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Embedding A_4 into left–right flavor symmetry: Tribimaximal neutrino mixing and fermion hierarchy

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Abstract

We address two fundamental aspects of flavor physics: the mass hierarchy and the large lepton mixing angles. On one side, left–right flavor symmetry realizes the democratic mass matrix patterns and explains why one family is much heavier than the others. On the other side, discrete flavor symmetry such as A_4 leads to the observed tribimaximal mixing for the leptons. We show that, by explicitly breaking the left–right flavor symmetry into the diagonal A_4, it is possible to explain both the observed charged fermion mass hierarchies and quark and lepton mixing angles. In particular we predict a heavy 3rd family, the tribimaximal mixing for the leptons, and we suggest a possible origin of the Cabibbo and other mixing angles for the quarks.

1. Introduction

The recent experimental developments in neutrino physics allowed us to intensify the studies of the flavor structure of the Standard Model (SM) and its extensions. The hardest task was the understanding of the relation between the mass hierarchies and the large lepton mixing angle between the 2nd and 3rd families. In particular the no-go theorem [1] shows that contrary to expectations, a maximal mixing angle θ_{23} can never arise in the symmetric limit of whatever flavor symmetry (global or local, continuous or discrete), provided that such a symmetry also explains the hierarchy among the fermion masses and is only broken by small effects, as we expect for a meaningful symmetry. A milestone in these studies has been the discovery that mass hierarchies and mixing angles can be not directly correlated among them in the flavor symmetry breaking [2,3]. In particular, while the mass hierarchies are in general obtained by using continuous flavor symmetries, such as non-Abelian or $U(1)$ flavor symmetry á la Froggatt–Nielsen, the neutrino experimental data indicate that the lepton mixing angles may be explained by discrete flavor symmetries. This complementarity between hierarchy and mixing angles allow us to escape from the hypothesis of the theorem previously outlined [4,5]. The idea is that the flavor symmetry that predicts a large mass for the 3rd family does not make any prediction on the mixing angles. However, once the symmetry is broken into a discrete one, then the mixing angles are naturally generated. Another guideline in flavor physics is given by the unification of the gauge groups. This ingredient forces the field transformations under the flavor symmetry to be related among them, and strongly reduce the degrees of freedom in the model building.

The finite group of even permutations of 4 objects, A_4, is the smaller non-Abelian finite group that contains a triplet irreducible representation. It is the first alternating group that is not isomorphic to any modulo n group, Z_n, or to any direct product of permutation groups, S_n. It has been used in the last years [6–15] to build a huge number of models that predict for the lepton sector the tribimaximal mixing matrix [16] with maximal atmospheric angle [17,18], $\theta_{13}=0$ [19] and $\sin^2\theta_{\rm sol}=1/3$ [20–24] that agree with neutrino oscillation data. In [5] a non-supersymmetric $SO(10)\times A_4$ grand unified model, which successfully preserves tribimaximal leptonic mixing and can accommodate all known fermion masses, has been discussed.

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In this Letter we will show how the embedding of the discrete group A_4 into a left–right symmetry allows us to explain the large hierarchy between the 3rd and the first two families of quarks and charged leptons. At the same time the charged fermion masses of two light families, that of the neutrinos, and the fermion mixing matrices are related to the explicitly breaking of the left–right symmetry into the diagonal A_4 and are generated when A_4 is spontaneously broken. In particular the Cabibbo angle in the quark sector is induced by higher order operators that explicitly break $SO(3)_L \times SO(3)_R$ but preserve the diagonal A_4. Our final aim would be to introduce a gauge unification group $SO(10)$-like. Since in $SO(10)$ all the Standard Model (SM) matter fields of one family belong to the same multiplet, namely a **16**-plet, as starting point we will consider a model based on the discrete flavor symmetry A_4 in which left-handed and right-handed fermions belong to the same representation of A_4.

The group A_4 has four irreducible representations, three singlets $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and a triplet $\mathbf{3}$. Several extensions of the SM are presented in the literature, depending on the A_4 family symmetry realization and the assignments for left-handed and right-handed fermion fields. As we motivated before, we are interested in a realization where both left-handed and right-handed fields have the same A_4 assignment, in such a way to be able to perform an embedding into a gauge grand unified group like $SO(10)$. Therefore in this Letter we will consider a model similar to that proposed in [4,5] where both left-handed and right-handed fields are in the triplet representation of A_4.

2. Mass of the 3rd family from the left–right flavor symmetry

The study of models based on the flavor symmetry $U(3)_L \times U(3)_R$ [25] or its subgroups both continuous [26] or discrete [27–29] has a long history. Usually, by imposing a discrete symmetry like $S_{3_L} \times S_{3_R}$, the charged fermion mass matrix obtained is the so-called democratic mass matrix [30] given by

$$M_{0f} = \frac{m_f}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{1}$$

This matrix has only one eigenvalues different from zero, m_f, and can be assumed to be the mass of the 3rd family. The unitary matrix that diagonalizes the symmetric matrix M_{0f} has one angle and the three phases undeterminated. One possible parametrization is given by

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\cos\theta\, e^{i\alpha} & \sqrt{2}\sin\theta e^{i(\beta+\gamma)} & 1 \\ -e^{i\alpha}(\frac{\cos\theta}{\sqrt{2}} + \sqrt{\frac{3}{2}}\sin\theta e^{-i\gamma}) & e^{i\beta}(\sqrt{\frac{3}{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta e^{i\gamma}) & 1 \\ -e^{i\alpha}(\frac{\cos\theta}{\sqrt{2}} - \sqrt{\frac{3}{2}}\sin\theta e^{-i\gamma}) & -e^{i\beta}(\sqrt{\frac{3}{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta e^{i\gamma}) & 1 \end{pmatrix}. \tag{2}$$

The unknown angle and phases are fixed only after breaking the democratic structure of M_{0f} with a small perturbation δM_f, i.e.,

$$M_f = M_{0f} + \delta M_f.$$

For instance, in [31] δM_f is given by

$$\delta M_f \sim \begin{pmatrix} i\delta & 0 & 0 \\ 0 & -i\delta & 0 \\ 0 & 0 & \epsilon \end{pmatrix} \tag{3}$$

and M_f is diagonalized by

$$U^f = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix},$$

obtained by substituting $\alpha = \beta = \gamma = 0$ and $\theta = \pi/6$ in Eq. (2).

The effect of δM_f is to give a small mass to the first and second families and to fix the mixing angles. Another feature of the models based on a symmetry that gives democratic charged fermion mass matrices is that up and down quarks are diagonalized by almost identical matrices and therefore the CKM can be fitted to be close to the identity. Some attempts of including the neutrinos in this kind of models are quite successful and can fit with good agreement the data [31]. Nevertheless, models that have a democratic mass matrix for the charged fermions and at the same time predict the tribimaximal mixing matrix for the leptons are still missing.

In the following we will build a "supersymmetry inspired" model, in the sense that the scalars and the SM matter fields we introduce belong to supermultiplets and the Lagrangian arises by a superpotential. In the supersymmetric model proposed in [32] the correct alignment of the vevs in the lepton sector, that gives the tribimaximal mixing matrix, has been successfully obtained. However it is difficult to obtain the same result in a context that is non-supersymmetric. By the way, in order to focus on the origin of the mass hierarchies and mixing angle and to make lighter the reading we will report only the Yukawa Lagrangian involving the SM-like fields.

2.1. Explicitly breaking of $SO(3)_L \times SO(3)_R$ into A_4

Let us now extend the flavor symmetry and let us think to A_4 as a discrete subgroup of the continuous global group $SO(3)_L \times SO(3)_R$. To implement the idea of explaining both the hierarchy and the mixing angles by starting with the same flavor symmetry, we use two kinds of symmetry breaking: the explicit one and the spontaneous one (see Fig. 1).

We impose that the fermion weak doublets L, Q transform with respect to $SO(3)_L \times SO(3)_R$ as $\sim (\mathbf{3}, \mathbf{1})$ while the right-handed fermions E, U, D as $\sim (\mathbf{1}, \mathbf{3})$. As explained in [26], by assuming that a discrete $S_{3_L} \times S_{3_R}$ is left survived in the spontaneous breaking of $SO(3)_L \times SO(3)_R$, the charged fermion mass matrices must have the democratic structure. To write down an invariant term, we introduce a weak scalar singlet $\Phi_{ij} \sim (\mathbf{3}, \mathbf{3})$ bi-triplet with respect to $SO(3)_L \times SO(3)_R$. The charged fermion masses arise from the following Lagrangian

$$\begin{aligned}\mathcal{L}_0 = {} & h^l \epsilon_{\alpha\beta} H_d^\alpha L_i^\beta E_j \frac{\Phi_{ij}}{\Lambda} + h^u \epsilon_{\alpha\beta} H_u^\beta Q_i^\alpha U_j \frac{\Phi_{ij}}{\Lambda} \\ & + h^d \epsilon_{\alpha\beta} H_d^\alpha Q_i^\beta D_j \frac{\Phi_{ij}}{\Lambda},\end{aligned} \tag{4}$$

where H_u, H_d are the scalar components of usual weak doublets of the MSSM. The constants h^l, h^u, and h^d are of order

$$\begin{tikzcd}
SO(3)_L \times SO(3)_R \arrow[rr, "\text{explicitly}"] \arrow[d, "\text{spontaneously}"', "\text{by } \langle\Phi\rangle"'] & & A_4 \arrow[d, "\text{spontaneously}"', "\text{by } \langle\phi\rangle"'] \\
\begin{array}{c} S_{3_L} \times S_{3_R} \\ \textit{(charged fermion 3rd family masses)} \end{array} & & \begin{array}{c} Z_3 \\ \textit{(charged fermion 1st and 2nd family masses and mixing)} \end{array} \arrow[d, "\text{spontaneously}"', "\text{by } \langle\phi'\rangle"'] \\
& & \begin{array}{c} Z_2 \\ \textit{(Neutrino masses and mixing, Cabibbo angle)} \end{array} \arrow[d, "\text{spontaneously}"'] \\
& & \begin{array}{c} \text{Nothing} \\ \textit{(Sub-dominant quark mixing angles)} \end{array}
\end{tikzcd}$$

Fig. 1. Diagrammatic representation of the flavor symmetry structure of the model. The horizontal arrow indicates the explicit global symmetry breaking $SO(3)_L \times SO(3)_R \to A_4$ due to the Yukawa terms induced by a hidden scalar sector. The vertical arrows show the spontaneous breaking. The hierarchy among the masses is not directly related to the mixing angles.

Table 1
The supermultiplet content of the model. We have denoted ω_5 the discrete charge that satisfy $\omega_5^5 = 1$

	MSSM fields								Fields of the explicit breaking into A_4				
	$\hat{L}$	$\hat{Q}$	$\hat{E}$	$\hat{U}$	$\hat{D}$	$\hat{H}_u$	$\hat{H}_d$	Φ	$\hat{\Delta}$	$\hat{\tilde{\Delta}}$	$\hat{\phi}$	$\hat{\phi}'$	$\hat{\xi}'$
Weak $SU(2)$	**2**	**2**	**1**	**1**	**1**	**2**	**2**	**1**	**3**	**3**	**1**	**1**	**1**
$SO(3) \times SO(3)$	**(3, 1)**	**(3, 1)**	**(1, 3)**	**(1, 3)**	**(1, 3)**	**(1, 1)**	**(1, 1)**	**(3, 3)**					
A_4	**3**	**3**	**3**	**3**	**3**	**1**	**1**		**1**	**1**	**3**	**3**	$\mathbf{1}'$
Z_5	1	1	1	1	1	1	1	1	ω_5^2	ω_5^3	1	ω_5^3	ω_5^2

one while Λ is a cut-off. The α, β and i, j are weak and flavor indices, respectively, and $\epsilon_{\alpha\beta}$ is the antisymmetric tensor. In Ref. [26] has been shown that the minimization of a potential, invariant with respect to $SO(3)_L \times SO(3)_R$, leads $S_{3_L} \times S_{3_R}$ invariant vev. The scalar field Φ_{ij} will take its vev in the direction

$$\langle\Phi\rangle \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{5}$$

and the resulting charged fermion masses are the democratic mass matrix.

The masses of the first and second families and that of the neutrinos arise once we include, in the Yukawa Lagrangian of Eq. (4), terms that explicitly break the continuous $SO(3)_L \times SO(3)_R$ but that preserve the discrete diagonal subgroup A_4. For example, we can assume the presence of a hidden scalar sector that breaks spontaneously the continuous $SO(3)_L \times SO(3)_R$ into the diagonal A_4. The explicit breaking terms will be of the form

$$\begin{aligned}\mathcal{L}_1 = {} & \frac{\delta^l}{\Lambda}\epsilon_{\alpha\beta} H_d^\alpha \left(L^\beta E \phi\right) + \frac{\delta^u}{\Lambda}\epsilon_{\alpha\beta} H_u^\beta \left(Q^\alpha U \phi\right) \\ & + \frac{\delta^d}{\Lambda}\epsilon_{\alpha\beta} H_d^\alpha \left(Q^\beta D \phi\right) + \frac{x}{\Lambda}\xi' \left(L^\alpha \sigma^n_{\alpha\beta} L^\beta\right)'' \tilde{\Delta}_n \\ & + \frac{y}{\Lambda}\left(\phi' L^\alpha \sigma^n_{\alpha\beta} L^\beta\right)\Delta_n,\end{aligned} \tag{6}$$

where the fields in Eq. (6) transform according to Table 1 and Λ is the cut-off scale of the model. The labels α, β are again weak indices, σ^n are the Pauli matrices and Δ_n, $\tilde{\Delta}_n$ are weak triplets, as reported in Table 1. We have introduced an additional Z_5 symmetry that affects only the scalar sector and avoids the presence of unwanted Yukawa couplings as done for instance in [8,14]. We remember that, if $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are two A_4 triplets, then $\mathbf{1} \sim (ab) = (a_1 b_1 + a_2 b_2 + a_3 b_3)$, $\mathbf{1}' \sim (ab)' = (a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3)$ and $\mathbf{1}'' \sim (ab)'' = (a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3)$ [5].

Under the hypothesis that the breaking of $SO(3)_L \times SO(3)_R$ into A_4 happens in a hidden scalar sector and then it is transmitted to the fermions through the integration of the heavy fields, it is quite natural to assume that the explicit breaking terms in Eq. (6), to be added to the Lagrangian of Eq. (4), are small. To get more familiar with the embedding of A_4 into $SO(3)_L \times SO(3)_R$, we report the decomposition of some representations of $SO(3)_L \times SO(3)_R$ into the representations of A_4 in Table 2. The correspondences for the fundamental representations are obvious. We can spend few words on the bi-fundamental. The $(\mathbf{3}, \mathbf{3})$ representation of $SO(3)_L \times SO(3)_R$ gives the irreducible representations $\mathbf{1} + \mathbf{3} + \mathbf{5}$ when the group is broken to the diagonal $SO(3)$ that in turn give $\mathbf{1} + \mathbf{3} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3}'$ when $SO(3)$ is broken into A_4 as explained in [33].

Table 2
Decomposition of some representations of $SO(3)_L \times SO(3)_R$ into the representations of A_4. To clarify the decompositions, we also report the representations under $SO(3)$ diagonal

$SO(3)_L \times SO(3)_R$	$SO(3)$	A_4
$(\mathbf{3},\mathbf{1})$	$\mathbf{3}$	$\mathbf{3}$
$(\mathbf{1},\mathbf{3})$	$\mathbf{3}$	$\mathbf{3}$
$(\mathbf{3},\mathbf{3})$	$\mathbf{1}+\mathbf{3}+\mathbf{5}$	$\mathbf{1}+\mathbf{3}+\mathbf{1}'+\mathbf{1}''+\mathbf{3}'$

When ϕ takes vev as $\langle\phi\rangle = v_\phi(1,1,1)$ we have for the charged leptons

$$\frac{\delta^l}{\Lambda}\epsilon_{\alpha\beta}H_d^\alpha\big(L^\beta E\phi\big) = \epsilon_{\alpha\beta}H_d^\alpha\big[\gamma_1^l\big(L_2^\beta E_3 + L_3^\beta E_1 + L_1^\beta E_2\big) + \gamma_2^l\big(L_3^\beta E_2 + L_1^\beta E_3 + L_2^\beta E_1\big)\big], \tag{7}$$

with $\gamma_i^l = \delta_i^l v_\phi/\Lambda$ and the two δ_i arise by the two different contractions of A_4. Similar expressions are obtained for the quarks. The effect of the explicit breaking terms in the mass matrices is translated in a perturbation of the democratic mass matrix of Eq. (5), that is

$$M^f = \frac{m_3^f}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} \to \frac{m_3^f}{3}\begin{pmatrix}1 & 1+\gamma_1^f & 1+\gamma_2^f\\ 1+\gamma_2^f & 1 & 1+\gamma_1^f\\ 1+\gamma_1^f & 1+\gamma_2^f & 1\end{pmatrix} \equiv v_f\begin{pmatrix}h_0^f & h_1^f & h_2^f\\ h_2^f & h_0^f & h_1^f\\ h_1^f & h_2^f & h_0^f\end{pmatrix}, \tag{8}$$

with the obvious correspondences $m_3^f/3 = v_f h_0^f = v_f v_\Phi/\Lambda$, $v_f h_{1,2}^f = (1+\gamma_{1,2}^f)\cdot m_3^f/3$ and $v_f = v_{u,d}$. The mass matrix of Eq. (8) is diagonalized by

$$\tilde U_\omega = \frac{1}{\sqrt3}\begin{pmatrix}\omega & \omega^2 & 1\\ \omega^2 & \omega & 1\\ 1 & 1 & 1\end{pmatrix}, \tag{9}$$

corresponding to the U of Eq. (2) with $\theta = \pi/4$, $\alpha = 2\pi/3$, $\beta = 5\pi/6$ and $\gamma = \pi/2$. M^f of Eq. (8) gives an heavy 3rd family mass m_3^f and small 1st and 2nd family masses satisfying

$$\frac{m_1^f}{m_3^f} = \frac{\omega\gamma_1^f + \omega^2\gamma_2^f}{3}, \qquad \frac{m_2^f}{m_3^f} = \frac{\omega^2\gamma_1^f + \omega\gamma_2^f}{3}. \tag{10}$$

2.2. Neutrino sector

The Yukawa interactions for the neutrinos are the following

$$\mathcal{L}_\nu = \frac{x}{\Lambda}\xi'\big(L^\alpha\sigma_{\alpha\beta}^n L^\beta\big)''\tilde\Delta_n + \frac{y}{\Lambda}\big(\phi' L^\alpha\sigma_{\alpha\beta}^n L^\beta\big)\Delta^n, \tag{11}$$

where the scalars ξ' and ϕ' are singlets of the weak $SU(2)_L$ and transform with respect to A_4 as $\mathbf{1}'$ and $\mathbf{3}$, respectively. The scalars Δ and $\tilde\Delta$ are singlets of A_4 and triplets of the weak $SU(2)_L$. When the triplet field ϕ' takes vev in the A_4 direction $\langle\phi\rangle \sim (0,0,1)$—notice that this alignment is different from the one used in many models as for example in [5,8], the resulting neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix}a & b & 0\\ b & \omega a & 0\\ 0 & 0 & \omega^2 a\end{pmatrix} = \tilde V_\nu\begin{pmatrix}\omega^2 a + b & 0 & 0\\ 0 & \omega^2 a & 0\\ 0 & 0 & -\omega^2 a + b\end{pmatrix}\tilde V_\nu^T,$$
$$\tilde V_\nu = \begin{pmatrix}\frac{\omega}{\sqrt2} & 0 & -i\frac{\omega^2}{\sqrt2}\\ \frac{\omega^2}{\sqrt2} & 0 & i\frac{\omega}{\sqrt2}\\ 0 & 1 & 0\end{pmatrix}. \tag{12}$$

The charged leptons are diagonalized by $L \to \tilde U_\omega L$, so we obtain a tribimaximal mixing for the lepton sector, that is

$$V_{\mathrm{PMNS}} = \tilde U^\dagger\cdot\tilde V_\nu = \begin{pmatrix}\frac{2}{\sqrt6} & \frac{1}{\sqrt3} & 0\\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt2}\\ -\frac{1}{\sqrt6} & \frac{1}{\sqrt3} & \frac{1}{\sqrt2}\end{pmatrix} \tag{13}$$

and the neutrino masses result to have the same expressions of [8].

2.3. The origin of the Cabibbo angle

In Eq. (6) we have reported the leading A_4 invariant terms that arise after the explicitly breaking of $SO(3)_L \times SO(3)_R$. We include now the higher order operators suppressed by powers of the cut-off scale Λ. The first terms at order $1/\Lambda^2$ that change the structure of the charged fermion mass matrices above are

$$\mathcal{L}_3 = g^l\epsilon_{\alpha\beta}H_d^\alpha\left(L^\beta E\frac{\phi'\xi'}{\Lambda^2}\right) + g^u\epsilon_{\alpha\beta}H_u^\beta\left(Q_\alpha U\frac{\phi'\xi'}{\Lambda^2}\right) + g^d\epsilon_{\alpha\beta}H_d^\alpha\left(Q_\beta D\frac{\phi'\xi'}{\Lambda^2}\right). \tag{14}$$

With the inclusion of these contributions the charged fermion mass matrices of Eq. (8) become

$$M^f = v_f\begin{pmatrix}h_0^f & h_1^f & h_2^f\\ h_2^f & h_0^f & h_1^f\\ h_1^f & h_2^f & h_0^f\end{pmatrix} \to M_{\mathrm{eff}}^f = v_f\begin{pmatrix}h_0^f & h_1^f + 3\rho_1^f & h_2^f\\ h_2^f + 3\rho_2^f & h_0^f & h_1^f\\ h_1^f & h_2^f & h_0^f\end{pmatrix},$$

where $v_f = v_{u,d}$, $3\rho_i^f \sim g_i^f v_{\phi'}v_\xi/\Lambda^2$. The g_i^f, $i = 1,2$, arise from the possible different contractions of $\mathbf{3}$-plet of A_4 to give a singlet $\mathbf{1}''$ and the factor 3 is introduced to simplify the subsequent formulas. In the basis rotated by $\tilde U_\omega$ of Eq. (9), namely $\tilde M^f \equiv \tilde U_\omega^\dagger M_{\mathrm{eff}}^f\tilde U_\omega$, the charged fermion mass matrices are now approximately given by

$$\tilde M^f \approx \tilde v_f\begin{pmatrix}r_1^f + \epsilon_1^f\omega + \epsilon_2^f\omega^2 & \epsilon_1^f + \epsilon_2^f & \epsilon_1^f\omega^2 + \epsilon_2^f\omega\\ \epsilon_1^f + \epsilon_2^f & r_2^f + \epsilon_1^f\omega^2 + \epsilon_2^f\omega & \epsilon_1^f + \omega^2\epsilon_2^f\\ \epsilon_1^f\omega^2 + \epsilon_2^f\omega & \epsilon_1^f + \omega^2\epsilon_2^f & r_3^f + \epsilon_1^f + \epsilon_2^f\end{pmatrix}, \tag{15}$$

where $r_i^f = m_i^f/\tilde{v}_f$, $\tilde{v}_f = v_f v_\Phi/\Lambda$ and $\epsilon_i^f = \rho_i^f/\tilde{v}_f$. Let us assume that the ϵ_i^f are arbitrary parameters of $O(\lambda^5)$, where λ is the Cabibbo angle. The crucial point is that this assumption has the consequences that the higher order operators give negligible effects in the down and charged lepton sectors, since for the down and charged leptons we have $(r_1^{d,l}, r_2^{d,l}, r_3^{d,l}) \sim (\lambda^4, \lambda^2, 1)$ and $\tilde{M}^{d,l}$ may be considered diagonals. On the contrary for the up quarks we have that $(r_1^u, r_2^u, r_3^u) \sim (\lambda^7, \lambda^4, 1)$ and therefore the off-diagonal entries $(1,2)$ and $(2,1)$ cannot be neglected: the matrix $\tilde{M}^u$ is diagonalized by a rotation in the 12 plane with $\sin\theta_{12} \approx \lambda$. This rotation produces the Cabibbo angle in the CKM. In fact while M^d is still diagonalized by U_ω, we have that M^u is diagonalized by $V_L^{u^\dagger} U_\omega^\dagger M^u U_\omega V_R^u$ where V_{LR}^u are unitary matrix, rotations in the 12 plane, and therefore the CKM mixing matrix is given by

$$V_{\mathrm{CKM}} = V_L^{u^\dagger} U_\omega^\dagger U_\omega \equiv V_L^{u^\dagger}.$$

The charm and top quark masses are almost unaffected by the corrections and still are given by $\tilde{v}_u r_2^u$ and $\tilde{v}_u r_3^u$, respectively. The up quark mass is obtained by tuning the ϵ_i^u and is given more or less by

$$m_u \approx \tilde{v}_u\left(\epsilon_1^u \omega + \epsilon_2^u \omega^2\right).$$

In [13] the full CKM was obtained by breaking the Z_2 symmetry that survives when a triplet of A_4 takes vev in the direction $(1,0,0)$. In our model we suggest that the origin of the Cabibbo angle is instead in the A_4 invariant subleading corrections to the Yukawa interactions. The breaking of the residual Z_2 symmetry allows instead to generate the complete CKM. The main difference between our model and some previous models, where the subleading corrections in the charged fermion matrix are too small to generate a Cabibbo angle in order to keep the lepton mixing angles inside the bounds given by the experimental data, is related to the different assignment and the $U(1)$ flavor symmetry one introduces in order to explain the mass hierarchies. For example, in [32] the left-handed fields belonged to a triplet of A_4, while at the right-handed fields was given the assignment $\mathbf{1}, \mathbf{1}'', \mathbf{1}'$ and they have $U(1)$ charges $(2q, q, 0)$ where q is a real number.

3. Grand unified group $SO(10) \times SU(3)$

As already explained in the introduction our final aim would be the construction of a grand unified $SO(10)$-like model. Let us assume the group A_4 as flavor symmetry and the "constrain" of assigning right and left-handed fermion fields to the same representations. Since A_4 has four irreducible representations, three singlets $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$, and a triplet $\mathbf{3}$, clearly we have just few possibilities. For example if we assign the three $\mathbf{16}$-plets to $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ we obtain a mass matrix for the charged fermions of the form

$$M_f = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \beta \\ 0 & \beta & 0 \end{pmatrix}, \tag{16}$$

where α and β are arbitrary parameters, that gives for instance the wrong prediction $m_c = m_t$. The situation is better only when the three $\mathbf{16}$-plets transform as a triplet of A_4. Indeed, it has been showed in [5] that the assignment of both left-handed and right-handed SM fields to triplets of A_4, that is therefore compatible with $SO(10)$, can be lead to the charged fermion textures proposed by Ma [4] and given by

$$M_f = \begin{pmatrix} h_0^f & h_1^f & h_2^f \\ h_2^f & h_0^f & h_1^f \\ h_1^f & h_2^f & h_0^f \end{pmatrix}, \tag{17}$$

with h_0^f, h_1^f and h_2^f distinct parameters. In [5], in order to obtain a mass matrix of the form of M_f in Eq. (17) without spoiling the predictions of the neutrino sector, higher order operators were introduced containing simultaneously the $SO(10)$ representations $\mathbf{45}_{T3R}$ and $\mathbf{45}_Y$ that took vevs in the isospin and hypercharge directions respectively. A renormalizable $SO(10) \times A_4$ model has been recently studied in [34] where however the A_4 flavor symmetry does not enforce a tribimaximal mixing in the lepton sector.

The group $SO(3)_L \times SO(3)_R$ is not compatible with $SO(10)$ since the $\mathbf{16}$-plet contains both left-handed and right-handed fields that belong to different representations of $SO(3)_L \times SO(3)_R$. We have therefore to search for a continuous group larger than $SO(3)_L \times SO(3)_R$, with rank bigger than $2+2=4$, and that has a triplet as fundamental representation. The group $SU(3)$ seems us a good candidate. The scalar field $\Phi_{ij} \sim (\mathbf{3}, \mathbf{3})$ of the model we have just considered will correspond to the $\bar{\mathbf{6}}$ representation of $SU(3)$ whose vev is compatible with the democratic mass matrix.

Without entering into the details of the realization of an $SO(10) \times SU(3)$ model [35–39] that we leave for a future work, we want to suggest how its realization could be achieved using non-renormalizable operators. We can think that such operators arise by integrating out some heavy extra fermions that are coupled to the matter fields at the renormalizable level, for instance see [40–43]. The effective $SO(10)$ invariant Lagrangian could be

$$\mathcal{L} = \mathcal{L}_{SU(3)} + \delta\mathcal{L}_{A_4},$$

where $\mathcal{L}_{SU(3)}$ is $SO(10) \times SU(3)$ invariant and $\delta\mathcal{L}_{A_4}$ is the explicit breaking term of the $SU(3)$ symmetry that, at this level, leaves $SO(10)$ unbroken. In particular the $SU(3)$ invariant term is

$$\mathcal{L}_{SU(3)} = \lambda\, \mathbf{16}\,\mathbf{16}\,\mathbf{10}_D \mathbf{45}_{T_{3R}} \mathbf{45}_Y,$$

where $\mathbf{10}_D$ transforms as $(\mathbf{10}, \bar{\mathbf{6}})$ with respect to $SO(10) \times SU(3)$. The scalar fields $\mathbf{45}_{T_{3R}}$ and $\mathbf{45}_Y$ are singlets of $SU(3)$ and their vevs are proportional to the right-handed isospin and to the hypercharge respectively. Thanks to the $\mathbf{45}_{T_{3R}}$ and $\mathbf{45}_Y$ scalar fields, the above operator does not give any contribution to the neutrino sector, while all the charged fermion mass matrices are of democratic form if $\mathbf{10}_D$ takes vev along the direction that preserves an $S_3 \times S_3$ subgroup of $SU(3)$. At this stage only the 3rd family acquire a mass. The neutrino mass matrix and the first and second families masses arise when we switch on the explicitly breaking terms of $SU(3)$

$$\delta\mathcal{L}_{A_4} = \mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_{s,t} + \mathbf{16}\,\mathbf{16}\,\mathbf{10}\,\mathbf{45}'_{T_{3R}}\mathbf{45}'_Y,$$

where the scalar fields $\overline{\mathbf{126}}_{s,t}$ are a singlet $\mathbf{1}'$ and a triplet of A_4, respectively, the $\mathbf{45}'_{T_{3R}}$, $\mathbf{45}'_Y$ are other scalars that transform as $\mathbf{45}$ of $SO(10)$, singlet and triplet of A_4, respectively. The $\mathbf{10}$ is a singlet of A_4. It is not difficult to show that when type-II seesaw is dominant, the first term in $\delta\mathcal{L}_{A_4}$ generates the light neutrino mass matrix of the form of Eq. (12). The second term in $\delta\mathcal{L}_{A_4}$ gives a contribution like in Eq. (6) and, after the breaking of A_4 into Z_3, it generates the first and second family masses, see Eqs. (8)–(10).

4. Conclusions

In this Letter we have proposed an embedding of the discrete A_4 flavor symmetry in the larger continuous group $SO(3)_L \times SO(3)_R$ that explains in a natural way the huge hierarchy between the 3rd family charged fermion masses and the others two. This is a consequence of the fact that $SO(3)_L \times SO(3)_R$ breaks spontaneously into $S_{3L} \times S_{3R}$ and gives a democratic mass matrix that has only one massive eigenstate. If such eigenstate is assumed to be the 3rd family state, we still have an undeterminated 12 angle in the charged fermion sector that is fixed by breaking the democratic mass matrix. The crucial feature of our model is that once we break explicitly $SO(3)_L \times SO(3)_R$ into A_4 we automatically generate first and second family charged fermion masses $m_{1,2} \ll m_3$. In order to fit the hierarchy between the masses of the first and second families, we require a tuning. Assuming that the light neutrino Yukawa interactions come from the couplings with an A_4 singlet $\xi \sim \mathbf{1}'$ and an A_4 triplet ϕ' that are scalar electroweak singlets and that ϕ' acquires vev in the direction $(0, 0, 1)$, we have showed that the lepton mixing matrix is the tribimaximal one. The CKM is given by the identity matrix. Afterward we suggest how to generate the Cabibbo angle in the quark sector through the introduction of higher order corrections. In particular in our model higher order operators give corrections of the same magnitude in each entries of all charged fermion mass matrices. Assuming that the ratio between the correction and m_c is of the order of the Cabibbo angle λ, we obtain that a rotation of order λ in the 12 plane appears in the up mass matrix. However the down and charged lepton mass matrices are almost unaffected by corrections. This mismatching gives up to the Cabibbo angle.

Finally we have briefly discussed an $SO(10)$ realization of our model where the flavor group $SO(3)_L \times SO(3)_R$ is enlarged to $SU(3)$ and the democratic structure should arise from the vev of a scalar that transform as a $\bar{\mathbf{6}}$ of the flavor group $SU(3)$.

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