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Synthesis of highly directive endfire arrays using modified Chebyshev polynomials

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Abstract

In this paper an additional parameter in the Chebyshev polynomial is introduced so as to design an endfire array by controlling two factors of the radiation characteristics- namely the Half Power Beam Width(HPBW) and the side lobe level(SLL) simultaneously. It has been shown that for a given SLL improved HPBW and First Null Beam Width (FNBW) can be achieved. Individual elements in the array are assumed to be isotropic. Simulations were carried out to find bounds on the choice of HPBW and SLL. Simulation results also shows that the directivity can be improved over the Dolph-Chebyshev array. Improvement in the roll off characteristics are also shown. Implementation of the array using the coefficients obtained by this synthesized procedure can result in a highly directive array.

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Keywords: Half power beam width, first null beam width, side lobe level, Chebyshev variant polynomial, excitation coefficient ;

1. Introduction

A simple case of an endfire array is the Uniform Linear Array (ULA), where array elements are excited with uniform amplitude and a constant progressive phase shift. With the uniform excitation, the maximum directivity that can be achieved is $4L/\lambda$. Where L is length of array. The directivity is further improved in the Hansen Woodyard array [1 2]. In the present work an additional parameter is introduced in the Chebyshev polynomial. These Chebyshev Variant(CV) polynomials can give directivities higher than that obtained for the Hansen Woodyard case. Endfire arrays of this type can be used as feeds in parabolic reflector antennas. Low SLL as well as low HPBW will result in reduced spill over losses and noise pickup.

In an antenna design, as HPBW is reduced, the SLL will rise. Conversely, if SLL is reduced, the HPBW rises. In the usual Chebyshev array design, an array of any SLL can be synthesised. However, as SLL is lowered, HPBW will increase since there is control over SLL only. Therefore designing a high directive antenna requires optimisation in

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both SLL and HPBW. It requires meticulous design of excitation coefficients, array geometry, inter element spacing and progressive phase shifts, to achieve the high directivity.

In the proposed CV array design there is simultaneous control over both SLL and HPBW, which is an improvement over the usual Chebyshev array design. Also CV polynomial can find its application wherever Chebyshev polynomial is being used as in[4 5 6].

2. Chebyshev variant polynomial

Consider the Chebyshev polynomial[1],

$$T_M(\theta) = \begin{cases} \cos\left(M \cos^{-1} x\right) & |x| \leq 1 \\ \cosh\left(M \cosh^{-1} x\right) & |x| \geq 1 \end{cases} \tag{1}$$

Where

$$x = x_m \cos\left(\frac{\psi(\theta)}{2}\right), \textit{ For Chebyshev array design} \tag{2}$$

$$\psi(\theta) = kd(\cos\theta - 1), \textit{ For endfire arrays} \tag{3}$$

x_m is a constant which decides SLL. d is the inter element spacing which is taken to be uniform. M is the order of Chebyshev polynomial. N is the number of array elements and $M = N - 1$.
In the array synthesis, Eq(1) is viewed as array factor, The main beam is contributed by cosh (.) and side lobes are by cos (.). Here $|x|$ in Eq(2) decides the magnitude of the main beam with respect to the constant SLL of unity. Eq(1) takes cosh (.) for $|x| \geq 1$, and cos (.) for $|x| \leq 1$. Here the values of θ for which $|x| \geq 1$ decides HPBW and SLL. Hence Eq(2) controls the main beam, SLL position and magnitude. Therefore by modifying Eq(2) an array factor of desired SLL and HPBW can be obtained. Hence, the original characteristics of the Chebyshev ploynomial can be modified to have additional features as in[3]. In Fig.1, the blue line is the plot of $|x|$ v/s θ as in Eq(2). To control the HPBW, the variant δ in Eq(2) is introduced. This controls the values of θ for which $|x| \geq 1$. In the later section, the physical meaning of the variant is explained. And it is also shown that this variant also controls x_m. So the modified equation is

$$x = x_m \cos\left(\frac{\psi(\theta)}{2} + \delta\right) \tag{4}$$

The plot of $|x|$ v/s θ for different δ is shown in Fig.1. The normalised plot of $|T_M(\theta)|$ v/s θ for the endfire array is shown in Fig.2 for the same δ, for which $x_m = 1.5$, $N = 5$, $d = 0.25\lambda$. $\delta = 0$ corresponds to the Chebyshev polynomial plot, and other values of δ corresponds to the CV plot.

3. Formulation of the problem

Referring to Fig.2, by choosing appropriate values of x_m and δ the required HPBW and SLL can be achieved. In order to establish a relation between SLL, HPBW, x_m, and δ, the following parameters are defined.
1. Side Lobe Level in absolute scale

$$SLL = b$$

2. Half Power Point (HPP) ratio $'p'$

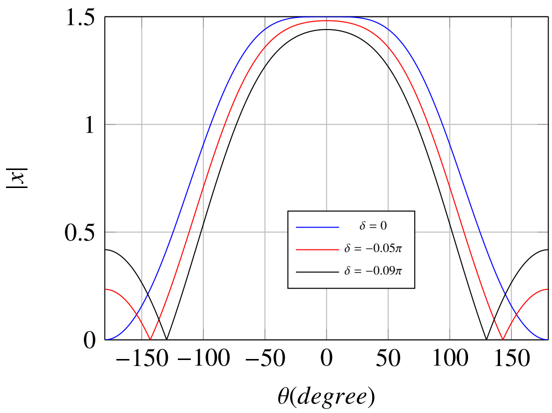

Fig. 1: $|x|$ for various δ

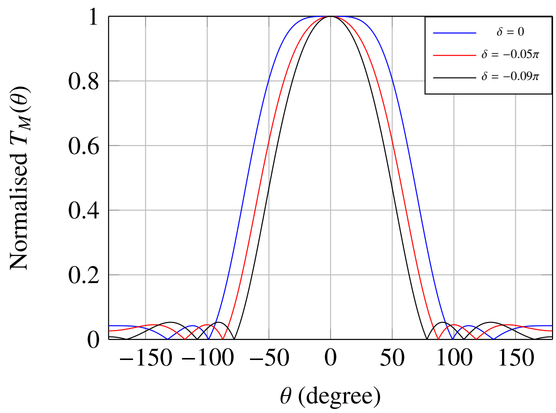

Fig. 2: Chebyshev Variant polynomial for various δ

$$p = \frac{HPP\,of\,non-ULA}{HPP\,of\,ULA\,with\,same\,N} \leq 1$$

Here as a specification of High-directive array, $'p'$ can also be taken as the fraction of HPBW of ULA to be achieved. Taking Eq(1) as the array-factor, from Eq(3) at $\psi(0) = 0$, Eq(1) takes the maximum value of $1/b$

$$T_M(\theta)|_{max} = T_M(0) = \frac{1}{b}$$

$$\cosh\left[M\cosh^{-1}(x_m\cos\delta)\right] = \frac{1}{b} \tag{5}$$

b, p can be chosen as per the specified directivity. $p\theta_h$ is the desired HPP. Where θ_h is HPP of ULA with the same number of array elements. At the HPP of the CV polynomial, using Eq(1) and Eq(5) give the conditions:

$$T_M(p\theta_h) = \frac{T_M(\theta)|_{max}}{\sqrt{2}} = \frac{T_M(0)}{\sqrt{2}} \quad and$$

$$\cosh\left[M\cosh^{-1}\left\{x_m\cos\left(\frac{\psi(p\theta_h)}{2} + \delta\right)\right\}\right] = \frac{1}{b\sqrt{2}} \tag{6}$$

Solving for δ and x_m using Eq(5) and Eq(6) will give:

$$\delta = \tan^{-1}\left[\frac{A_1 - A_3}{A_2}\right] \tag{7}$$

$$x_m = \frac{B_1}{\cos\delta} \tag{8}$$

Where

$$A_1 = \cos\left[\frac{\psi(p\theta_h)}{2}\right]$$

$$A_2 = \sin\left[\frac{\psi(p\theta_h)}{2}\right]$$

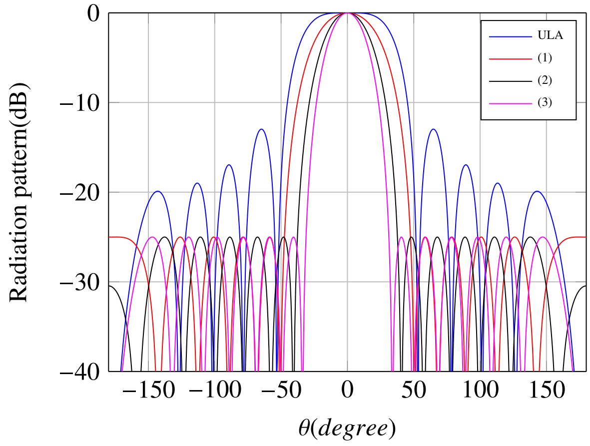

Fig. 3: Radiation pattern of 10 element array with $d = 0.25\lambda$

$$A_3 = \frac{B_2}{B_1}$$

$$B_1 = \cosh\left[\frac{1}{M}\cosh^{-1}\left(\frac{1}{b}\right)\right] \tag{9}$$

$$B_2 = \cosh\left[\frac{1}{M}\cosh^{-1}\left(\frac{1}{b\sqrt{2}}\right)\right] \tag{10}$$

Since $b < 1$, $B_1 > B_2$. Referring to Eq(3) it is seen that $A_2 \leq 0$. Furthermore it is observed that $A_1 > A_3$ for $b, p < 1$. Therefore it may be inferred that:

$$\begin{aligned} x_m \geq B_1 > B_2 > 1 \\ \delta < 0 \end{aligned}$$

The equality holds only for $p = 0$. Fig.3 shows the radiation patterns computed from the CV polynomial for the different values of SLL and HPBW as in Table 1.

Table 1: Specifications of radiation pattern for different SLL and HPBW with $N = 10$, $d = 0.25\lambda$

S.L.No.	$SLL(dB)$	p	$HPBW(degree)$
1	−25	0.7	48.47
2	−25	0.5	34.62
3	−25	0.4	27.69

The excitation coefficients of the array are found by the coefficients of zeros of CV polynomial by Schelkunoff polynomial method[1]

$$AF = \prod_{i=1}^{N} \left(Z - e^{j\psi_i}\right) \tag{11}$$

Here ψ_i is the i^{th} value of ψ for which magnitude of the polynomial is zero. The coefficients of Z gives the excitation coefficients of the linear endfire array.
The excitation coefficients can also be found by well known Fourier series method[1] .

4. Minimum achievable HPBW and FNBW for a given SLL

Examining the CV polynomial, it can be observed that the values of p and b cannot be arbitrarily given.This is because as more stringent values for $HPBW$ and SLL are chosen, $|x|$ takes the values greater than unity for values of θ close to 180^0. This results in back lobes greater than the side lobes. Table 2 shows a few values of p and b for which this is observed. Using these values the plot of $|x|$ v/s θ and plot of normalised radiation pattern v/s θ can be seen in Fig.4 and Fig.5 respectively. Hence there exists a minimum possible value of $HPBW$ for a given SLL. Fig. 6 shows the simulation result of minimum achievable HPBW($HPBW_{min}$) for a given SLL and Fig. 7 shows the simulation result of minimum achievable FNBW($FNBW_{min}$) for a given SLL for CV polynomial(bold line plot) and Chebyshev polynomial(dashed line plot) for different inter element spacing respectively.

Table 2: Specifications of radiation pattern for different SLL and HPBW with $N = 10$, $d = 0.25\lambda$

S.L.No.	$SLL(dB)$	p	$HPBW(degree)$
1	−50	0.41	28.38
2	−60	0.45	31.16
3	−40	0.37	25.62

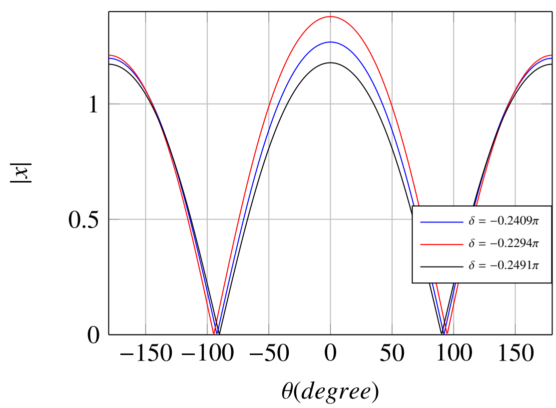

Fig. 4: $|x|$ for various δ corresponding to the specifications as in table 2

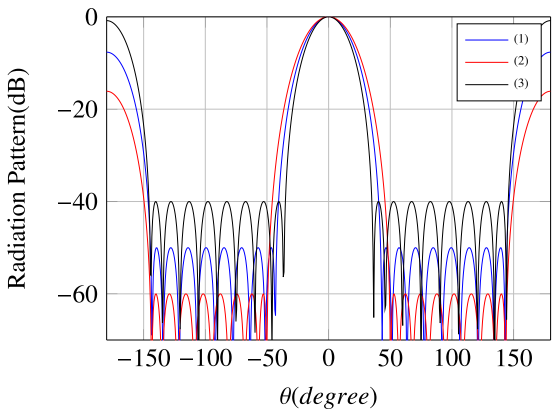

Fig. 5: Radiation Pattern of 10 element array with $d = 0.25\lambda$ for specifications as in table 2

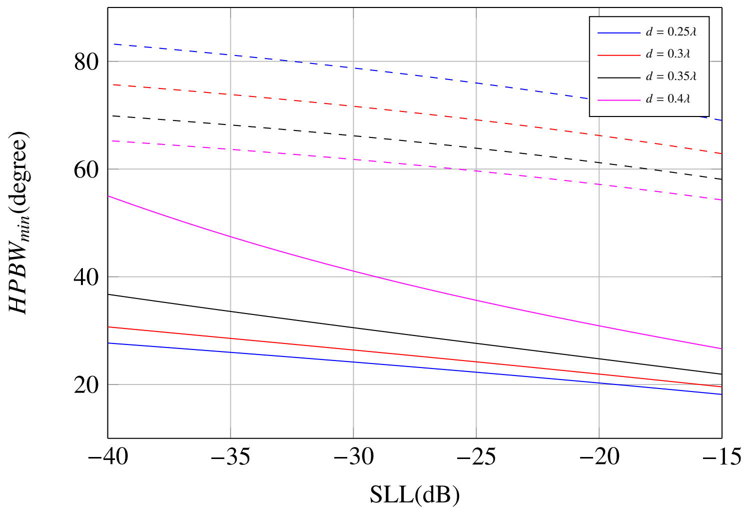

Fig. 6: Minimum achievable HPBW for a given SLL for the 10 element array

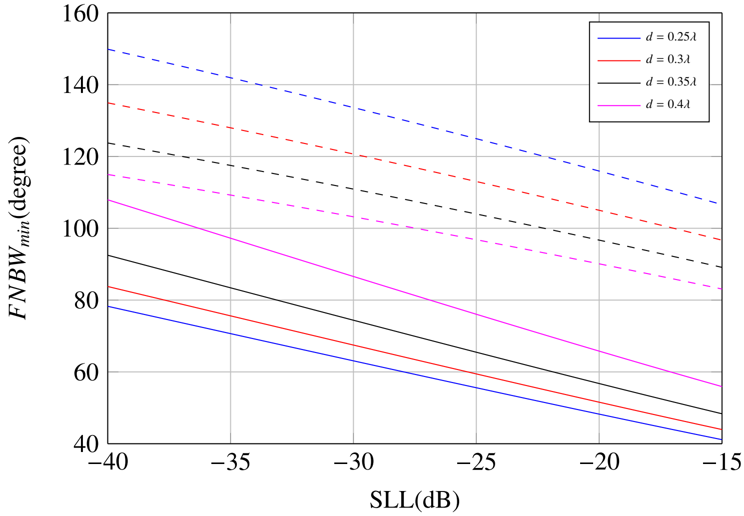

Fig. 7: Minimum achievable FNBW for a given SLL for the 10 element array

5. Results and discussions

Matlab simulations were carried out for various values of b and p. Table 3 shows the comparisons of the $HPBW_{min}$ and $FNBW_{min}$ obtained from the CV polynomials and HPBW($HPBW_C$), FNBW ($FNBW_C$) (in degrees) obtained from the Chebyshev polynomials for different SLL for an interelement spacing of 0.25λ. Fig.8 shows the radiation pattern of the CV array as per the specifications in Table 3.
Table 4 compares the directivity of the ULA, the Hansen Woodyard endfire array(HWA)[2], the Chebyshev array (CA) and the CV array(CVA) for the specifications of table 3. And Fig.9 shows the radiation pattern of different types of endfire arrays as per S.L.No.2 of Table 3. It is seen that the CV array has the highest directivity among all the endfire arrays. It has been seen that for an inter-element spacing of 0.5λ or more the CV polynomial yields back lobes greater than the SLL.

Fig.10 and Fig.11 shows the normalised magnitude of excitation coefficients ($|E|$) and the phase of excitation coefficients ($\angle E$)obtained from the Chebyshev and the CV polynomial as per the specification of S.L.No.2 of Table 3 respectively. Comparing both the polynomials, pattern of the magnitudes of excitation are similar whereas the phase of excitation of the CV array is linear with the finite discontinuities. This is observed for any possible values of N, d, p, b. The variant δ causes this change in phase of excitation, which makes high directivity possible.

Table 3: Comparison of Chebyshev and CV polynomial with $N = 10$, $d = 0.25\lambda$

S.L.No.	$SLL(dB)$	$HPBW_{min}$	$FNBW_{min}$	$HPBW_C$	$FNBW_C$
1	−20	20.28	48.24	72.76	115.94
2	−25	22.28	55.59	75.98	124.95
3	−30	24.17	63.08	78.76	133.62
4	−35	25.97	70.65	81.18	141.92

Table 4: Camparison of directivity

ULA	$HWarray$	$Chebyshev\,array$	$CVarray$
10	17.95	9.62	67.78
10	17.95	9.04	73.10
10	17.95	8.47	66.97
10	17.95	7.98	58.97

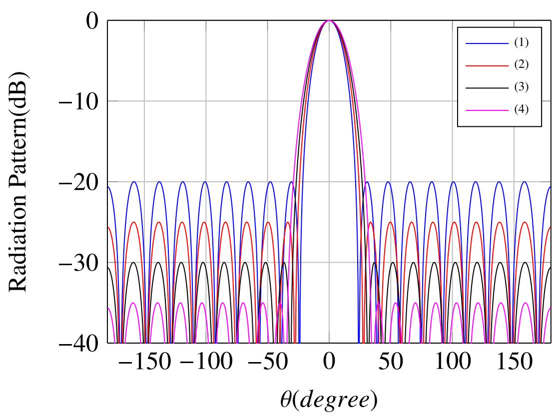

Fig. 8: Radiation Pattern of 10 element CV array with $d = 0.25\lambda$ for the specifications as in Table 3

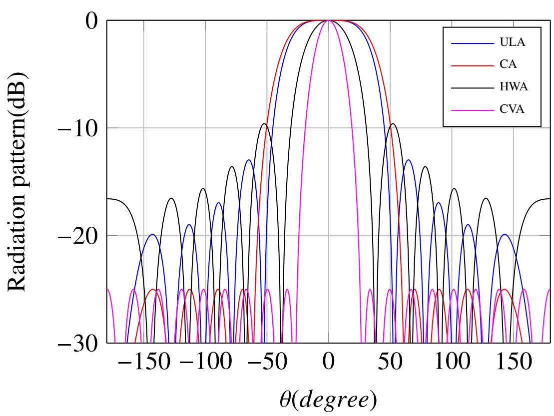

Fig. 9: Radiation Pattern of different types of endfire arrays as per S.L.No.2 of Table 3

6. Conclusion

In this work a variant of the Chebyshev polynomial (CV) was used to synthesize a linear, endfire array. In the conventional Chebyshev polynomial synthesis only the SLL can be controlled. In the proposed approach the SLL as well as the HPBW can be controlled. Simulations were carried out to show that the directivity can be improved

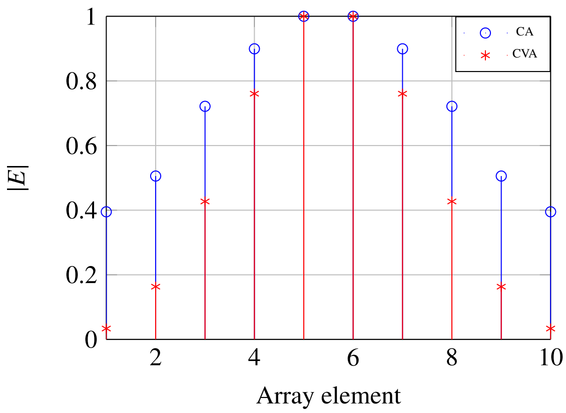

Fig. 10: Magnitude of Excitation coefficients of chebyshev and CV array as per S.L.No.2 of Table 3

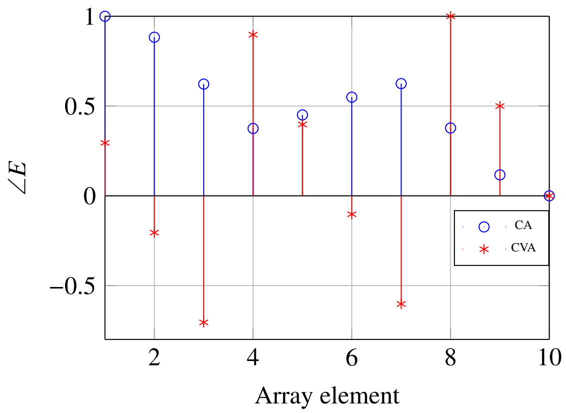

Fig. 11: Phase of Excitation coefficients of Chebyshev and CV array as per S.L.No.2 of Table 3

over the ULA. Improvement in the roll off characteristics were also shown. A practical implementation applying this synthesis procedure can result in super directive arrays when the inter element spacing is less than 0.5λ.

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