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On the extremal number of edges in hamiltonian connected graphs[☆]

Tung-Yang Ho[a,∗], Cheng-Kuan Lin[b], Jimmy J.M. Tan[b], D. Frank Hsu[c], Lih-Hsing Hsu[d]

[a] *Department of Information Management, Ta Hwa Institute of Technology, Hsinchu, 30740, Taiwan, ROC*
[b] *Department of Computer Science, National Chiao Tung University, Hsinchu, 30010, Taiwan, ROC*
[c] *Department of Computer and Information Science, Fordham University, New York, NY 10023, USA*
[d] *Department of Computer Science and Information Engineering, Providence University, Taichung, 43301, Taiwan, ROC*

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ABSTRACT

Assume that n and δ are positive integers with $3 \le \delta < n$. Let $hc(n, \delta)$ be the minimum number of edges required to guarantee an n-vertex graph G with minimum degree $\delta(G) \ge \delta$ to be hamiltonian connected. Any n-vertex graph G with $\delta(G) \ge \delta$ is hamiltonian connected if $|E(G)| \ge hc(n, \delta)$. We prove that $hc(n, \delta) = C(n - \delta + 1, 2) + \delta^2 - \delta + 1$ if $\delta \le \lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1$, $hc(n, \delta) = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$ if $\lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1 < \delta \le \lfloor \frac{n}{2} \rfloor$, and $hc(n, \delta) = \lceil \frac{n\delta}{2} \rceil$ if $\delta > \lfloor \frac{n}{2} \rfloor$.

1. Introduction

In this paper, we use $C(a, b)$ to denote the combination of "a" numbers taking "b" numbers at a time, where a, b are positive integers and $a \ge b$. For the graph definitions and notations, we follow [1]. Let $G = (V, E)$ be a graph if V is a finite set and E is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. Two vertices u and v are *adjacent* if $(u, v) \in E$. The *complete graph* K_n is the graph with n vertices such that any two distinct vertices are adjacent. The *degree* of a vertex u in G, denoted by $\deg_G(u)$, is the number of vertices adjacent to u. We use $\delta(G)$ to denote $\min\{\deg_G(u) \mid u \in V(G)\}$. A *path* of length $m - 1$, $\langle v_0, v_1, \ldots, v_{m-1}\rangle$, is an ordered list of distinct vertices such that v_i and v_{i+1} are adjacent for $0 \le i \le m - 2$. A *cycle* is a path with at least three vertices such that the first vertex is the same as the last one. A *hamiltonian cycle* of G is a cycle that traverses every vertex of G exactly once. A graph is *hamiltonian* if it has a hamiltonian cycle. A *hamiltonian path* is a path of length $|V(G)| - 1$. A graph G is *hamiltonian connected* if there exists a hamiltonian path between any two distinct vertices of G. It is easy to see that a hamiltonian connected graph with at least three vertices is hamiltonian.

It is proved by Moon [2] that the degree of any vertex in a hamiltonian connected graph with at least four vertices is at least 3. Therefore, it is natural to consider the n-vertex graph G with $n \ge 4$ and $\delta(G) \ge 3$. Assume that n and δ are positive integers with $3 \le \delta < n$. Let $hc(n, \delta)$ be the minimum number of edges required to guarantee an n-vertex graph with minimum degree $\delta(G) \ge \delta$ to be hamiltonian connected. Any n-vertex graph G with $\delta(G) \ge \delta$ is hamiltonian connected if $|E(G)| \ge hc(n, \delta)$. We will prove the following main theorem.

Theorem A. *Assume that n and δ are positive integers with $3 \le \delta < n$. Then*

$$hc(n,\delta) = \begin{cases} C(n-\delta+1, 2) + \delta^2 - \delta + 1 & \text{if } \delta \le \left\lfloor \dfrac{n + 3 \times (n \textbf{ mod } 2)}{6} \right\rfloor + 1, \\ C\left(n - \left\lfloor \dfrac{n}{2} \right\rfloor + 1, 2\right) + \left\lfloor \dfrac{n}{2} \right\rfloor^2 - \left\lfloor \dfrac{n}{2} \right\rfloor + 1 & \text{if } \left\lfloor \dfrac{n + 3 \times (n \textbf{ mod } 2)}{6} \right\rfloor + 1 < \delta \le \left\lfloor \dfrac{n}{2} \right\rfloor, \\ \lceil n\delta/2 \rceil & \text{if } \delta > \left\lfloor \dfrac{n}{2} \right\rfloor. \end{cases}$$

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∗ Corresponding author.
E-mail address: hoho@thit.edu.tw (T.-Y. Ho).

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We will defer the proof of Theorem A to Section 4. In Section 2, we describe an application of Theorem A, which is the original motivation of this paper. In particular, we establish the relationship between $hc(n, g)$ and g-conditional edge-fault tolerant hamiltonian connectivity of the complete graph K_n. In Section 3, we present some preliminary results. Section 4 gives the proof of Theorem A.

2. An application

A hamiltonian graph G is *k edge-fault tolerant hamiltonian* if $G - F$ remains hamiltonian for every $F \subset E(G)$ with $|F| \leq k$. The *edge-fault tolerant hamiltonicity*, $\mathcal{H}_e(G)$, is defined as the maximum integer k such that G is k edge-fault hamiltonian if G is hamiltonian and is undefined otherwise. It is proved by Ore [3] that any n-vertex graph with at least $C(n, 2) - (n - 3)$ edges is hamiltonian. Moreover, there exists an n-vertex non-hamiltonian graph with $C(n, 2) - (n - 2)$ edges. In other words, $\mathcal{H}_e(K_n) = n - 3$ for $n \geq 3$. In Latifi et al. [4], it is proved that $\mathcal{H}_e(Q_n) = n - 2$ for $n \geq 2$ where Q_n is the n-dimensional hypercube. In Li et al. [5], it is proved that $\mathcal{H}_e(S_n) = n - 3$ for $n \geq 3$ where S_n is the n-dimensional star graph.

Chan and Lee [6] began the study of the existence of a hamiltonian cycle in a graph such that each vertex is incident with at least a number of nonfaulty edges. In particular, they have obtained results on hypercubes. A graph G is *g-conditional k edge-fault tolerant hamiltonian* if $G - F$ is hamiltonian for every $F \subset E(G)$ with $|F| \leq k$ and $\delta(G - F) \geq g$. The *g-conditional edge-fault tolerant hamiltonicity*, $\mathcal{H}_e^g(G)$, is defined as the maximum integer k such that G is g-conditional k edge-fault tolerant hamiltonian if G is hamiltonian and is undefined otherwise. Chan and Lee [6] proved that $\mathcal{H}_e^g(Q_n) \leq 2^{g-1}(n - g) - 1$ for $n > g \geq 2$ and the equality holds for $g = 2$.

Recently, Fu [7] study the 2-conditional edge-fault tolerant hamiltonicity of the complete graph. In the paper by the authors, Ho et al. [8] extend Fu's result by studying the g-conditional edge-fault tolerant hamiltonicity of the complete graph for $g \geq 2$.

Several results (Lick [9], Moon [2], and Ore [10]) have studied hamiltonian connected graphs and some good sufficient conditions for a graph to be hamiltonian connected. Fault tolerant hamiltonian connectivity is another important parameter for graphs as indicated in [11]. A graph G is *k edge-fault tolerant hamiltonian connected* if $G - F$ remains hamiltonian connected for any $F \subset E(G)$ with $|F| \leq k$. The *edge-fault tolerant hamiltonian connectivity* of a graph G, $\mathcal{HC}_e(G)$, is defined as the maximum integer k such that G is k edge-fault tolerant hamiltonian connected if G is hamiltonian connected and is undefined otherwise. Again, Ore [10] proved that $\mathcal{HC}_e(K_n) = n - 4$ for $n \geq 4$.

Similarly, a graph G is *g-conditional k edge-fault tolerant hamiltonian connected* if $G - F$ is hamiltonian connected for every $F \subset E(G)$ with $|F| \leq k$ and $\delta(G - F) \geq g$. The *g-conditional edge-fault tolerant hamiltonian connectivity*, $\mathcal{HC}_e^g(G)$, is defined to be the maximum integer k such that G is g-conditional k edge-fault tolerant hamiltonian connected if G is hamiltonian connected and is undefined otherwise.

With the inspiration of the work by Fu [7] in the study of 2-conditional edge-fault tolerant hamiltonicity of the complete graph, Ho et al. [12] begin the study on 3-conditional edge-fault tolerant hamiltonian connectivity of the complete graph. The following result was obtained in [12]:

Let $n \geq 4$ and $F \subset E(K_n)$ with $\delta(K_n - F) \geq 3$. Then $K_n - F$ is hamiltonian connected if $|F| \leq 2n - 10$ for $n \notin \{4, 5, 8, 10\}$, $|F| = 0$ for $n = 4$, $|F| \leq 2$ for $n = 5$, and $|F| \leq 2n - 11$ for $n \in \{8, 10\}$.

We restate this result using our terminology.

Theorem 1. *$\mathcal{HC}_e^3(K_n) = 2n - 10$ for $n \notin \{4, 5, 8, 10\}$ and $n \geq 5$, $\mathcal{HC}_e^3(K_4) = 0$, $\mathcal{HC}_e^3(K_5) = 2$, $\mathcal{HC}_e^3(K_8) = 5$, and $\mathcal{HC}_e^3(K_{10}) = 9$.*

Now, we extend the result in [12] and use our main result Theorem A to compute $\mathcal{HC}_e^g(K_n)$ for $3 \leq g < n$.

Theorem 2. *$\mathcal{HC}_e^g(K_n) = C(n, 2) - hc(n, g)$ for $3 \leq g < n$.*

Proof. Let F be any faulty edge set of K_n with $|F| \leq C(n, 2) - hc(n, g)$ such that $\delta(K_n - F) \geq g$. Obviously, $|E(K_n - F)| \geq hc(n, g)$. By Theorem A, $K_n - F$ is hamiltonian connected. Thus, $\mathcal{HC}_e^g(K_n) \geq C(n, 2) - hc(n, g)$.

Now, we prove that $\mathcal{HC}_e^g(K_n) \leq C(n, 2) - hc(n, g)$. Assume that $\mathcal{HC}_e^g(K_n) \geq C(n, 2) - hc(n, g) + 1$. Let G be any graph with $hc(n, g) - 1$ edges such that $\delta(G) \geq g$. Let $F = E(K_n) \setminus E(G)$. In other words, $G = K_n - F$. Obviously, $|F| = C(n, 2) - hc(n, g) + 1$. Since $\mathcal{HC}_e^g(K_n) \geq C(n, 2) - hc(n, g) + 1$, G is hamiltonian connected. This contradicts to the definition of $hc(n, g)$. Thus, $\mathcal{HC}_e^g(K_n) \leq C(n, 2) - hc(n, g)$.

Therefore, $\mathcal{HC}_e^g(K_n) = C(n, 2) - hc(n, g)$ for $3 \leq g < n$. □

3. Preliminary results

The following theorem is proved by Ore [10].

Theorem 3 ([10]). *Let G be an n-vertex graph with $\delta(G) > \lfloor \frac{n}{2} \rfloor$. Then G is hamiltonian connected.*

The following theorem is given by Lick [9].

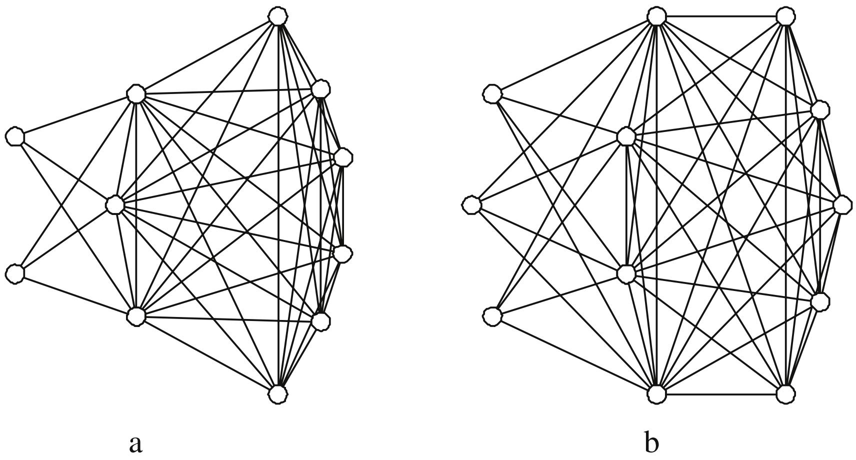

Fig. 1. The graphs (a) $H_{3,11}$ and (b) $H_{4,12}$.

Theorem 4 ([9]). *Let G be an n-vertex graph. Assume that the degree d_i of G satisfy $d_1 \leq d_2 \leq \ldots \leq d_n$. If $d_{j-1} \leq j \leq n/2 \Rightarrow d_{n-j} \geq n-j+1$, then G is hamiltonian connected.*

To our knowledge, no one has ever discussed the sharpness of the above theorem. In the following, we give a logically equivalent theorem.

Theorem 5. *Let G be an n-vertex graph. Assume that the degree d_i of G satisfy $d_1 \leq d_2 \leq \ldots \leq d_n$. If G is non-hamiltonian connected, then there exist at least one integer $2 \leq m \leq n/2$ such that $d_{m-1} \leq m \leq n/2$ and $d_{n-m} \leq n-m$.*

To discuss the sharpness of Theorem 5, we introduce the following family of graphs. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The *union* of G_1 and G_2, written $G_1 + G_2$, has edge set $E_1 \cup E_2$ and vertex set $V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$. The *join* of G_1 and G_2, written $G_1 \vee G_2$, obtained from $G_1 + G_2$ by joining each vertex of G_1 to each vertex of G_2.

The *degree sequence* of an n-vertex graph is the list of vertices degree, in nondecreasing order, as $d_1 \leq d_2 \leq \ldots \leq d_n$. For $2 \leq m \leq n/2$, let $H_{m,n}$ denote the graph $(\bar{K}_{m-1} + K_{n-2m+1}) \vee K_m$. The graphs $H_{3,11}$ and $H_{4,12}$ are shown in Fig. 1. Obviously, the degree sequence of $H_{m,n}$ is

$$(\underbrace{m, m, \ldots, m}_{m-1}, \underbrace{n-m, n-m, \ldots, n-m}_{n-2m+1}, \underbrace{n-1, n-1, \ldots, n-1}_{m})$$

A sequence of real numbers $(p_1, p_2, \ldots, p_n)$ is said to be *majorised* by another sequence $(q_1, q_2, \ldots, q_n)$ if $p_i \leq q_i$ for $1 \leq i \leq n$. A graph G is *degree-majorised* by a graph H if $|V(G)| = |V(H)|$ and the nondecreasing degree sequence of G is majorised by that of H. For instance, the 5-cycle is degree-majorised by the complete bipartite graph $K_{2,3}$ because $(2, 2, 2, 2, 2)$ is majorised by $(2, 2, 2, 3, 3)$.

Lemma 1. *Let $G = (V, E)$ be a graph, X be a subset of V, and u, v be any two distinct vertices in X. Suppose that there exists a hamiltonian path between u and v. Then there are at most $|X| - 1$ connected components of $G - X$.*

Let S be the subset of $V(H_{m,n})$ corresponding to the vertex of K_m. Since $2 \leq m \leq n/2$, $|S| \geq 2$. Let u and v be any two distinct vertices in S. Obviously, there are m connected components of $H_{m,n} - S$. By Lemma 1, $H_{m,n}$ does not have a hamiltonian path between u and v. Thus, $H_{m,n}$ is not hamiltonian connected. In other words, the result in Theorem 5 is sharp.

So we have the following corollary.

Corollary 1. *The graph $H_{m,n}$ is not hamiltonian connected where n and m are integers with $2 \leq m \leq n/2$.*

Thus, the following theorem is equivalent to Theorem 5.

Theorem 6. *If G is an n-vertex non-hamiltonian connected graph, then G is degree-majorised by some $H_{m,n}$ with $2 \leq m \leq n/2$.*

Corollary 2. *Let $n \geq 6$. Assume that G is an n-vertex non-hamiltonian connected graph. Then $\delta(G) \leq \lfloor \frac{n}{2} \rfloor$ and $|E(G)| \leq \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\}$.*

Proof. Let G be any n-vertex non-hamiltonian connected graph. With Theorem 3, $\delta(G) \leq \lfloor \frac{n}{2} \rfloor$. By Theorem 6, G is degree-majorised by some $H_{m,n}$. Since $\delta(H_{m,n}) = m$, $\delta(G) \leq m \leq \lfloor \frac{n}{2} \rfloor$. Therefore $|E(G)| \leq \max\{|E(H_{m,n})| \mid \delta(G) \leq m \leq \lfloor \frac{n}{2} \rfloor\}$. Since $|E(H_{m,n})| = \frac{1}{2}(m(m-1) + (n-2m+1)(n-m) + m(n-1))$ is a quadratics function with respect to m and the maximum value of it occurs at the boundary $m = \delta(G)$ or $m = \lfloor \frac{n}{2} \rfloor$, $|E(G)| \leq \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\}$. $\square$

By Corollary 2, we have the following corollary.

Corollary 3. *Let G be an n-vertex graph with* $n \geq 6$. *If* $|E(G)| \geq \max\{|E(H_{\delta(G),n})|, |E(H_{\lfloor \frac{n}{2} \rfloor,n})|\} + 1$, *then G is hamiltonian connected.*

Lemma 2. *Let n and k be integers with* $n \geq 6$ *and* $3 \leq k \leq \lfloor \frac{n}{2} \rfloor$. *Then* $|E(H_{k,n})| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$ *if and only if* $3 \leq k \leq \lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1$ *or* $k = \lfloor \frac{n}{2} \rfloor$.

Proof. We first prove the case that n is even. We claim that $|E(H_{k,n})| \geq |E(H_{\frac{n}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n}{6} \rfloor + 1$ or $k = \frac{n}{2}$. Suppose that $|E(H_{k,n})| < |E(H_{\frac{n}{2},n})|$. Then $|E(H_{k,n})| = \frac{1}{2}(k(k-1) + (n-2k+1)(n-k) + k(n-1)) < |E(H_{\frac{n}{2},n})| = \frac{1}{2}((\frac{n}{2}-1)(\frac{n}{2})) + (\frac{n}{2})(n-1) + (\frac{n}{2})$. This implies $3k^2 - (2n+3)k + (\frac{1}{4}n^2 + \frac{3}{2}n) < 0$, which means $(k - \frac{n}{2})(3k - \frac{n}{2} - 3) < 0$. Thus $|E(H_{k,n})| < |E(H_{\frac{n}{2},n})|$ if and only if $\frac{n}{6} + 1 < k < \frac{n}{2}$. Note that n and k are integers with n is even, $n \geq 6$, and $3 \leq k \leq \frac{n}{2}$. Therefore, $|E(H_{k,n})| \geq |E(H_{\frac{n}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n}{6} \rfloor + 1$ or $k = \frac{n}{2}$.

For odd integer n, using the same method, we can prove that $|E(H_{k,n})| < |E(H_{\frac{n-1}{2},n})|$ if and only if $\frac{n+3}{6} + 1 < k < \frac{n-1}{2}$. Given that $n \geq 7$, and $3 \leq k \leq \frac{n-1}{2}$, then $|E(H_{k,n})| \geq |E(H_{\frac{n-1}{2},n})|$ if and only if $3 \leq k \leq \lfloor \frac{n+3}{6} \rfloor + 1$ or $k = \frac{n-1}{2}$. Therefore, the result follows. $\square$

4. Proof of Theorem A

By brute force, we can check that $hc(4,3) = 6$, $hc(5,3) = 8$, and $hc(5,4) = 10$. Therefore, the theorem holds for $n = 4, 5$. Next, we consider the cases that $3 \leq \delta \leq \lfloor \frac{n}{2} \rfloor$ and $n \geq 6$.

Suppose that $3 \leq \delta \leq \lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1$ or $\delta = \lfloor \frac{n}{2} \rfloor$. By Lemma 2, $|E(H_{\delta,n})| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$. Let G be any n-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(H_{\delta,n})| + 1$. By Corollary 3, G is hamiltonian connected. We note that $|E(H_{\delta,n})| + 1 = C(n-\delta+1, 2) + \delta^2 - \delta + 1$. Therefore, $hc(n,\delta) \leq C(n-\delta+1,2) + \delta^2 - \delta + 1$. By Corollary 1, $H_{\delta,n}$ is not hamiltonian connected. Thus, $hc(n,\delta) > |E(H_{\delta,n})| = C(n-\delta+1,2) + \delta^2 - \delta$. Hence, $hc(n,\delta) = C(n-\delta+1,2) + \delta^2 - \delta + 1$.

Suppose that $\lfloor \frac{n+3\times(n \textbf{ mod } 2)}{6} \rfloor + 1 < \delta < \lfloor \frac{n}{2} \rfloor$. By Lemma 2, $|E(H_{\delta,n})| < |E(H_{\lfloor \frac{n}{2} \rfloor,n})|$. Let G be any n-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(H_{\lfloor \frac{n}{2} \rfloor,n})| + 1$. By Corollary 3, G is hamiltonian connected. We note that $|E(H_{\lfloor \frac{n}{2} \rfloor,n})| + 1 = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$. Therefore, $hc(n,\delta) \leq C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$. By Corollary 1, $H_{\lfloor \frac{n}{2} \rfloor,n}$ is not hamiltonian connected. Thus, $hc(n,\delta) > |E(H_{\lfloor \frac{n}{2} \rfloor,n})| = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor$. Hence, $hc(n,\delta) = C(n - \lfloor \frac{n}{2} \rfloor + 1, 2) + \lfloor \frac{n}{2} \rfloor^2 - \lfloor \frac{n}{2} \rfloor + 1$.

Finally, we consider the case that $\delta > \lfloor \frac{n}{2} \rfloor$ and $n \geq 6$. Let G be any graph with $\delta(G) \geq \delta > \lfloor \frac{n}{2} \rfloor$. By Theorem 3, G is hamiltonian connected. Obviously, $|E(G)| \geq \lceil \frac{n\delta}{2} \rceil$. Thus, $hc(n,\delta) = \lceil \frac{n\delta}{2} \rceil$.

The proof of our main result, Theorem A, is complete. $\square$

References

[1] J.A. Bondy, U.S.R. Murty, Graph Theory with Applications, North-Holland, New York, 1980.
[2] J.W. Moon, On a problem of Ore, Mathematical Gazette 49 (1965) 40–41.
[3] O. Ore, Note on Hamilton circuits, The American Mathematical Monthly 67 (1960) 55.
[4] S. Latifi, S.Q. Zheng, N. Bagherzadeh, Optimal ring embedding in hypercubes with faulty links, in: Proceedings of the IEEE Symposium on Fault-Tolerant Computing, 1992, pp. 178–184.
[5] T.K. Li, J.J.M. Tan, L.H. Hsu, Hyper hamiltonian laceability on the edge fault star graph, Information Sciences 165 (2004) 59–71.
[6] M.Y. Chan, S.J. Lee, On the existence of Hamiltonian circuits in faulty hypercubes, SIAM Journal on Discrete Mathematics 4 (1991) 511–527.
[7] J.S. Fu, Conditional fault hamiltonicity of the complete graph, Information Processing Letters 107 (2008) 110–113.
[8] T.Y. Ho, C.K. Lin, J.J.M. Tan, D.F. Hsu, L.H. Hsu, On the extremal number of edges in hamiltonian graphs, Journal of Information Science and Engineering (2009) (submitted for publication).
[9] D.R. Lick, A sufficient condition for hamiltonian connectedness, Journal of Combinatorial Theory 8 (1970) 444–445.
[10] O. Ore, Hamilton connected graphs, Journal of Mathematic Pures Application 42 (1963) 21–27.
[11] W.T. Huang, J.J.M. Tan, C.N. Hung, L.H. Hsu, Fault-tolerant hamiltonicity of twisted cubes, Journal of Parallel and Distributed Computing 62 (2002) 591–604.
[12] T.Y. Ho, Y.K. Shih, J.J.M. Tan, L.H. Hsu, Conditional fault hamiltonian connecticity of the complete graph, Information Processing Letters 109 (2009) 585–588.