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On coefficient inequalities of functions associated with conic domains

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ABSTRACT

In this paper, the concepts of Janowski functions and the conic regions are combined to define a new domain which represents the conic type regions. Different views of this modified conic domain for specific values are shown graphically for better understanding of the behavior of this domain. The class of such functions which map the open unit disk E onto this modified conic domain is defined. Also the classes of k-uniformly Janowski convex and k-Janowski starlike functions are defined and their coefficient inequalities are formulated. The coefficient bound for a certain class of analytic functions, proved by Owa et al. (2006) in [16], has also been improved.

1. Introduction and preliminaries

Let A be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $E = \{z : |z| < 1\}$. The classes $S^*(\alpha)$ and $C(\alpha)$ are the well-known classes of starlike and convex univalent functions of order α $(0 \le \alpha < 1)$ respectively; for details, see [1].

A function $h(z)$ is said to be in the class $P[A, B]$ if it is analytic in E with $h(0) = 1$ and

$$h(z) \prec \frac{1 + Az}{1 + Bz}, \quad -1 \le B < A \le 1,$$

where $\prec$ stands for subordination. Geometrically, a function $h(z) \in P[A, B]$ maps the open unit disk E onto the disk defined by the domain

$$\Omega[A, B] = \left\{ w : \left| w - \frac{1 - AB}{1 - B^2} \right| < \frac{A - B}{1 - B^2} \right\}.$$

The class $P[A, B]$ is connected with the class P of functions with positive real parts by the relation

$$h(z) \in P \Leftrightarrow \frac{(A + 1)h(z) - (A - 1)}{(B + 1)h(z) - (B - 1)} \in P[A, B].$$

This class was introduced by Janowski [2] and then studied by several authors, for example see [3–5].

Kanas and Wisniowska [6,7] introduced and studied the class k-*UCV* of k-uniformly convex functions and the corresponding class k-*ST* of k-starlike functions. These classes were defined subject to the conic domain Ω_k, $k \ge 0$ which was defined by Kanas and Wisniowska [6,7] as

$$\Omega_k = \{u + iv : u > k\sqrt{(u - 1)^2 + v^2}\}.$$

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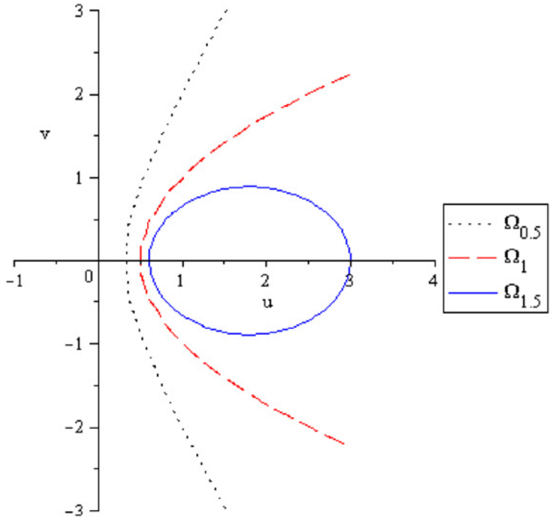

Fig. 1. The curve $u = k\sqrt{(u-1)^2 + v^2}$.

This domain represents the right half plane for $k = 0$, a hyperbola for $0 < k < 1$, a parabola for $k = 1$ and an ellipse for $k > 1$ as shown in Fig. 1.

The functions which play the role of extremal functions for these conic regions are given as

$$p_k(z) = \begin{cases} \dfrac{1+z}{1-z}, & k = 0, \\ 1 + \dfrac{2}{\pi^2}\left(\log \dfrac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & k = 1, \\ 1 + \dfrac{2}{1-k^2}\sinh^2\left[\left(\dfrac{2}{\pi}\arccos k\right)\arctan h\sqrt{z}\right], & 0 < k < 1, \\ 1 + \dfrac{1}{k^2-1}\sin\left(\dfrac{\pi}{2R(t)}\displaystyle\int_0^{\frac{u(z)}{\sqrt{t}}} \dfrac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}\mathrm{d}x\right) + \dfrac{1}{k^2-1}, & k > 1, \end{cases} \tag{1.2}$$

where $u(z) = \frac{z-\sqrt{t}}{1-\sqrt{tz}}$, $t \in (0, 1)$, $z \in E$ and z is chosen such that $k = \cosh\left(\frac{\pi R'(t)}{4R(t)}\right)$, $R(t)$ is Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral of $R(t)$; for more details, see [6,7]. If $p_k(z) = 1 + \delta_k z + \cdots$, then it is shown in [8] that from (1.2), one can have

$$\delta_k = \begin{cases} \dfrac{8(\arccos k)^2}{\pi^2(1-k^2)}, & 0 \le k < 1, \\ \dfrac{8}{\pi^2}, & k = 1, \\ \dfrac{\pi^2}{4(k^2-1)\sqrt{t}(1+t)R^2(t)}, & k > 1. \end{cases} \tag{1.3}$$

These conic regions are being studied by several authors, for example see [9–12].

The classes *k-UCV* and *k-ST* are defined as follows.

A function $f(z) \in A$ is said to be in the class *k-UCV*, if and only if,

$$\frac{(zf'(z))'}{f'(z)} \prec p_k(z), \quad z \in E,\ k \ge 0.$$

A function $f(z) \in A$ is said to be in the class *k-ST*, if and only if,

$$\frac{zf'(z)}{f(z)} \prec p_k(z), \quad z \in E,\ k \ge 0.$$

These classes were then generalized to $KD(k, \alpha)$ and $SD(k, \alpha)$ respectively by Shams et al. [13] subject to the conic domain $G(k, \alpha)$, $k \ge 0$, $0 \le \alpha < 1$, which is

$$G(k, \alpha) = \{w : \operatorname{Re} w > k|w-1| + \alpha\}.$$

Now using the concepts of Janowski functions and the conic domain, we define the following.

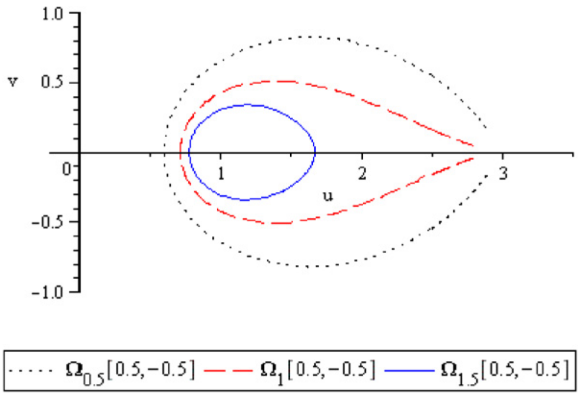

Fig. 2. The curve of domain $\Omega_k[0.5, -0.5]$.

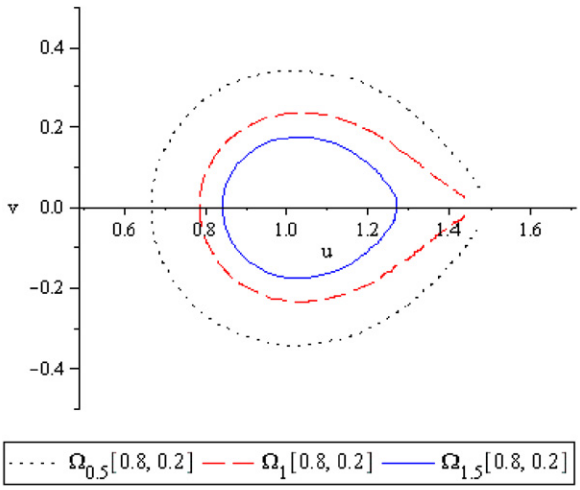

Fig. 3. The curve of domain $\Omega_k[0.8, 0.2]$.

Definition 1.1. A function $p(z)$ is said to be in the class $k - P[A, B]$, if and only if,

$$p(z) \prec \frac{(A+1)p_k(z) - (A-1)}{(B+1)p_k(z) - (B-1)}, \quad k \geq 0, \tag{1.4}$$

where $p_k(z)$ is defined by (1.2) and $-1 \leq B < A \leq 1$.

Geometrically, the function $p(z) \in k - P[A, B]$ takes all values from the domain $\Omega_k[A, B], \ -1 \leq B < A \leq 1, \ k \geq 0$ which is defined as

$$\Omega_k[A, B] = \left\{ w : \mathrm{Re}\left(\frac{(B-1)\,w(z) - (A-1)}{(B+1)w(z) - (A+1)} \right) > k \left| \frac{(B-1)w(z) - (A-1)}{(B+1)w(z) - (A+1)} - 1 \right| \right\} \tag{1.5}$$

or equivalently

$$\begin{aligned} \Omega_k[A, B] = {} & \{u + \mathrm{i}v : [(B^2-1)(u^2+v^2) - 2(AB-1)u + (A^2-1)]^2 \\ & > k^2[(-2(B+1)(u^2+v^2) + 2(A+B+2)u - 2(A+1))^2 + 4(A-B)^2v^2]\}. \end{aligned}$$

The domain $\Omega_k[A, B]$ represents the conic type regions as shown in the figures below.

The domain $\Omega_k[A, B]$ retains the conic domain Ω_k inside the circular region defined by $\Omega[A, B]$. The impact of $\Omega[A, B]$ on the conic domain Ω_k changes the original shape of the conic regions. The ends of hyperbola and parabola get closer to each other but never meet anywhere and the ellipse gets the oval shape as shown in Figs. 2 and 3. When $A \longrightarrow 1, \ B \longrightarrow -1$, the radius of the circular disk defined by $\Omega[A, B]$ tends to infinity, consequently the arms of hyperbola and parabola expand and the oval turns into ellipse as shown in Figs. 4 and 5.

It can be seen that $\Omega_k[1, -1] = \Omega_k$, the conic domain defined by Kanas and Wisniowska [6]. Here are some basic facts about the class $k - P[A, B]$.

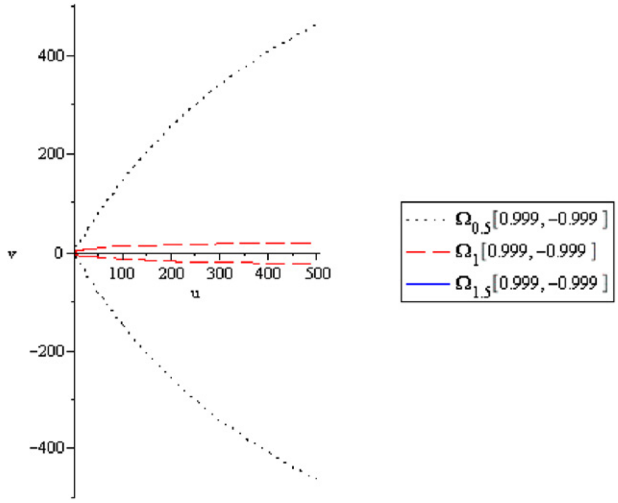

Fig. 4. The curve of domain $\Omega_k[0.999, -0.999]$.

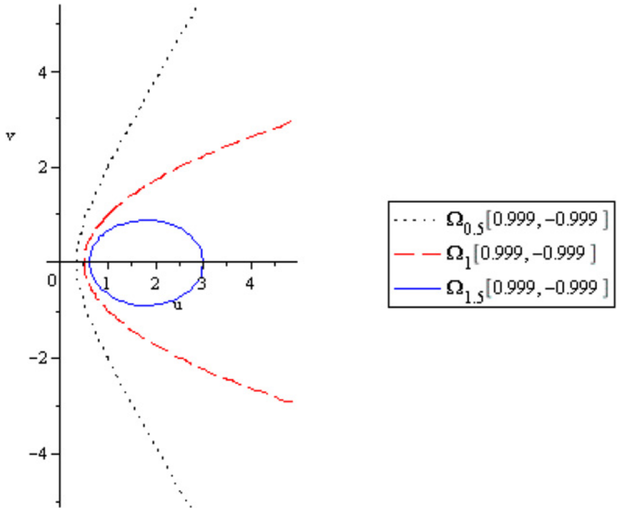

Fig. 5. Close view of Fig. 4.

Remark 1.2. (1) $k-P[A,B]\subset P\left(\frac{2k+1-A}{2k+1-B}\right)$, the well-known class of functions with real part greater than $\frac{2k+1-A}{2k+1-B}$.
(2) $k-P[1,-1]=P(p_k)$, the well-known class introduced by Kanas and Wisniowska [6].
(3) $0-P[A,B]=P[A,B]$, the well-known class introduced by Janowski [2].

Definition 1.3. A function $f(z)\in A$ is said to be in the class k-$UCV[A,B]$, $k\geq 0$, $-1\leq B<A\leq 1$, if and only if,

$$\mathrm{Re}\left(\frac{(B-1)\frac{(zf'(z))'}{f'(z)}-(A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)}-(A+1)}\right)>k\left|\frac{(B-1)\frac{(zf'(z))'}{f'(z)}-(A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)}-(A+1)}-1\right|,$$

or equivalently,

$$\frac{(zf'(z))'}{f'(z)}\in k-P[A,B]. \tag{1.6}$$

Definition 1.4. A function $f(z)\in A$ is said to be in the class k-$ST[A,B]$, $k\geq 0$, $-1\leq B<A\leq 1$, if and only if,

$$\mathrm{Re}\left(\frac{(B-1)\frac{zf'(z)}{f(z)}-(A-1)}{(B+1)\frac{zf'(z)}{f(z)}-(A+1)}\right)>k\left|\frac{(B-1)\frac{zf'(z)}{f(z)}-(A-1)}{(B+1)\frac{zf'(z)}{f(z)}-(A+1)}-1\right|,$$

or equivalently,

$$\frac{zf'(z)}{f(z)} \in k - P[A, B]. \tag{1.7}$$

It can be easily seen that

$$f(z) \in k\text{-}UCV[A, B] \iff zf'(z) \in k\text{-}ST[A, B]. \tag{1.8}$$

Special cases.

i. k-$ST[1, -1] = k$-ST, k-$UCV[1, -1] = k$-UCV, the well-known classes of k-uniformly convex and k-starlike functions respectively, introduced by Kanas and Wisniowska [7,6].
ii. k-$ST[1 - 2\alpha, -1] = SD(k, \alpha)$, k-$UCV[1 - 2\alpha, -1] = KD(k, \alpha)$, the classes, introduced by Shams et al. in [13].
iii. 0-$ST[A, B] = S^*[A, B]$, 0-$UCV[A, B] = C[A, B]$, the well-known classes of Janowski starlike and Janowski convex functions respectively, introduced by Janowski [2].

Lemma 1.5 ([14]). *Let $h(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$ be subordinate to $H(z) = 1 + \sum_{n=1}^{\infty} C_n z^n$. If $H(z)$ is univalent in E and $H(E)$ is convex, then*

$$|c_n| \le |C_1|, \quad n \ge 1.$$

2. Main results

Theorem 2.1. *A function $f \in A$ and of the form (1.1) is in the class k-$ST[A, B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \{2(k+1)(n-1) + |n(B+1) - (A+1)|\}|a_n| < |B - A|, \tag{2.1}$$

where $-1 \le B < A \le 1$ and $k \ge 0$.

Proof. Assuming that (2.1) holds, then it suffices to show that

$$k\left|\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1\right| - \operatorname{Re}\left[\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1\right] < 1.$$

We have

$$\begin{aligned}
&k\left|\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1\right| - \operatorname{Re}\left[\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1\right] \\
&\quad \le (k+1)\left|\frac{(B-1)zf'(z) - (A-1)f(z)}{(B+1)zf'(z) - (A+1)f(z)} - 1\right| \\
&\quad = 2(k+1)\left|\frac{f(z) - zf'(z)}{(B+1)zf'(z) - (A+1)f(z)}\right| \\
&\quad = 2(k+1)\left|\frac{\sum_{n=2}^{\infty}(1-n)a_n z^n}{(B-A)z + \sum_{n=2}^{\infty}\{n(B+1) - (A+1)\}a_n z^n}\right| \\
&\quad \le 2(k+1)\frac{\sum_{n=2}^{\infty}|1-n||a_n|}{|B-A| - \sum_{n=2}^{\infty}|n(B+1) - (A+1)||a_n|}.
\end{aligned}$$

The last expression is bounded above by 1 if

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1) + |n(B+1) - (A+1)|\}|a_n| < |B - A|$$

and this completes the proof. $\square$

When $A = 1,\ B = -1$, then we have the following known result, proved by Kanas and Wisniowska in [7].

Corollary 2.2. *A function $f \in A$ and of the form (1.1) is in the class k-ST, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n + k(n-1)\}|a_n| < 1, \quad k \geq 0. \tag{2.2}$$

When $A = 1 - 2\alpha,\ B = -1$ with $0 \leq \alpha < 1$, then we have the following known result, proved by Shams et al. in [13].

Corollary 2.3. *A function $f \in A$ and of the form (1.1) is in the class $SD(k, \alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n(k+1) - (k+\alpha)\}|a_n| < 1 - \alpha, \tag{2.3}$$

where $0 \leq \alpha < 1$ and $k \geq 0$.

When $A = 1 - 2\alpha,\ B = -1$ with $0 \leq \alpha < 1$ and $k = 0$, then we have the following known result, proved by Selverman in [15].

Corollary 2.4. *A function $f \in A$ and of the form (1.1) is in the class $S^*(\alpha)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}(n-\alpha)|a_n| < 1 - \alpha, \quad 0 \leq \alpha < 1. \tag{2.4}$$

Theorem 2.5. *A function $f \in A$ and of the form (1.1) is in the class k-$UCV[A, B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n\{2(k+1)(n-1) + |n(B+1) - (A+1)|\}|a_n| < |B - A|, \tag{2.5}$$

where $-1 \leq B < A \leq 1$ and $k \geq 0$.

The proof follows immediately by using Theorem 2.1 and (1.8).

Theorem 2.6. *Let $f(z) \in k$-$ST[A, B]$ and is of the form (1.1). Then, for $n \geq 2$,*

$$|a_n| \leq \prod_{j=0}^{n-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)}, \tag{2.6}$$

where δ_k is defined by (1.3).

Proof. By definition, for $f(z) \in k$-$ST[A, B]$, we have

$$\frac{zf'(z)}{f(z)} = p(z), \tag{2.7}$$

where

$$\begin{aligned} p(z) &\prec \frac{(A+1)p_k(z) - (A-1)}{(B+1)p_k(z) - (B-1)} \\ &= [(A+1)p_k(z) - (A-1)][(B+1)p_k(z) - (B-1)]^{-1} \\ &= \frac{A-1}{B-1}\left[1 - \frac{A+1}{A-1}p_k(z)\right]\left[1 + \sum_{n=1}^{\infty}\left(\frac{B+1}{B-1}p_k(z)\right)^n\right] \\ &= \frac{A-1}{B-1} + \left(\frac{(A-1)(B+1)}{(B-1)^2} - \frac{A+1}{B-1}\right)p_k(z) + \left(\frac{(A-1)(B+1)^2}{(B-1)^3} - \frac{(A+1)(B+1)}{(B-1)^2}\right)(p_k(z))^2 \\ &\quad + \left(\frac{(A-1)(B+1)^3}{(B-1)^4} - \frac{(A+1)(B+1)^2}{(B-1)^3}\right)(p_k(z))^3 + \cdots. \end{aligned}$$

If $p_k(z) = 1 + \delta_k z + \cdots$, then we have after suitable simplification

$$p(z) \prec \sum_{n=1}^{\infty} \frac{-2(B+1)^{n-1}}{(B-1)^n} + \left\{\sum_{n=1}^{\infty} \frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}\right\}\delta_k z + \cdots.$$

Now we see that the series $\sum_{n=1}^{\infty}\frac{-2(B+1)^{n-1}}{(B-1)^n}$ and $\sum_{n=1}^{\infty}\frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}$ are convergent and converge to 1 and $\frac{A-B}{2}$ respectively. Therefore,

$$p(z) \prec 1+\frac{1}{2}(A-B)\delta_k z+\cdots.$$

Now if $p(z)=1+\sum_{n=1}^{\infty}c_n z^n$, then by Lemma 1.5, we have

$$|c_n| \le \frac{1}{2}(A-B)\delta_k, \quad n\ge 1. \tag{2.8}$$

Now from (2.7), we have

$$zf'(z)=f(z)p(z),$$

which implies that

$$z+\sum_{n=2}^{\infty}na_n z^n=\left(z+\sum_{n=2}^{\infty}a_n z^n\right)\left(1+\sum_{n=1}^{\infty}c_n z^n\right).$$

Equating coefficients of z^n on both sides, we have

$$(n-1)a_n=\sum_{j=1}^{n-1}a_{n-j}c_j, \quad a_1=1.$$

This implies that

$$|a_n| \le \frac{1}{n-1}\sum_{j=1}^{n-1}|a_{n-j}||c_j|, \quad a_1=1.$$

Using (2.8), we have

$$|a_n| \le \frac{|\delta_k|(A-B)}{2(n-1)}\sum_{j=1}^{n-1}|a_j|, \quad a_1=1. \tag{2.9}$$

Now we prove that

$$\frac{|\delta_k|(A-B)}{2(n-1)}\sum_{j=1}^{n-1}|a_j| \le \prod_{j=0}^{n-2}\frac{|\delta_k(A-B)-2jB|}{2(j+1)}. \tag{2.10}$$

For this, we use the induction method.

For $\underline{n=2}$: from (2.9), we have

$$|a_2| \le \frac{|\delta_k|(A-B)}{2}.$$

From (2.6), we have

$$|a_2| \le \frac{|\delta_k|(A-B)}{2}.$$

For $\underline{n=3}$: from (2.9), we have

$$\begin{aligned}|a_3| &\le \frac{|\delta_k|(A-B)}{4}(1+|a_2|)\\ &\le \frac{|\delta_k|(A-B)}{4}\left(1+\frac{|\delta_k|(A-B)}{2}\right).\end{aligned}$$

From (2.6), we have

$$\begin{aligned}|a_3| &\le \frac{|\delta_k|(A-B)}{2}\frac{|\delta_k(A-B)-2B|}{4}\\ &\le \frac{|\delta_k|(A-B)}{2}\frac{|\delta_k|(A-B)+2|B|}{4}\\ &\le \frac{|\delta_k|(A-B)}{2}\left(\frac{|\delta_k|(A-B)}{2}+1\right).\end{aligned}$$

Let the hypothesis be true for $n=m$.

From (2.9), we have

$$|a_m| \le \frac{|\delta_k|(A-B)}{2(m-1)} \sum_{j=1}^{m-1} |a_j|, \quad a_1 = 1.$$

From (2.6), we have

$$\begin{aligned} |a_m| &\le \prod_{j=0}^{m-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)} \\ &\le \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)}. \end{aligned}$$

By the induction hypothesis, we have

$$\frac{|\delta_k|(A-B)}{2(m-1)} \sum_{j=1}^{m-1} |a_j| \le \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)}.$$

Multiplying both sides by $\frac{|\delta_k|(A-B)+2(m-1)}{2m}$, we have

$$\begin{aligned} \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)} &\ge \frac{|\delta_k|(A-B)}{2(m-1)} \frac{|\delta_k|(A-B) + 2(m-1)}{2m} \sum_{j=1}^{m-1} |a_j| \\ &= \frac{|\delta_k|(A-B)}{2m} \left\{ \frac{|\delta_k|(A-B)}{2(m-1)} \sum_{j=1}^{m-1} |a_j| + \sum_{j=1}^{m-1} |a_j| \right\} \\ &\ge \frac{|\delta_k|(A-B)}{2m} \left\{ |a_m| + \sum_{j=1}^{m-1} |a_j| \right\} \\ &= \frac{|\delta_k|(A-B)}{2m} \sum_{j=1}^{m} |a_j|. \end{aligned}$$

That is,

$$\frac{|\delta_k|(A-B)}{2m} \sum_{j=1}^{m} |a_j| \le \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)},$$

which shows that inequality (2.10) is true for $n = m + 1$. Hence the required result. □

Corollary 2.7. *When $A = 1,\ B = -1$, then (2.1) reduces to*

$$|a_n| \le \prod_{j=0}^{n-2} \frac{|\delta_k + j|}{j+1}, \quad n \ge 2, \tag{2.11}$$

which is the coefficient inequality of the class k-ST, introduced by Kanas and Wisniowska [7].

Corollary 2.8. *When $A = 1 - 2\alpha,\ B = -1$ with $0 \le \alpha < 1$, then (2.1) reduces to*

$$|a_n| \le \prod_{j=0}^{n-2} \frac{|\delta_k(1-\alpha) + j|}{j+1}, \quad n \ge 2, \tag{2.12}$$

which is the coefficient inequality of the class $SD(k, \alpha)$, introduced by Shams et al. [13]. Inequality (2.12) gives the better result as compared with that, proved in [16].

When $k = 0$, then $\delta_k = 2$ and we get the following known result, proved in [2].

Corollary 2.9. *Let $f(z) \in S^*[A, B]$ and is of the form (1.1). Then, for $n \ge 2$,*

$$|a_n| \le \prod_{j=0}^{n-2} \frac{|(A-B) - jB|}{j+1}, \quad -1 \le B < A \le 1. \tag{2.13}$$

When $A = 1 - 2\alpha,\ B = -1$ with $0 \le \alpha < 1$ and $k = 0$, then we have the following known result; see [1].

Corollary 2.10. *Let $f(z) \in S^*(\alpha)$ and is of the form (1.1). Then, for $n \ge 2$,*

$$|a_n| \le \frac{\prod_{j=2}^{n}(j - 2\alpha)}{(n-1)!}, \quad 0 \le \alpha < 1. \tag{2.14}$$

Theorem 2.11. *Let $f(z) \in k$-$UCV[A, B]$ and is of the form (1.1). Then, for $n \ge 2$,*

$$|a_n| \le \frac{1}{n} \prod_{j=0}^{n-2} \frac{|\delta_k(A - B) - 2jB|}{2(j+1)}, \tag{2.15}$$

where δ_k is defined by (1.3).

The proof follows immediately by using Theorem 2.1 and (1.8).

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