromechanics of damages, localization of damages, size effects and the question either to apply the phenomenological approach in solving those problems or to resort to the micromechanical approach. In this paper authors were focusing on the last two problems: size effects (explained by various scaling laws) and decision making what would be the best way in solving problems of modeling the fracture processes in concrete and concrete structures*.

Numerous macroscopic models based on plasticity theory, fracture or continuum damage mechanics, have been proposed and implemented in finite element codes with varying degrees of success. They usually use various parameters, which are difficult to determine and no model has been able to fully simulate all the aspects of the complex nature of fracture in concrete. Actually, each of those theories supplies only partial explanation for fracture processes in concrete, i.e., only some individual aspects of those processes. This is why some researchers have tried to apply more complex procedures, the so-called complex systems theories, like the statistical physics, theory of dynamical systems, bifurcation theory, etc.

The well known fact in modeling any process in quasi-brittle materials, like rocks, sea ice, fiber reinforced composites or concrete, is the influence of sizes – the so-called size effects, which has dual nature, namely both sizes of a structure (or specimen) and of defects (voids, cracks, inhomogeneities) influence the structural behavior under loads and in presence of defects. The problem of size effect is particularly important to structural and geotechnical engineers and to geophysicists. The problem is practically of lesser interest to mechanical and aerospace engineers because most structures are tested at full size. The main problem for them is the prediction of the structure life, i.e., extrapolation in time rather than size. In solving civil engineering problems, engineers must inevitably extrapolate from reduced-scale laboratory tests to real structures, which are too large to be tested systematically.

Modeling of fracture in concrete can be classified in the two categories: macroscopic models based on the phenomenological approach and models based on micromechanical solutions. The former models are based on theory of plasticity and fracture mechanics, while the latter models are establishing the relationship between the microstructure of concrete and its macroscopic behavior. Which of those two approaches should be applied in solving certain problems related to fracture process in concrete structure, depends on the type of structure, type and way of loading, material properties, etc. In some cases the former approach has advantages, while for some other problems the latter approach is the better solution. Combination of both approaches can also be considered, though it poses problems related to too complex mathematical apparatus or impossibility of numerical simulations.

2. Size effect in modeling the concrete fracture

Concrete may be considered as the composite material consisting of the three main components: the cement matrix, the aggregates and the interface – the transition layer the so-called transition halo. This transition zone is the most porous part of the composite and is considered as the weakest. Initiation of the concrete damages always appears in this zone. The nature of concrete is somewhat particular, since even before any load is applied to a concrete structure, the material contains many micro cracks. They are the consequence of concrete shrinkage and the heat development in the process of concrete hydration. Once the load is applied those preexisting micro cracks start to propagate and interact, creating a network of cracks, thus forming macro cracks, which cause the phenomenon

* The first two problems are discussed in the first part of this study: "Fundamental problems of modeling the fracture processes in concrete I: Micromechanics and localization of damages" by the same authors.

known as strain softening or the damage localization. The experimentally observed structural size effect, as well as the related spurious effect of the mesh size, presents the most potent argument for the application of fracture mechanics to concrete structures.

The quasi brittle materials, due to the fact that they do not have enough plastic reserve, are characterized by the gradual softening in the fracture process zone. With mentioned formation of macro cracks, this process zone becomes of significant size with respect to the concrete specimen's or concrete structure's size. This problem is present, regardless of the type of load the concrete is subjected to, i.e., whether it is loaded in tension or compression. This is the so-called scaling problem, when the size of a defect (in this case crack and its process zone with prominent plasticity) becomes comparable to dimensions of the specimen.

The problem of scaling was first noticed and extensively studied in fluid dynamics. In solid mechanics it was neglected for a long time, since in plasticity theory the dominant was the concept of critical stress, which is the material property and it is not affected by the size of specimen. With development of fracture mechanics, where the dominant is the concept of the critical crack size, the scaling, i.e., the size effect became subject of interest. However, this was not the case when the initial state of fracture is considered, namely when laws of linear elastic fracture mechanics (LEFM) are valid. The size effect becomes the crucial point when the nonlinear fracture mechanics (NLFM) is involved, i.e., when the crack size becomes significant. Today there exist two theories in explaining the size effect in solid mechanics: theory based on energetic-statistical scaling and theory based on fractal geometry.

"*The scaling, i.e., the change of response when the spatial dimensions are scaled up or down, while the geometry and all other characteristics are preserved, is a quintessential problem of every physical theory.* ***If the scaling is not understood, a viable theory does not exist***", Bažant [1].

The source of the energetic size effect is a mismatch between the size dependence of the energy release rate and the rate of energy consumption by fracture. A significant part of the former increases as the square of the structure size, while the latter increases in proportion. Therefore, the nominal stress must decrease to reduce the energy release rate of structure so as to achieve a match. Two simple kinds of size effect may be distinguished: the size effect in structures with notches or large cracks at the maximum load and the size effect when the failure occurs at the initiation of fracture from a smooth surface [1]. The first size effect is typical for reinforced concrete structures where the reinforcement makes possible stable growth of large cracks before the maximum load. It also occurs when there is a large compressive stress parallel to the crack, for instance in the fracture of dams. The second effect occurs when the maximum load in a material with a large fracture process zone is reached at fracture initiation from the surface, for instance the modulus of rupture test. The capability to correctly reproduce the size effect is an important check on the validity of any computational model for a quasi brittle structure. To simulate the quasi brittle size effect transitional between plasticity and LEFM, nonlocal material models must be used for the description of softening damage in finite element programs.

When the material exhibits the time-dependent behavior such as viscoelasticity or viscoplasticity (creep), a different type of size effect is caused, which is varying with time. This is due to the fact that the presence of viscosity in the material model implies a characteristic length of the material, as well as a characteristic time. The characteristic length poses a limit on the damage localization within a fixed time interval and thus may produce what looks as a quasi brittle size effect bridging plasticity and LEFM, which actually is not quite true. The localization limiting properties, as well as the size effect caused by material viscosity, existsonly within a certain limited range of loading rates and durations of loading and those properties disappear when this range is exceeded by a factor of 10 or more. On the other hand, various quasi brittle materials, like concrete, exhibit size effect and damage bands of finite thickness over an extremely broad range of delay times (load durations) or loading rates, spanning over about ten orders of magnitude. Such a behavior cannot be captured by viscosity, and the energetic size effect is the proper approach, again [2].

The size effect is defined by comparing geometrically similar structures of different sizes. In the case of notched or fractured structures, the geometric similarity means that the notches or initial cracks are also geometrically similar, Figure 1. Here is presented a brief review of the scaling laws, [3]. The response quantity whose size dependence is to be determined is denoted as Y. This can be the nominal strength, the maximum deflection, or the maximum strain. Bažant was focusing on comparing the nominal strength (or nominal stress at failure) $Y = \sigma_N$, which is defined for the 2D and 3D cases as

$$\sigma_N = c_N \frac{P_u}{bD} \tag{1}$$

and

$$\sigma_N = c_N \frac{P_u}{bD^2} \ , \tag{2}$$

respectively, where P_u is the maximum (ultimate) load; b is the structure thickness for the case two-dimensional similarity; D is the characteristic dimension (or characteristic size), which can be chosen arbitrarily (for instance, as the depth of beam, the span, the half span, the notch depth, and so forth) and c_N is the coefficient introduced for convenience if σ_N is to correspond to some commonly used stress formulas.

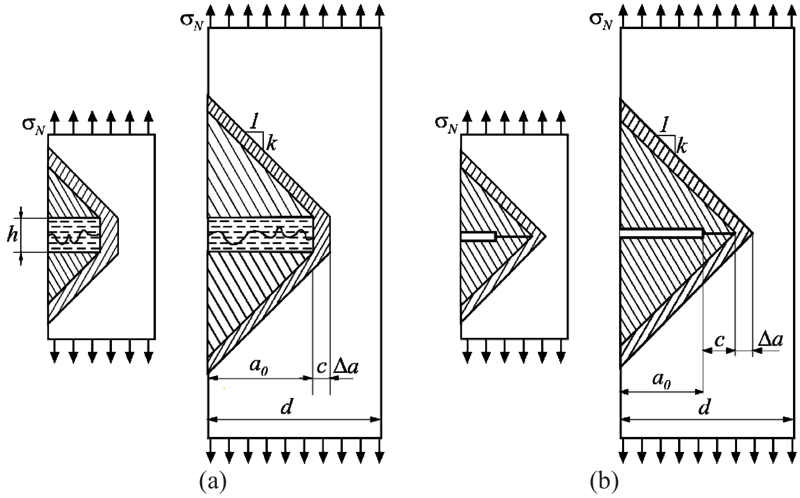


Fig. 1. Areas of energy release in similar small and large specimens: (a) panel with the densely cross-hatched slip; (b) and with the line crack

2.1. General scaling law in absence of characteristic length

This scaling law is applied in theories where there is no characteristic length. This means that the scaling ratio of the corresponding responses $\bar{Y}/Y$ depends only on the size ratio $\lambda = \bar{D}/D$ of two different sizes $\bar{D}$ and D, but it is independent of the choice of the reference side D. Theories to which this scaling law is applicable are plasticity, elasticity with a strength limit, continuum damage mechanics (without nonlocal concepts) and linear elastic fracture mechanics (LEFM). The scaling law for all those theories is a power law:

$$\frac{\bar{Y}}{Y} = f(\lambda) = \lambda^m \ , \tag{3}$$

where m is a constant.

This power-scaling law must hold for every physical system in which there is no characteristic dimension. This includes plasticity or elasticity with a strength limit and also the LEFM. This is so despite the fact that the tensile strength $f_t^{'}$, Young's elastic modulus E, and fracture energy G_f can be combined to give a length quantity $l_0 = EG_f / f_t^{'2}$ which was called the characteristic length, but would be better to call it the characteristic fracture process-zone size.

2.2. Scaling law for boundary value problem of continuum mechanics

Geometrically similar structures, Figure 2, of different sizes are related by the affine transformation (affinity), which is the transformation of change of scale:

$$\bar{x}_i = \lambda x_i \, , \tag{4}$$

where x_i are the Cartesian coordinates for the reference structure of characteristic size D, $\bar{x}_i$ are coordinates for a geometrically similar scaled structure and $\lambda = \bar{D} / D$, with $\bar{D}$ being the characteristic size of the scaled structure. The overbars are denoting quantities related to the scaled structure.

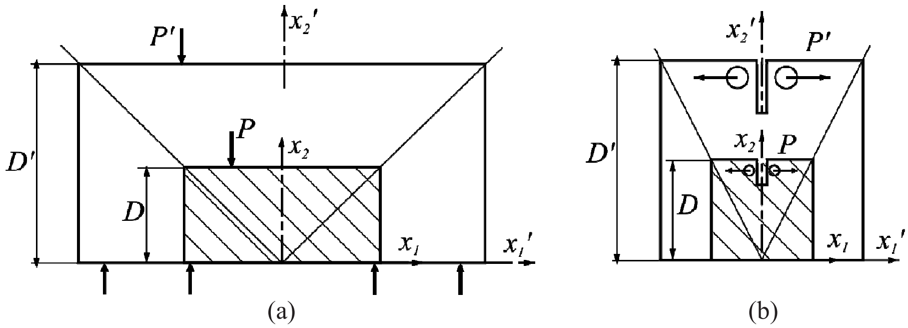


Fig. 2. Geometrically similar structures: (a) without cracks; (b) with similar cracks.

The scaling law in this case reads:

$$\bar{A} = A\lambda^{m+1} \, , \tag{5}$$

where A and $\bar{A}$ are characteristic variables for the scaled and reference structure, respectively. Values of coefficient m are different for various structures. For elastic-plastic structure $m = 0$. In linear elastic fracture mechanics the value is $m = -1/2$.

In Figure 3 are presented size effect laws for limiting cases of different theories [3].

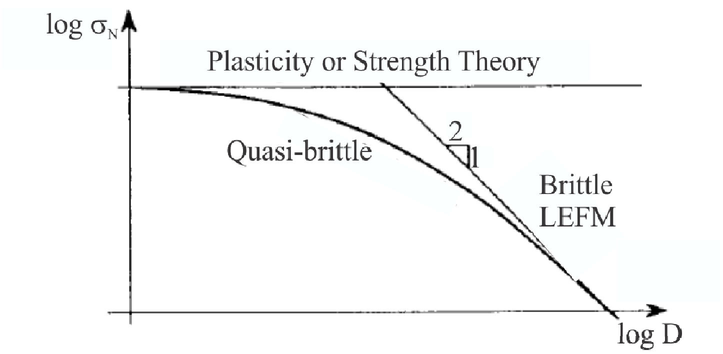


Fig. 3. Size-Effect Laws (asymptotes) for limiting cases of: (a) Strength Criterion; (b) LEFM;
(c) Transitional Size effect Curve for Nonlinear Fracture Mechanics (NLFM).

Application of the power scaling law in some cases produces some anomalous effects. For instance, in the case of the floating elastic plates with large bending fractures and a negligible process shown the coefficient in power law is $m = -3/8$ and it appears that the nominal strength is proportional to $(\text{size})^{-3/8}$. This is the case of the large ice plates floating in the ocean. The anomaly is caused by the fact that the plate thickness, which is the only geometric dimension present, is not a dimension in the 2D domain of the boundary problem. Another anomalous effect is for the breakout of borehole in rocks, where the size effect is of the type $(\text{size})^{-2/5}$. Here is assumed that the failure is due to a zone parallel splitting-compression cracks whose spacing is governed by buckling considerations and that those zones are similar for different sizes.

2.3. Bažant's approximate size effect law

The fracture resistance of many heterogeneous aggregate materials such as rocks, concretes and various ceramics, as well as some metals, is increased by a toughening mechanism, which is due to shielding of the crack tip by a nonlinear zone of distributed micro cracking or void formation. The fracture energy does not represent the sole material characteristics of fracture behavior. Another essential characteristics is the size of the nonlinear fracture process zone at the crack tip. The size of the fracture process zone, in which micro cracking or void formation takes place, is essentially the material property. It is determined by the size of the inhomogeneities in the microstructure, such as the maximum grain size in rock or the maximum aggregate size in concrete. If the size of this zone is negligible compared to the specimen or structure dimensions, the fracture behavior approaches that of linear elastic fracture mechanics. If the size of this zone encompasses all or the most of the specimen or structure volume, the failure is determined by a strength or yield criterion. If the size of this zone is intermediate, the fracture behavior is transitional between the strength criterion and the linear elastic fracture mechanics.

Bažant [4] showed that the scaling law, in the most general case can be written as:

$$\sigma_N = Bf_u \left\{ \beta \left[1 + \beta^{-1} + A_1 \beta^{-2} + A_2 \beta^{-3} + \ldots + \right] \right\}^{-1/2}, \quad \beta = d / d_0, \tag{6}$$

where d is the characteristic dimension of the specimen, B, d_0, A_1, A_2, … are empirical coefficients, f_u is some measure of tensile strength and σ_N is the tensile strength and parameter β characterizes the relative structure size. Equation (6) represents an asymptotic expansion with respect to an infinitely large specimen. For the size range up to 1: 20, this asymptotic series can be truncated after the first term and the truncated expression represents the ***Bažant's size effect law***:

$$\sigma_N = Bf_u \left(1 + \frac{d}{d_0} \right)^{-1/2}. \tag{7}$$

For the case $d \gg d_0$ equation yields $\sigma_N \propto d^{-1/2}$ which is the form of the size effect for every formula of linear elastic fracture mechanics. If $d \ll d_0$ expression (7) reduces to $\sigma_N = Bf_u$ which is the value of the nominal stress at failure according to the yield criterion. For the intermediate values of size d, expression (7) describes a gradual transition from the failures governed by the strength criterion (the plastic yield criterion) to the failures governed by the LEFM. Expression (7) is valid for specimens that are similar, not only in 2D, but also for specimens that are similar in 3D. The size effect law (7) can be transformed to a linear regression plot, as:

$$Y = AX + C \tag{8}$$

where A represents the slope of the regression line of the measured σ_N values in the plot Y in terms of d, while c is the Y-axis intercept of the regression line. Coefficients B and d_0 can be measured values of the maximum load. Linear regression plot for concrete is presented in Figure 4.

2.4. The brittleness number

To characterize the brittleness of the structural response quantitatively, various definitions of the so-called brittleness numbers have been proposed [5, 6]. The Bažant's brittleness number β, is the only one which is independent of the geometrical shape of the specimen, what was justified experimentally in [7]. This brittleness number is defined as $\beta = d / d_0$ and its value may be calculated as

$$\beta = \frac{g(\alpha_0)}{g'(\alpha_0)} \frac{d}{c_f} = \frac{D}{c_f}, \tag{9}$$

where $D = dg(\alpha_0) / g'(\alpha_0)$ is an effective structural dimension, α_0 is the initial notch length and $g(\alpha_0)$ is a function defining the energy release rate G.

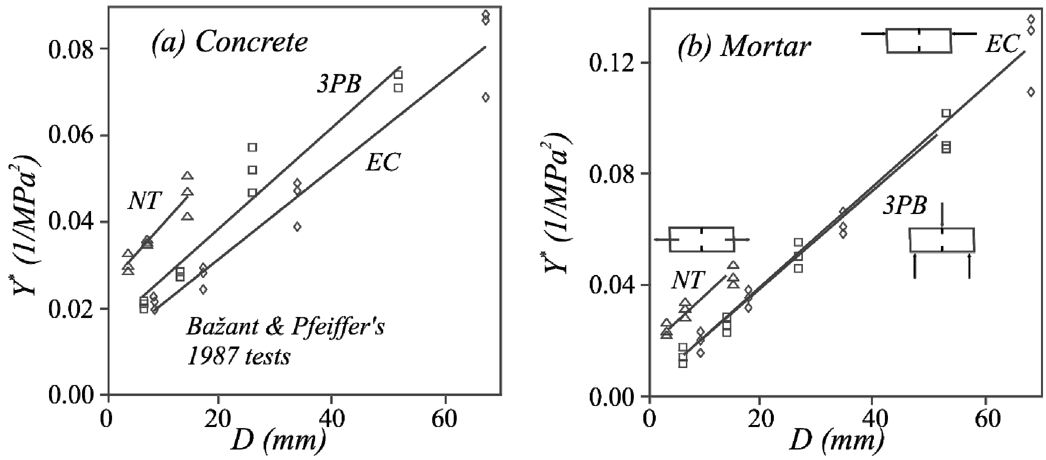


Fig. 4. Linear regression plots for concrete and mortar (NT - notched tension specimens, 3PB - three-point bend specimens, EC - eccentric compression specimens).

The functions of α_0 represent the necessary correction of the ratio d/c_f which makes β independent of the geometrical shape. When $\beta > 10$, the failure may be analyzed according to linear elastic fracture mechanics. When $\beta < 0.1$, the failure may be analyzed on the basis of the strength criterion or plastic limit analysis. For $0.1 < \beta < 10$, nonlinear fracture analysis is required. This greatly simplifies practical applications. Since various types of brittle failures of concrete structures, such as the diagonal shear of beams, punching shear of slabs, torsion of beams, pull-out of bars, and beam and ring failures of pipes, are within the range of nonlinear fracture mechanics and have been shown to follow Bažant's size effect law, the use of the brittleness number can considerably simplify failure calculations.

2.5. Crack propagation and size effect in concrete structures

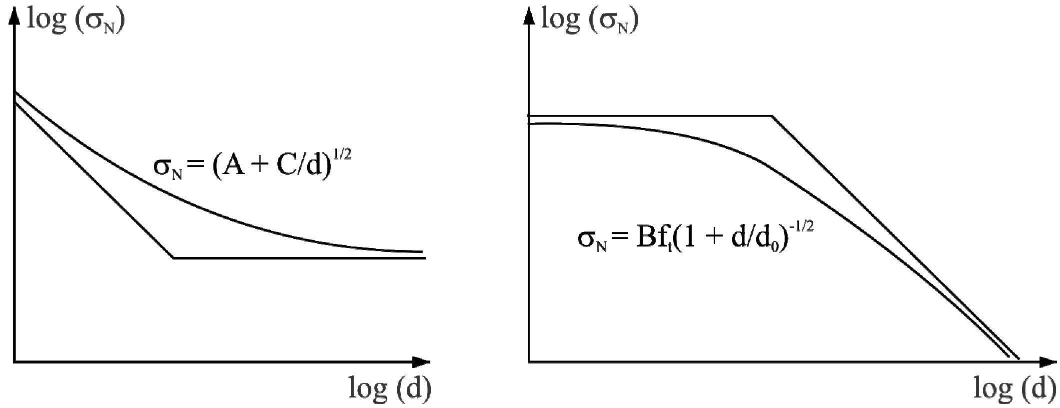
There is plenty of experimental evidence on the existence of size effect in concrete structures. The problem has two aspects - statistical and deterministic. Although the statistical aspects are not negligible, the scaling law is controlled by the structural energy release due to cracking. If a stable crack growth before reaching peak load was possible, strong size effect may be expected. For infinitely large structures of this type scaling law based on the linear elastic fracture mechanics must be used. However, concrete structures fail at crack initiation without any size effect, i.e. scaling law based on the strength criteria apply. Due to the finite size of the concrete fracture process zone, size effect for any small structure must exist.

Currently, two major completely opposite groups of deterministic scaling laws exist, which in a simple close form define the size effect phenomenon for different problems, [8, 9]. The first group of scaling laws is based on a multifractal aspects of damage. The fundamental assumption in multifractal damage concept is perfect homogeneity of the material when structure size $d \to \infty$ and scaling law is of the form:

$$\sigma_N = \left(A + \frac{C}{d} \right)^{1/2} , \tag{10}$$

where σ_N is the nominal strength (failure load divided by characteristic area), d is the structure size measure, A and C are two constants obtained by fitting test or calculated data, Figure 5 a). As can be seen, if $d \to \infty$, the nominal strength σ_N yields to a constant value different than zero. When $d \to 0$, $\sigma_N \to \infty$. This means that the size effect is strong only in limited size range, which may be larger or smaller, depending on the problem type. The second group of scaling laws are in the form of the Bažant's size effect law (7), which finds its physical background in balance between released and consumed fracture energy. These two laws are schematically presented in Figure 5 b).

In any concrete structure crack in a critical cross section starts when the tensile stress become larger than tensile strength ($\sigma > f_t$). This is necessary condition for crack initiation. Once the crack is initiated, its further propagation is controlled by energy balance between structural energy release rate (*dU/da*) and concrete energy consumption limit (G_f), where *U* is the energy accumulated in the structure and *a* is the crack length. Only two crack propagation possibilities exist: when $\left(dU / da\right) \geq G_f$ the crack propagation will be unstable and when $\left(dU / da\right) < G_f$ the crack propagation will be stable.



(a) (b)

Fig. 5. Scaling laws: (a) Multifractal damage approach; (b) Bažant's size effect law (right).

The structural energy release rate, and therefore cracking, is generally a function of geometry, loading type (problem type) and size. With this respect, two typical structure geometries exist: *Positive geometries*, where after crack initiation unstable crack propagation is taking place and *Negative geometries*, where the crack after initiation grows in a stable manner. In the case of positive geometry no crack propagation is possible and, therefore, there is no reason for size effect i.e. scaling law based on strength criteria must be employed. However, in the case of negative geometry stable crack propagation before reaching peak load is possible. As a consequence, size effect must be strong. Those geometries are two extreme cases; however, one has to take into account two additional aspects. The size of the concrete fracture process zone (FPZ) is a function of aggregate size and it has a finite dimension. Therefore, if the size of the FPZ is relatively large with respect to the structure size $d \to 0$, the lower scaling law boundary conditions must account for this effect. Due to the nonlinearity, the failure mode may be changed, i.e., in certain size range structure may act as a structure of positive geometry and in another as a structure of negative geometry.

The size effect in concrete structures is a research subject of many other authors, covering different aspects of this problem. Di Luzio et al. [10], for instance, considered size effect in thermally damaged concrete, while Oyewole et al. [11] considered effects of aggregate sizes on the physical and mechanical properties of concrete using artificial aggregates, etc.

3. Phenomenological or micromechanical solution?

Modeling of fracture in concrete can be classified in the two categories: macroscopic models based on the phenomenological approach and models based on micromechanical solutions. The former models are based on theory of plasticity and fracture mechanics, while the latter models are establishing the relationship between the microstructure of concrete and its macroscopic behavior [12-20].

The theory of plasticity and models based on it use a definition of the yield surface, the flow rule and the hardening function. For the small strain case, those models assume that the total strain can be split into the elastic and plastic parts, with a constitutive law for the elastic part. The yield surface defines the elastic domain in the stress space and its evolution is controlled by the hardening function, while the evolution of the plastic strain is governed by the flow rule.

The continuum damage mechanics describes the progressive degradation of material stiffness due to propagation of micro cracks. The degradation degree is defined by the damage parameters, which can be scalars, a family of vectors or even a fourth-order tensor. Similar to plasticity theory, damage mechanics also defines a damage function, which controls the initiation of damage, as well as the propagation functions that, by the damage parameters, define the damage function evolution.

The fracture mechanics concept is best illustrated by the so-called simple isotropic failure model [12], where one assumes that the degradation of material stiffness is isotropic and the Poisson's ratio remains unaffected; thus the failure is characterized by a single scalar parameter. Since the fracture in concrete is not an isotropic process, such a simplified model can not describe the difference between tensile and compressive behavior of concrete. The models based on the isotropic failure assumption have several deficiencies: they can not include the volumetric expansion which is observed in uniaxial compression tests, as well as the large tensile strains applied in one direction; the fact that the stiffness is completely lost in the loading direction and extremely reduced in the lateral directions. The anisotropic formulations, which have a higher level of complexity, are proposed in order to resolve those deficiencies of the simple models. Linear elastic fracture mechanics thus can not explain concrete fracture. However, when applied in the nonlinear form (NLFM), with taking into account the finite nonlinear zone (the so-called process zone) at the crack tip, i.e. at the crack front, some form of solutions can be obtained.

The energy dissipation during the fracture of concrete is influenced by the meso-structure of the material, the loading applied and the geometry of the specimen [20]. The spatial distribution of the dissipated energy density across the fracture process zone (FPZ) is governed by both statistics and mechanics. Depending on the type of loading, either statistical or mechanical processes dominate. For instance, the shape of shear bands in concrete is strongly influenced by the mechanical interaction of aggregates. For this type of loading, the deterministic contribution on the shape of the FPZ is significant.

For tensile fracture the tortuosity of the path of the main crack, which dissipates most of the energy, is predominantly determined by the statistics of the random arrangement of aggregates. Therefore, the crack paths in concrete subjected to the same loading conditions differ significantly, which implies that a purely deterministic meso-scale model, which does not consider the statistical variation of the fracture paths, cannot describe the FPZ of concrete subjected to tension. Hence, a direct determination of the mean FPZ by meso-scale analysis requires averaging of the results of meso-scale analyses.

Within the framework of the nonlinear finite element analysis, three main approaches to fracture modeling can be distinguished. Continuum approaches describe the fracture process by higher-order constitutive models, such as integral-type nonlocal models. In continuum models with discontinuities, cracks are described as displacement discontinuities, which are embedded into the continuum description. The discrete approaches describe the nonlinear fracture process as failure of discrete elements, such as trusses and beams.

The micromechanical modeling of fracture processes in concrete was described at length in the first part of this study and will not be repeated here. However, the question remains, which of the two approaches is better. Actually, such a question is somewhat artificial, or ill-posed. Both approaches are good, but can not be applied for solving all types of fracture processes in concrete. When considering the concrete fracture at the macro level the phenomenological approach is more plausible. When attempt is made to explain how the fracture starts, what causes the crack initiation and propagation, then the micromechanical approach provides for better answers. But those two approaches need also to be considered together, since the fracture at the meso or macro level has also to include considerations from the micromechanical analysis in order to obtain the complete picture.

Thus the dilemma in the title of this section actually does not exist, since the two approaches are both the best when applied combined.

4. Conclusion

This paper presents the second part of the review of the most important works in the area of modeling the fracture processes in concrete. Here are considered two out of several major issues the researchers are confronted with when studying this field: size effect and scaling laws and phenomenological versus micromechanical approach to solving the fracture processes in concrete. Works of Vile, Bažant, Stroeven, Ferretti and others represent major contributions in understanding the fracture process initiation and development in concrete. All the works published on those subjects until now are explaining some of the aspects in searching for the solution how to explain, understand and prevent the concrete fracture. The complete and final answer is still not in sight.

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References

[1] Z.P. Bažant: Size effect, Intern. Journal of Solids and Structures, Vol. 37, 2000, pp. 69-80.
[2] Z.P. Bažant, Fracture mechanics of concrete structures, Elsevier, New York, 1992.
[3] Z.P. Bažant, Scaling laws in Mechanics of Failure, Journal of Engineering Mechanics, Vol. 119, No. 9, 1993, pp.1828-1844.
[4] Z.P. Bažant, M.T. Kazemi, Determination of fracture energy, process zone length and brittleness number from size effect, with an application to rock and concrete, Intern. Journal of Fracture, Vol. 44, 1990, pp. 111-131.
[5] A. Carpinteri, Notch Sensitivity In Fracture Testing Of Aggregative Materials, Engineering Fracture Mechanics, Vol. 16, No .4, 1982, pp. 467-481.
[6] Z.P. Bažant, M. Kazemi, Size dependence of concrete fracture energy determined by RILEM work-of-fracture method, Intern. Journal of Fracture Vol. 51, 1991, pp. 121-138.
[7] Z.P. Bažant, P.A. Pfeiffer, ACI Material Journal, Vol. 84, No. 6, 1987, pp. 468-480.
[8] J. Ožbolt, R. Eligehausen, Scaling Laws in Concrete Structures, US-Europe Workshop on Fracture and Damage of Quasi-Brittle Materials, Prag, Czech Republic, 1994, pp. 311-318.
[9] R. Eligehausen, J. Ozbolt, Size Effect in Anchorage Behavior, 8th Biennial European Conf. On Fracture, ECF8, Oct 1-5, Torino, 1990, pp. 2671-2677.
[10] G. Di Luzio, G. Muciaccia, L. Luigi Biolzi, Size effect in thermally damaged concrete, Intern. Journal of Damage Mechanics, Vol. 19, No. 5, 2010, pp. 631-656.
[11] O. O. Oyewole, S. S. Arilewola, A.A. Jimoh, D.O. Oyejobi, Effects of Aggregate Sizes on the Physical and Mechanical Properties of Concrete Using Artificial Aggregates, 3rd Annual Conference of Civil Engineering, Ilorin, Nigeria, 2011, pp. 21-23.
[12] I.C. Mihai, Micromechanical constitutive models for cementitious composite materials, Ph.D. Thesis, School of Engineering, cardiff University, 2012.
[13] M. Plavšić, Open problems in modeling fracture processes of concrete, Structural Integrity and Life, Vol. 5, No. 2, 2005, pp. 101-106.
[14] K.M. Nemati, Generation and Interaction of Compressive Stress-Induced Microcracks in Concrete, Ph.D. thesis, University of California at Berkeley, 1994.
[15] A.D. Jefferson Constitutive modelling of aggregate interlock in concrete, International Journal for Numerical and Analytical Methods in Geomechanics 26, 2002, pp. 515-535.
[16] B. Pichler, C. Hellmich, H.A. Mang, A combined fracture-micromechanics model for tensile strain-softening in brittle materilas, based on propagation of interacting microcracks, Intern. Journal for Numerical and Analytical Methods in Geomechanics, Vol 31, 2007, pp. 111-132.
[17] L.N. Lowes, Finite element modeling of reinforced concrete beam-column bridge connections, Ph.D. Thesis, University of California, Berkeley, 1999.
[18] P. Stroeven, Some aspects of the Micromechanics of Concrete, Ph.D. Thesis, Delft University Press, Delft, 1973.
[19] P. Stroeven, J. Hu, Damage evolution in compressed concrete, 11th Intern. Conf. on Fracture, Turin, Italy, 2005, pp. 1536-1541.
[20] P. Grassi, M. Jirasek, Meso-scale approach to modeling the fracture process zone of concrete subjected to uniaxial tension, Intern. Journal of Solids and Structures, Vol. 47, No. 7–8, 2010, pp. 957-968.
[21] G.W.D. Vile, Behavior of concrete under simple and combined stresses, Ph.D. Thesis, University of London, 1965.