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Decoupling of pion coupling f_π from quarks at high density in three models, and its possible observational consequences

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Abstract

Chiral symmetry is restored at high density, quarks become nearly massless and pion, the Goldstone of the symmetry breaking decouples from the quarks. What happens at high density is important for finding the density dependence of Strange Quark Matter (SQM), which in turn is relevant for understanding the structure of compact stars.

Keywords: Quark matter; QCD; Chiral symmetry; Nambu and Jona-Lasinio model; Pion coupling; Quark condensate

1. Introduction

We investigate what happens to f_π, at high densities. In our convention, f_π is defined as follows (vacuum value of $f_\pi \sim 93$ MeV):

$$\langle 0|A^a_\mu(x)|\pi^b(q)\rangle = i q_\mu \delta_{ab} f_\pi\left(q^2\right) e^{-iqx}. \tag{1}$$

The Hellmann–Feynman theorem, applied to a nuclear many body model, gives the quark condensate in nuclear matter at high density [1]. Coupling this with theoretical one of Nambu and Jona-Lasinio (NJL) [2], one can extract $f_\pi(n_B)$, where n_B is the baryon number density. Chiral symmetry breaking and pion properties was discussed in the framework of NJL model by Bernard [3]. There was follow up of the work on $f_\pi(n_B)$, using the NJL model, by Bernard, Meissner and Zahed [4] and more recently by Caldas [5].

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Low temperature QCD sumrule results also give $f_\pi(n_B)$ upto $n_B \sim 4n_0$, where n_0 is normal density [6].

Again density dependent quark masses, used for SQM calculations [7], can be used to fix the parameters of the NJL model. This in turn enables one to get the pion coupling to the QCD vacuum $f_\pi(n_B)$.

The quark mass is given in the SQM [7] as:

$$M_i^* = m_i + M_Q \operatorname{sech}\left(\frac{n_B}{N n_0}\right), \quad i = u, d, s, \tag{2}$$

where $n_B = (n_u + n_d + n_s)/3$ is the baryon number density, $n_0 = 0.17\ \text{fm}^{-3}$ is the normal nuclear matter density; n_u, n_d, n_s are number densities of u, d and s quarks, respectively, and N is a parameter. The current quark masses (m_i) are taken as: $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV. M_Q is the constituent quark mass taken around ~ 325 MeV according to latest version of the model [8].

2. Nuclear matter model

In the relativistic σ–ω models of nucleon matter it is found that the quark condensate can be estimated using the Hellmann–Feynman theorem and this was investigated in detail in [1,9]. Interestingly, the title of [1] also referred to a decoupling, that of the nucleon mass and the quark condensate in the medium. The Walecka model, the pioneering one, implies an effective quark condensate that increases with density. This is contrary to common belief. The newer Zimanyi–Moskowski (ZM) model, has an edge over the Walecka model in satisfying the criterion that the quark condensate falls with increase in density as shown in [1].

Further relevance of the ZM has been recently pointed out by Sinha et al., who have shown that the velocity and the incompressibility of the ZM model also match onto a quark model [10].

According to Hellmann–Feynman theorem [9,11,12]

$$\langle\psi(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi(\lambda)\rangle = \frac{d}{d\lambda}\langle\psi(\lambda)|H(\lambda)|\psi(\lambda)\rangle, \tag{3}$$

where $H(\lambda)$ is any hermitian operator depends on a real parameter λ and $|\psi(\lambda)\rangle$ is a normalized eigenvector of $H(\lambda)$.

In QCD the Hamiltonian density is given by

$$\mathcal{H}_{\text{QCD}} = \mathcal{H}_0 + 2m_q \bar{q} q, \tag{4}$$

with the major part being the chirally symmetric $\mathcal{H}_0$. Here m_q is quark mass and q is the quark field.

Making the identification $H \to \int d^3x\, \mathcal{H}_{\text{QCD}}$ and $\lambda \to m_q$ one finds the Hellmann–Feynman theorem as:

$$\begin{aligned} &2m_q \langle\psi(\lambda)|\int d^3x\, \bar{q}q|\psi(\lambda)\rangle \\ &\quad = m_q \frac{d}{dm_q}\langle\psi(\lambda)|\int d^3x\, \mathcal{H}_{\text{QCD}}|\psi(\lambda)\rangle. \end{aligned} \tag{5}$$

The above equation may be applied to nuclear matter and vacuum with $|\psi(\lambda)\rangle = |n_B\rangle$ and $|\psi(\lambda)\rangle = |\text{vac}\rangle$, respectively. Here $|n_B\rangle$ denotes ground state of nuclear matter at rest with nucleon density n_B and $|\text{vac}\rangle$ denotes the vacuum state. Taking the difference of the above two cases and keeping in mind the uniformity of the system, one gets

$$2m_q\big(\langle\bar{q}q\rangle_{n_B} - \langle\bar{q}q\rangle_{\text{vac}}\big) = m_q \frac{d\mathcal{E}}{dm_q}, \tag{6}$$

where n_B is the number density in nuclear matter.

Here in general $\langle\Omega\rangle_{n_B} = \langle n_B|\Omega|n_B\rangle$ and $\langle\Omega\rangle_{\text{vac}} = \langle\text{vac}|\Omega|\text{vac}\rangle$ notations have been used for an arbitrary operator Ω.

The energy density $\mathcal{E}$ of nuclear matter is given by

$$\mathcal{E} = n_B M_N + \delta\mathcal{E}, \tag{7}$$

where $\delta\mathcal{E}$ is the contribution to energy density from the nucleon kinetic energy and nucleon–nucleon interaction energy. $\delta\mathcal{E}$ is of higher order in the nucleon density and is empirically small at low density.

At low density the quark condensate can be related to the nucleon σ term σ_N, which may be defined as [13]

$$\begin{aligned} \sigma_N = \frac{1}{3}\sum_{a=1}^{3}\big(&\langle N|\big[Q_A^a,\big[Q_A^a, H_{\text{QCD}}\big]\big]|N\rangle \\ &- \langle\text{vac}|\big[Q_A^a,\big[Q_A^a, H_{\text{QCD}}\big]\big]|\text{vac}\rangle\big), \end{aligned} \tag{8}$$

where Q_A^a is axial charge, H_{QCD} QCD Hamiltonian and $|N\rangle$ is state vector of nucleon at rest. Alternatively, σ_N can be expressed as:

$$\sigma_N = 2m_q \int d^3x\, \big(\langle N|\bar{q}q|N\rangle - \langle\text{vac}|\bar{q}q|\text{vac}\rangle\big), \tag{9}$$

where

$$\sigma_N = m_q \frac{dM_N}{dm_q}. \tag{10}$$

Hence, Eq. (6) can be written as (using Eq. (7))

$$2m_q\big(\langle\bar{q}q\rangle_{n_B} - \langle\bar{q}q\rangle_{\text{vac}}\big) = m_q n_B \frac{dM_N}{dm_q} + m_q \frac{d\delta\mathcal{E}}{dm_q} = n_B\sigma_N + m_q \frac{d\delta\mathcal{E}}{dm_q}. \tag{11}$$

Assuming translational invariance which makes quark condensate constant one can define

$$\sigma_A = 2m_q V\big(\langle\bar{q}q\rangle_{n_B} - \langle\bar{q}q\rangle_{\text{vac}}\big). \tag{12}$$

Using Eq. (11)

$$\sigma_{\text{eff}} = \frac{\sigma_A}{A} = \sigma_N\left(1 + \frac{d\delta(\mathcal{E}/n_B)}{dM_N}\right). \tag{13}$$

Now from Gell-Mann–Oakes–Renner relation we know

$$2m_q\langle\bar{q}q\rangle_{\text{vac}} = -m_\pi^2 f_\pi^2, \tag{14}$$

m_π and f_π being the pion mass and pion decay constant, respectively. From Eq. (6)

$$\frac{\langle\bar{q}q\rangle_{n_B}}{\langle\bar{q}q\rangle_{\text{vac}}} = 1 - n_B \frac{\sigma_{\text{eff}}}{m_\pi^2 f_\pi^2}. \tag{15}$$

In the ZM model the Lagrangian describes the motion of a baryon with an effective mass instead of bare mass. This information goes to modify the scalar coupling constant making it density dependent while the vector coupling remains the same. In contrast with the Walecka model $\langle\bar{q}q\rangle_{n_B}/\langle\bar{q}q\rangle_{\text{vac}}$ goes down with density [1].

3. The QCD sumrule method

This is a very elegant method devised by Shifman, Vainshtein and Zakharov [14] and consists of equating the coupling of an interpolating Lorentz invariant current for a meson or a baryon—with proper spin, parity and isospin degrees of freedom—to quark–antiquark for meson and three quark for baryons. The quarks or antiquarks are then allowed to mix into the QCD vacuum, which have condensates of quark–antiquark and gluons, and also exchange gluon lines through operator product expansion (OPE). Starting at high momentum transfer for finding the coefficients of the OPE by Borel transform one finds a 'window' where the sum rule becomes independent of the Borel mass parameter. The condensate values picked up from one set, say the ρ meson can be used for all the meson or baryon sumrules. For meson–baryon coupling one has to go over to three-point functions which is more complicated but straightforward in principle. Reviews are available by Reinders, Rubinstein and Yazaki [15], and Dey and Dey [16].

Working out the density and temperature dependence of the pion–nucleon coupling constant ($g_{\pi NN}$), within the framework of QCDSR techniques, Dey and Dey [6] deduced the f_π to be about half its value (44 MeV compared to 81 MeV) at four times normal density.[5] This agrees with the estimate of the present Letter using the NJL model. The sumrule model predicted that the Goldberger–Treiman relation $g_{\pi NN} = M_N\sqrt{2}/f_\pi$ is independent of density [17] and this was confirmed in a later calculation by [18].

4. Quark mass used in stellar calculation

Early suggestion of a cosmic separation of phases of hadronic and strange matter led to investigations properties of strange quark star, but were not very successful. This was because the star with maximum mass had a radius of about ∼9–10 km and this is comparable to that of a neutron star. One could not distinguish between the two. The density dependence of quark masses was not considered in these early models. At high density there is chiral symmetry restoration (CSR) and the masses approach the current quark mass values.

By putting CSR, in a simple tree level large N_c model [7], one can set up an equation of state (EOS) and seek to explain the properties of compact stars Her X-1 and 4U 1820-30. Li and others used this EOS to explain the properties of SAX J1808.4-3658 or 4U 1728-34 [19,20]. Compact stars are assumed to be composed of (u, d, s) matter that is very dense (typically 4.6 (surface) to 15 (core) times the normal nuclear density). In the model (u, d) matter has

[5] f_π was normalized to vacuum value 130 MeV in [6] and is readjusted here by the factor $\sqrt{2}$. It is somewhat low in a nucleon which already has a substantial hadron density.

less binding per baryon E/A, compared to Fe^{56} and (u,d,s) matter has more.

It is interesting to note that many X-ray emitters are rotating and shows periodicity. Only recently, however, six sources were discovered starting with SAX J1808.4-3658 (1998), which are accreting millisecond X-ray pulsars and an important question is raised by Wijnands [21]: why are those compact stars different from others for which no pulsations have been found? Perhaps, he comments, new ideas need to be explored to explain these six sources. Stability of the star may be a crucial point in resolving this issue, according to the present authors and the use of (u,d,s) matter with restored chiral symmetry may help. We must mention that the model leads to stars which are very stable as shown by Sharma et al. [22], by matching the external Schwarzchild metric to a realistic one at the boundary of the star. For details we refer the reader to [22]. Strange star models, with the above EOS, are also very stable when rotating fast, as shown by Gondek–Rosińska et al. [23] and Bombaci, Thampan and Datta [24]. The density dependence of the strong coupling constant α_s in this model was explored using the simple Schwinger–Dyson expansion advocated by Bailin, Cleymans and Scadron in Ray et al. [25].

Further, there are other interesting applications of this model enumerated below:

1. X-ray superbursts lasting for several hours thrice in 4U 1636-53 and once (so far) in KS 1731-260 [26], and also the phenomenon in general, seen in 7 stars altogether.
2. Occurrence of two quasi-periodic peaks in the X-ray power spectrum model of 4U 1636-53 and KS 1731-260 [20,27] and other stars.
3. Absorption in 1E1207.4-5209 [28] and emission [29] in various stars like 4U 0614+091, 2S 0918-549, 4U 1543-624, 4U 1850-087 from surface compressional modes.

In addition, the interesting model of quark nova of Ouyed et al. [30], employs the idea of contraction of normal matter when it is converted to (u,d,s) matter of the above model. Gravitational energy from matter falling onto a compact core, formed during a supernova explosion and consequent generation of a core remnant, can lead to gamma ray after glow according to [30].

The density dependent quark mass used in the (u,d,s) matter is used to generate the pion coupling to quarks in the present Letter.

5. The Nambu–Jona-Lasinio model

We recall that in the model of Nambu and Jona-Lasinio, one can calculate the quark mass M^*, f_π, the quark condensate $\langle\bar{q}q\rangle$ for a given coupling G, following the equations below in terms of a cut off Λ of 631 MeV (see [31]):

$$M^* = m_0 + 4G\left(N_c N_f + \frac{1}{2}\right) M^* \int^{\Lambda} \frac{d^3p}{(2\pi)^3}\frac{1}{E}, \qquad (16)$$

$$f_\pi^2 = N_c M^{*2} \int^{\Lambda} \frac{d^3p}{(2\pi)^3}\frac{1}{E^3}, \qquad (17)$$

$$\langle\bar{q}q\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -6M^* \int^{\Lambda} \frac{d^3p}{(2\pi)^3}\frac{1}{E}. \qquad (18)$$

Knowing the NJL coupling G, one can therefore relate the quantities M^*, f_π and $\bar{q}q$. We assume that G varies with density and find it (1) by fitting it to f_π in the QCDSR for which we do not need the NJL model,

Table 1
Variation of f_π, G, and $\langle\bar{q}q\rangle$ with density ratio ($n_0 = 0.17\ \text{fm}^{-3}$)

n_B/n_0	f_π (MeV)	G (MeV^{-2})	$\langle\bar{q}q\rangle^{1/3}$ (MeV)
1	90.9227	4.936×10^{-6}	−243
2	85.8209	4.682×10^{-6}	−232
3	77.5264	4.389×10^{-6}	−223
4	67.1299	4.13×10^{-6}	−208
5	56.1471	3.922×10^{-6}	−191
6	45.8124	3.755×10^{-6}	−174
7	36.8022	3.606×10^{-6}	−159
8	29.3312	3.458×10^{-6}	−144
9	23.3414	3.294×10^{-6}	−131
10	18.6502	3.103×10^{-6}	−120
11	15.0386	2.877×10^{-6}	−110
12	12.2944	2.616×10^{-6}	−102
13	10.2314	2.323×10^{-6}	−95
14	8.69422	2.01×10^{-6}	−89
15	7.55736	1.692×10^{-6}	−84
16	6.72196	1.386×10^{-6}	−80
17	6.11141	1.107×10^{-6}	−78
18	5.66718	8.645×10^{-7}	−75

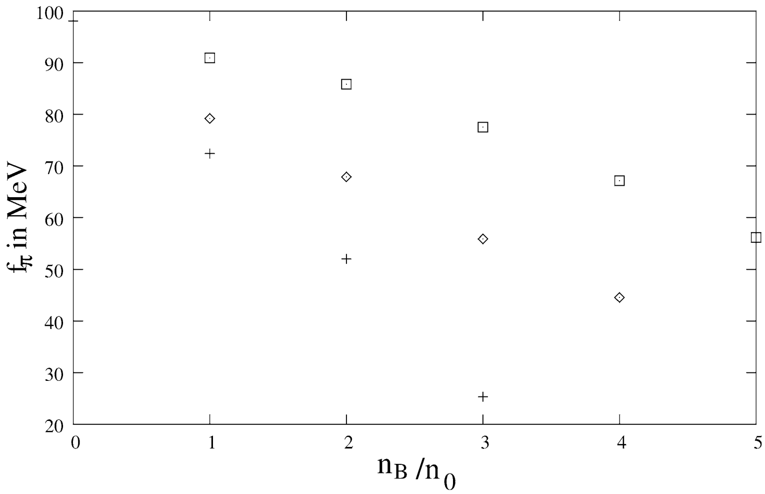

Fig. 1. Density f_π from different models upto ($\sim 4\rho$): diamonds corresponds to QCDSR results, + corresponds to the nuclear matter model of ZM, squares correspond to the SQM.

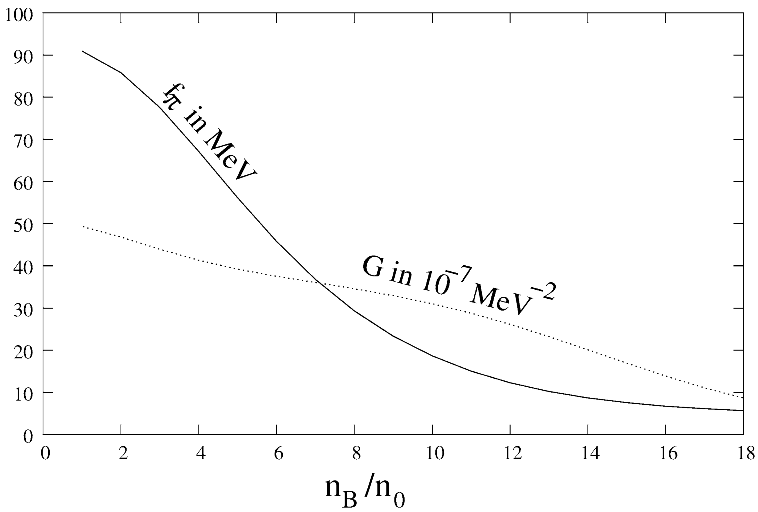

Fig. 2. Predictions from density-dependent quark mass of the SQM upto high density: density (ρ) dependence of f_π (full line), ρ dependence of the NJL coupling constant G (dotted line).

(2) using $\langle \bar{q}q \rangle$ in the σ–ω nuclear matter model and (3) by fitting density-dependent (u, d, s) quark mass in Eq. (16). From (17) and density dependence of G, f_π and the corresponding quark condensates are obtained and are tabulated in Table 1.

At high density, nucleon mass decreases very much with f_π in the Skyrme and other models and the nuclear radius becomes so large that there is no point in talking of a 'confined' nucleons, the quarks are percolating.

Fig. 1 shows that the σ–ω model predicts a zero f_π at about $\sim 4\rho_0$. The QCDSR fall-off is also sharp compared to SQM. We can thus claim that the CR in SQM is mild. The full n_B dependence is shown in Fig. 2 where the density dependence of NJL coupling G is also shown. Our result checks with [4]. For example, for number density five times n_0 the value of f_π is about 60 MeV. A much more mild density dependence of f_π is implied by Caldas [5] who display a number like 80 MeV. It will be very interesting to see

Table 2
Coefficients for density expansion of f_π, G and $\langle\bar{q}q\rangle$

Coeff.	f_π	G	$\langle\bar{q}q\rangle$
a_1	164.51	9.025×10^{-6}	-2.65893×10^{7}
a_2	-104.31	-5.832×10^{-6}	1.72333×10^{7}
a_3	31.08	1.780×10^{-6}	-5.11645×10^{6}
a_4	-5.21	-3.006×10^{-7}	8.49758×10^{5}
a_5	0.513	2.959×10^{-8}	-8.30147×10^{4}
a_6	-0.029	-1.690×10^{-9}	4.7245×10^{3}
a_7	-0.0009	5.187×10^{-11}	-1.44734×10^{2}
a_8	-1.156×10^{-5}	-6.611×10^{-13}	1.843

if the photon width increase, predicted in this Letter, is indeed found in heavy-ion collisions. The photon momentum resolution of STAR experiment does not allow any decisive conclusion about the possible enhancement of the π^0 width, for details see [5].

For future use we have fitted all the quantities by the equation

$$y = a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8, \quad (19)$$

where y represents the variables (f_π, G, $\langle\bar{q}q\rangle$, respectively) and x is the density ratio n_B/n_0. The coefficients for each quantity are tabulated in Table 2.

6. The problem of relativistic heavy ion collisions (RHIC)

Recently exciting new results have been reported by several groups from the gold on gold nuclear collisions in Brookhaven. It appears that there is thermalization and a high temperature is reached. The problem with the experimental results is that although the system is not describable by hadronic models, the nearly non-interacting quark gluon model also does not seem to work. In the language of the protagonists 'the interpretation of current data relies heavily on theoretical input and modeling, in particular, on the apparent necessity to include partonic degrees of freedom in order to arrive at a consistent description of many of the phenomenon observed in experimental data. Seen from a purely experimental point of view this situation is somewhat unsatisfying, but probably not unexpected, not avoidable, considering the complexity of the reaction and associated processes' [32]. Quoting another group to conclude 'the data from RHIC collisions provide strong evidence for the creation of high energy density, low baryonic chemical potential, medium which cannot simply be described in terms of hadrons and whose constituents experience significant interactions with each other' [33].

In conclusion, from high temperature RHIC data, it is not clear that either of the features of QCD like chiral symmetry restoration (CSR) or asymptotic freedom (AF) is actually realized due to the complexity of the system and the system may display strong interacting coherent partonic interactions. The system that one can observe in stars may in fact yield a clearer signature of CSR and AF. We are grateful to the referee for allowing us to comment on this feature.

In the next section we shall discuss the nature of the density dependence that one expects from heuristic considerations given by various authors.

7. Discussion

In the model [34], the radius of the pion is:

$$R_\pi = 0.4\sqrt{z}/f_\pi, \quad (20)$$

where z is the probability of finding a purely $\bar{q}q$ component in the pion. The decrease of f_π with increasing density signifies increase in the radii of the hadrons. This in fact ultimately leads to the percolation of the quarks. Assuming the nucleon radius $R_N = (c\ \text{MeV fm})/f_\pi$, in [6] the constant c is adjusted to get the radius of the nucleon at normal density:

$$R_N = (86.12\ \text{MeV fm})/f_\pi. \quad (21)$$

One can review QCD scales following Bailin, Cleymans and Scadron [35].

$$m_{\text{dyn}} \simeq \Lambda_{\overline{\text{MS}}}e^{1/6} \simeq 300\ \text{MeV}, \quad (22)$$

where the minimal subtraction overall energy scale of QCD, $M_{\overline{\text{MS}}} \simeq 250$ MeV for the 3 flavour case. This is close to 325 MeV of (2). One can go on to get

$$f_\pi = \frac{\sqrt{3}}{2\pi}m_{\text{dyn}} \approx 87\ \text{MeV}, \quad (23)$$

and the string tension

$$\sigma \approx \sqrt{\frac{\pi}{2}}\,m_{\text{dyn}} \approx 400\ \text{MeV}. \quad (24)$$

As density increases f_π and quark condensate decreases with $m_{\rm dyn}$ and this is borne out by the NJL model of the present Letter.

f_π is a parameter in chiral models, the pioneering one being the Skyrme model,

$$\mathcal{L} = \frac{1}{8} f_\pi^2 \,\mathrm{Tr}\big(\partial_\mu U \partial_\mu U^\dagger\big) + \frac{1}{32e^2} \mathrm{Tr}\big(\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\big)^2, \tag{25}$$

where it is the **only** parameter depending on temperature and density [36]. The consequences of $f_\pi(\rho)$ was first analyzed by Rho [37] and Meissner [38].

The nucleon mass can be calculated using the Skyrme model:

$$M_N = \frac{f_\pi \mu}{e\sqrt{2}}, \tag{26}$$

where $\mu = 73.0$ is an integral over the chiral angle of the Skyrmion [39] and e is the dimensionless Skyrme parameter taken to be 5.78.

In this context it is interesting to emphasize the suggestion by Dosch and Narison [40], from QCD Sum Rule (QCDSR) method, that e is independent of the quark condensate. Based on this [36] found that indeed the parameter e, being independent of temperature and density, could in fact be 2π, as suggested by Skyrme to represent a spin current. Incidentally the nucleon radius R_N is proportional to its inverse $R_N = (c\ \mathrm{MeV\,fm})/f_\pi$, where c is a constant. In general, all chiral model properties scale with f_π, as in the Skyrme model.

These values have the support of tentative observations made for compact stars. The importance of the results can be anticipated, since a convincing proof for the existence of such compact stars may soon emerge, from the copious flow of recent astrophysical observations.

In particular, it will be interesting to see if there is any change in Ouyed's model for Skyrmion star [41] with a density-dependent f_π.

8. Conclusions and summary

We have calculated the variation of the pion coupling $f_\pi(\rho)$ with density in the Nambu–Jona-Lasinio model and it is satisfying to see that this matches with expectations of other models. $f_\pi(\rho)$ and the constant G are parametrized as polynomials of density in the hope that the results may be used in future calculations.

In summary, we have calculated the pion coupling constant f_π from the density-dependent (u, d, s) masses employed in compact star models and the results are qualitatively matching with other models, namely, (1) QCD sum rule and (2) nuclear matter models. Results may be useful for chiral models where use of $f_\pi(\rho)$ will produce significant difference at high density.

To conclude, in our opinion, observations on high density matter, perhaps possible in compact stars in an indirect manner, may yield signatures of asymptotically free and nearly chirally symmetric matter. These signatures are elusive in present day RHIC data.

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