Parity and time reversal in J/Ψ decay

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Abstract

With the prospect of large numbers of J/Ψ decay events becoming available in the near future, it is interesting to search for symmetry violating effects as probes of new physics and tests of the standard model. J/Ψ decay events could provide the first observation of weak effects in otherwise strongly decaying particles. We calculate a T-odd asymmetry in the J/Ψ decay into photon plus lepton pair due to Z boson exchange. Extensions to hadronic final states are also discussed.

1. Introduction

In the US and China it is possible that current accelerators could be adopted to become J/Ψ factories with an initial annual yield of 10^9 J/Ψ. Such machines could provide the first laboratory for the interplay of weak and strong forces leading to parity and ultimately time reversal violating effects in the hadronic final states. Although parity odd correlations should be easily observable in such machines, the observation of true standard model time reversal violations will be quite challenging and probably will require further luminosity upgrades beyond those of the first generation machines. Some consideration of weak effects in J/Ψ decay have already been made in final states other than those considered here [1]. Tests of T-violation will require a careful separation of pseudo-time-reversal violation coming, not from T-violating terms in the La-

grangian, but from final state interactions and particle width effects.

In leptonic decays final state interactions are expected to be negligible. However the fact that unstable particles decrease in amplitude rather than increase causes a possible T-odd asymmetry to appear in leptonic final states even in the absence of T-odd terms in the Lagrangian. In the current article we calculate the J/Ψ decay to photon plus lepton pair including Z boson intermediate states.

In Section 2 we calculate the standard model electroweak decay of the J/Ψ to photon plus lepton pair and present the distribution in photon energy. Section 3 is devoted to a discussion of the T-odd asymmetry defined by a forward–backward asymmetry of the normal to the plane defined by the lepton pair. In the current calculation this asymmetry is proportional to the Z width and is below the sensitivity of the first generation J/Ψ factory proposals. Nevertheless it serves as a background calculation for possible new physics searches such as an electric dipole moment of the charm quark as well as a model for larger though theoretically more complicated hadronic final

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state effects. Conclusions and some discussion of related opportunities at J/Ψ factories are presented in Section 4.

2. Standard model amplitudes for $J/\Psi \to$ photon plus lepton pair

The lowest order electroweak Feynman graphs for J/Ψ are shown in Figs. 1 and 2. C invariance requires that in the amplitude $\mathcal{M}_2$ the lepton pair is produced by an intermediate Z boson with axial coupling at the charm quark line. Therefore, in Fig. 2, the virtual photon and the vector coupling of the Z do not contribute. The matrix element is

$$
\mathcal{M}_2 = \frac{4\mathcal{M}_0 e_q a_{Aq} i\varepsilon(\epsilon, \lambda, \epsilon^\gamma, k)}{k \cdot P D_Z(q)}
$$
$$
\times \bar{u}(p_1)\gamma_\lambda(a_{Vl} + a_{Al}\gamma_5)v(p_2). \tag{2.1}
$$

Here we neglect the lepton mass in numerator factors. We have defined in terms of the totally antisymmetric Levi-Civita symbol

$$
\varepsilon(a, b, c, d) = \varepsilon_{\mu\nu\alpha\beta}a^\mu b^\nu c^\alpha d^\beta. \tag{2.2}
$$

We employ the Bjorken–Drell sign conventions in which

$$
\varepsilon_{0123} = +1. \tag{2.3}
$$

The electric charges of the quark and lepton are $e_q = 2/3$, $e_l = -1$. The Z couplings to a fermion f are written

$$
\mathcal{L} = eZ_\lambda \bar{f}\gamma_\lambda(a_{Vf} + a_{Af}\gamma_5)f, \tag{2.4}
$$

with standard model values

$$
a_{Vf} = \frac{I_{3f} - 2e_f(\sin\theta_W)^2}{\sin(2\theta_W)}, \tag{2.5}
$$

$$
a_{Af} = \frac{-I_{3f}}{\sin(2\theta_W)}. \tag{2.6}
$$

We write for the denominator of the Z propagator

$$
D_Z(q) = M_Z^2 - q^2 - iM_Z\Gamma_Z. \tag{2.7}
$$

Here P, p_1, p_2, k, respectively, are the four momenta of the J/Ψ, the negatively charged lepton, the positively charged (anti)lepton, and the photon. In addition we have put $q = p_1 + p_2$. We use the non-relativistic

color singlet model for the J/Ψ where the quark momentum and mass in the J/Ψ are $P/2$ and $M/2$, respectively. The model, based on the positronium formalism, was extended to quarkonia some twenty years ago and has been successfully employed in a number of applications [2]. The non-relativistic quark model specifies the insertion of the wave function factor

$$
F_W = \frac{\Psi(0)T^0}{\sqrt{12M}}(\gamma \cdot P + M)\gamma \cdot \epsilon, \tag{2.8}
$$

into the charm quark trace. In Fig. 1 a trace is taken over the Dirac and color matrices, T^0 being the 3×3 unit matrix in color space.

The wave function at the origin is then related to the electronic decay rate of the J/Ψ.

$$
\left|\Psi(0)\right|^2 = \frac{\Gamma(J/\Psi \to e^+e^-)M^2}{(16\pi\alpha^2 e_q^2)} \approx 0.0044M^2 \text{ GeV}. \tag{2.9}
$$

The wave function at the origin squared has dimensions $[E]^3$. The empirical scaling with mass is indicated allowing our result to be also extended to other vector quarkonia. In terms of the fine structure constant, α, and the J/Ψ wave function at the origin, the normalization constant $\mathcal{M}_0$ is given by a trace around the quark line in Fig. 1 together with a collection of coupling constants. Namely

$$
\mathcal{M}_0\epsilon_\mu = 2(4\pi\alpha)^{3/2}\text{Tr}\, F_W\gamma_\mu
$$
$$
= (4\pi\alpha)^{3/2}\sqrt{3M}\,\Psi(0)\epsilon_\mu. \tag{2.10}
$$

The corresponding amplitude for the Feynman graphs of Fig. 1 is

$$
\mathcal{M}_1 = \mathcal{M}_0 e_l \bar{u}(p_1)
$$
$$
\times \left(2\left(\frac{p_2 \cdot \epsilon^\gamma}{p_2 \cdot k} - \frac{p_1 \cdot \epsilon^\gamma}{p_1 \cdot k}\right)\gamma \cdot \epsilon \right.
$$
$$
\left. + \frac{\gamma \cdot \epsilon\gamma \cdot k\gamma \cdot \epsilon^\gamma}{p_2 \cdot k} - \frac{\gamma \cdot \epsilon^\gamma\gamma \cdot k\gamma \cdot \epsilon}{p_1 \cdot k}\right)
$$
$$
\times (A + B\gamma_5)v(p_2). \tag{2.11}
$$

Here A contains the vector couplings of the photon and Z while B is the axial term.

$$
A = -\frac{e_q e_l}{M^2} + \frac{a_{Vq}a_{Vl}}{D_Z(P)},
$$
$$
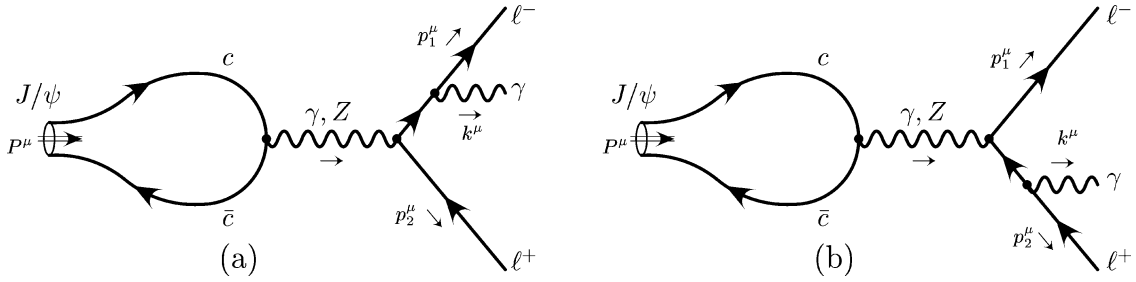B = \frac{a_{Vq}a_{Al}}{D_Z(P)}. \tag{2.12}
$$

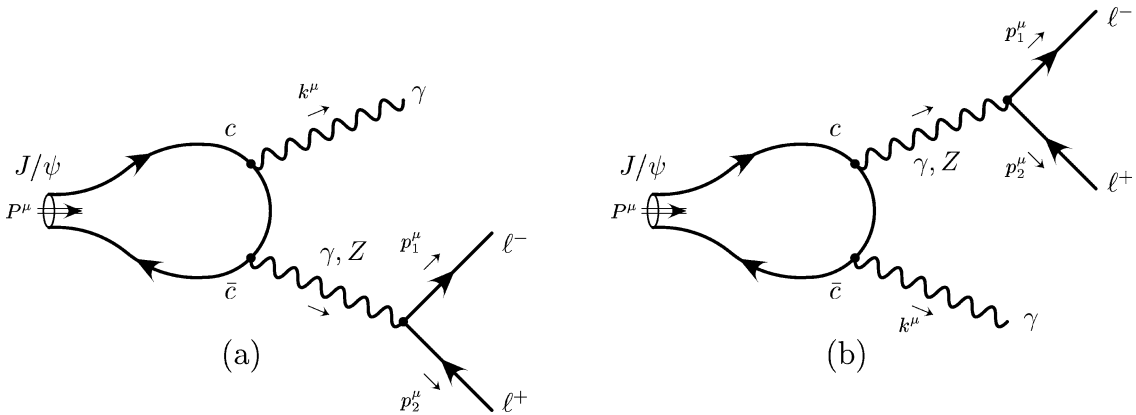Fig. 1. Feynman diagrams with vector couplings only on the quark line.

Fig. 2. Feynman diagrams with an axial coupling on the quark line.

In electron–positron annihilation the J/Ψ is produced transversely polarized. Consequently the average over the J/Ψ helicities is given by

$$\frac{1}{2} \sum_\lambda \epsilon_\beta(\lambda) \epsilon_\gamma^*(\lambda)$$
$$= \frac{1}{2} \left(-g_{\beta\gamma} + \frac{P_\beta P_\gamma}{M^2} - \frac{\delta_\beta \delta_\gamma}{M^2} \right)$$
$$= -\frac{1}{2} g_{\beta\gamma} + \frac{p_\beta^e \overline{p_\gamma^e} + \overline{p_\beta^e} p_\gamma^e}{M^2}, \qquad (2.13)$$

where P is the J/Ψ momentum and p^e, $\overline{p^e}$ are the initial state electron and positron momenta, respectively. In addition we have put

$$\delta = p^e - p^{\bar{e}}. \qquad (2.14)$$

Because of the symmetry of the spin summation one can choose a basis in which the J/Ψ polarization vector is real.

We neglect here a small departure from transverse polarization due to production through the Z whose effect on the distributions we calculate is subleading in $(M/M_Z)^2$. Similarly we treat the J/Ψ as a pure vector state neglecting any possible weak mixing with the axial vector quarkonium state that is several hundred MeV higher in mass. To adequately treat these P wave states one must allow some relative momentum between the quarks in the charmonium wave function.

The matrix elements squared summed over final state spins are then

$$|\mathcal{M}_1|^2 = \frac{16(|A|^2 + |B|^2)}{p_1 \cdot k \, p_2 \cdot k} \mathcal{M}_0^2 e_l^2$$
$$\times \left(-\epsilon \cdot \epsilon \left[(p_1 \cdot P)^2 + (p_2 \cdot P)^2 \right] \right.$$
$$\left. - M^2 \left[(p_1 \cdot \epsilon)^2 + (p_2 \cdot \epsilon)^2 \right] \right), \qquad (2.15)$$

$$|\mathcal{M}_2|^2 = \frac{128\mathcal{M}_0^2 e_q^2 a_{Aq}^2 (a_{Vl}^2 + a_{Al}^2)}{(P \cdot k)^2 |D_Z(q)|^2}$$
$$\times \left(-\epsilon \cdot \epsilon \, p_1 \cdot k \, p_2 \cdot k \right.$$
$$\left. + k \cdot \epsilon [p_1 \cdot \epsilon \, p_2 \cdot k + p_2 \cdot \epsilon \, p_1 \cdot k] \right),$$
$$(2.16)$$

$$2\,\mathrm{Re}\,\mathcal{M}_2\mathcal{M}_1^*$$
$$= \frac{64\mathcal{M}_0^2 a_{Aq} e_l e_q}{k \cdot P}$$
$$\times \left(D_2 \varepsilon(P, \epsilon, p_1, p_2) \left(\frac{p_1 \cdot \epsilon}{p_1 \cdot k} + \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \right) \right.$$
$$+ F_2 \left(\epsilon \cdot \epsilon \left[k \cdot P - M^2 \right] \right.$$
$$- k \cdot P \left[\frac{(p_1 \cdot \epsilon)^2}{p_1 \cdot k} + \frac{(p_2 \cdot \epsilon)^2}{p_2 \cdot k} \right]$$
$$+ k \cdot \epsilon \left[\frac{p_1 \cdot P \, p_1 \cdot \epsilon}{p_1 \cdot k} \right.$$
$$\left. \left. \left. + \frac{p_2 \cdot P \, p_2 \cdot \epsilon}{p_2 \cdot k} \right] \right) \right).$$
$$(2.17)$$

Here

$$D_2 = \mathrm{Re}\, \frac{i(a_{Vl}A^* + a_{Al}B^*)}{D_Z(q)}, \qquad (2.18)$$

and

$$F_2 = \mathrm{Re}\, \frac{a_{Vl}B^* + a_{Al}A^*}{D_Z(q)}. \qquad (2.19)$$

Performing the spin average and partially integrating over the phase space, we find the double distribution in photon energy and angle relative to the incident electron as a sum of three terms corresponding to the two direct terms and one interference term

$$\frac{d^2\Gamma_{11}(\Psi \to l^+l^-\gamma)}{dk_0\, d(\cos\theta)}$$
$$= E_{11}\left(2\sin^2\theta_k \left(\frac{M}{k_0} - 2 \right) \right.$$
$$+ \left(1 + \cos^2\theta_k\right) \left(-1 + \ln\frac{M(M - 2k_0)}{m_l^2} \right)$$
$$\left. \times \left(-2 + 2\frac{k_0}{M} + \frac{M}{k_0} \right) \right), \qquad (2.20)$$

where

$$E_{11} = 12\Psi(0)^2\alpha^3\left(|A|^2 + |B|^2\right)M, \qquad (2.21)$$

$$\frac{d^2\Gamma_{22}(\Psi \to l^+l^-\gamma)}{dk_0\, d(\cos\theta)}$$
$$= E_{22}\frac{k_0}{3M}\left(1 + \sin^2\theta\left(\frac{1}{2} - 2\frac{k_0}{M} \right) \right), \qquad (2.22)$$

where

$$E_{22} = 96\alpha^3 M |\Psi(0)|^2 e_q^2 a_{Aq}^2 \frac{(a_{Vl}^2 + a_{Al}^2)}{|D_Z(q)|^2}, \qquad (2.23)$$

$$\frac{d^2\Gamma_{12}(\Psi \to l^+l^-\gamma)}{dk_0\, d(\cos\theta)}$$
$$= E_{12}\left(1 + \left(\frac{1}{2} - 2\frac{k_0}{M} \right) \sin^2\theta \right), \qquad (2.24)$$

with

$$E_{12} = 2a_{Aq}e_l e_q \frac{|\mathcal{M}_0|^2}{(2\pi)^3} F_2$$
$$= 48\alpha^3 M |\Psi(0)|^2 e_l e_q a_{Aq} F_2. \qquad (2.25)$$

In spite of the presence of the Z, there is no forward–backward asymmetry in the photon direction. This is due to the assumed pure vector nature of the J/Ψ which results in its spin summation being symmetric under the interchange $p^e \leftrightarrow p^{\bar{e}}$.

3. T-odd asymmetry

In principle one could search for T-violation in the $l^+l^-\gamma$ final state in J/Ψ decay. A possible T-odd effect might be a forward backward asymmetry of the normal to the plane of the final state leptons. This could come from an electric dipole moment of the charm quark or other new physics effects. In the standard model, there is no T-violation in the Z couplings, however T-odd effects could come from final state interactions or particle instabilities which are not, of course, evidence for T-violation in the Lagrangian. By restricting our attention to non-hadronic final states, we avoid strong final state interactions. However, since unstable particles decrease in amplitude rather than increase, width effects in propagators can mimic T-violation. Thus one could expect a T-odd asymmetry in J/Ψ decay or in other processes due to intermediate Z bosons. Such effects will be proportional to the Z width to mass ratio. In this section we calculate this T-odd asymmetry in the standard model

for its own sake and as a background to new physics searches.

The appearance of the Levy-Civita tensor in (2.17) is such a T-odd effect since the coefficient D_2 is proportional to the Z width.

Averaging this term over J/Ψ polarizations using (2.13) we have

$$|\overline{\mathcal{M}}|^2 = -\frac{-32|\mathcal{M}_0|^2 a_{Al} e_l e_q D_2}{M^2} \frac{\varepsilon(P, \delta, p_1, p_2)}{k \cdot P}$$
$$\times \left(\frac{p_1 \cdot \delta}{p_1 \cdot k} + \frac{p_2 \cdot \delta}{p_2 \cdot k}\right). \tag{3.1}$$

The contribution to the J/Ψ decay rate is

$$d\Gamma = \frac{1}{2M}|\overline{\mathcal{M}}|^2 d\Omega_3, \tag{3.2}$$

with the standard Lorentz invariant phase space differential:

$$d\Omega_3 = \frac{d^3k}{2k_0} \frac{d^3p_1}{2p_{10}} \frac{d^3p_2}{2p_{20}} \frac{\delta^4(P - p_1 - p_2 - k)}{(2\pi)^5}. \tag{3.3}$$

One may use the 4D delta function to eliminate p_2 and afterwards it is convenient to do the p_1 integral in the rest frame of $P - k$ with the k direction defining the z axis and the δ direction taken to be in the xz plane. Relative to this coordinate system the p_1 polar and azimuthal angles are θ_1 and ϕ_1, respectively. The result can be put back into a manifestly Lorentz invariant form allowing one to then do the k integral in the J/Ψ rest frame with the δ (or p^e) direction defining the z axis. In this latter frame the energy and polar angle of k are k_0 and θ_k, respectively. There can be no azimuthal dependence.

In terms of these variables and $w = 2k_0/M$ we have

$$d\Omega_3 = \frac{M^2}{64(2\pi)^4} w \, dw \, d(cos(\theta_k)) \, d(cos(\theta_1)) \, d\phi_1, \tag{3.4}$$

and

$$d\Gamma = \frac{M}{4} \frac{|\mathcal{M}_0|^2 a_{Al} e_l e_q D_2}{(2\pi)^4} \sqrt{1-w}$$
$$\times \sin(\theta_k)\sin(\phi_1) \, dw \, d(cos(\theta_k)) \, d\theta_1 \, d\phi_1 \, f, \tag{3.5}$$

where

$$f = w\cos(\theta_k)\sin^2(\theta_1) - 2\sqrt{1-w}\cos(\theta_1)$$
$$\times \left(\cos(\theta_k)\cos(\theta_1) + \sin(\theta_k)\sin(\theta_1)\cos(\phi_1)\right). \tag{3.6}$$

A T-odd parameter is any function, y, of the final state momenta that is odd in $\varepsilon(P, \delta, p_1, p_2)$. In the J/Ψ rest frame this is proportional to the cosine of the angle between the normal to the final state lepton plane and the initial state electron direction. From an experimental point of view $\varepsilon(P, \delta, p_1, p_2)$ itself is not the optimum variable to consider since the corresponding distribution has an (integrable) singularity at the origin. A better variable from this point of view is

$$y = \sin(\theta_k)\sin(\phi) = -\frac{\varepsilon(P, \delta, p_1, p_2)}{M\sqrt{2p_1 \cdot k p_2 \cdot k p_1 \cdot p_2}}. \tag{3.7}$$

One may then define

$$\frac{d\Gamma}{dy} = \int d\Gamma \, \delta\big(y - \sin(\theta_k)\sin(\phi)\big)\epsilon(k \cdot \delta). \tag{3.8}$$

The factor $\epsilon(k \cdot \delta)$ implies that the events with the photon momentum in the forward hemisphere relative to the incident electron are to be counted negatively. Without such a factor the distribution would vanish identically upon integration over the photon direction.

The integral can be easily done by Monte Carlo methods. In Fig. 3 we plot $\frac{1}{\Gamma_{J/\Psi}} \frac{d\Gamma}{dy}$ versus y. The distribution is too small to be seen at the first generation J/Ψ factories where of order of 10^9 events are projected. This means that a search for a y asymmetry is sensitive to new physics such as, perhaps, an electric
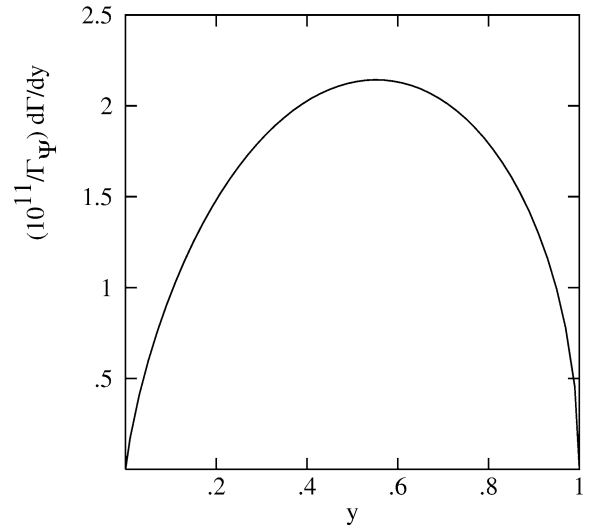

Fig. 3. Asymmetry distribution (times 10^{11}) in the region of positive y (see text).

dipole moment of the charm quark. Most models for electric dipole moments, such as supersymmetry, predict an effect proportional to the mass of the fermion. Thus the electric dipole moment of the charm quark can be expected to be orders of magnitude greater than any effect in the light quarks [3]. We leave a full calculation of the T-odd asymmetry in J/Ψ decay in the presence of a charm quark electric dipole moment to a later paper. At present we know of no experimental limits on charm quark electric dipole moments.

It is clear from Eq. (3.8) that the distribution is antisymmetric in y. To leading order in $(1/M_Z)$ it can be written analytically.

$$
\frac{d\Gamma}{dy} = \frac{11\alpha^3 |\Psi(0)|^2 a_{Vl} a_{Aq} (e_l e_q)^2 \Gamma_Z}{5 M_Z^3}
$$
$$
\times \ y \ln \frac{1 + \sqrt{1 - y^2}}{1 - \sqrt{1 - y^2}}. \tag{3.9}
$$

Integrated over positive values of y and expressed relative to the J/Ψ total width this is

$$
\frac{1}{\Gamma_\Psi} \int_0^1 dy \, \frac{d\Gamma}{dy} = \frac{11\alpha^3 |\Psi(0)|^2 a_{Vl} a_{Aq} (e_l e_q)^2 \Gamma_Z}{5 M_Z^3 \Gamma_\Psi}
$$
$$
= 1.617 \times 10^{-11}. \tag{3.10}
$$

4. Conclusions

We have calculated the photon spectrum and angular distribution in the standard model decay of the J/Ψ into photon plus lepton pair including the effects of an intermediate Z boson. In addition we have plotted the T-odd asymmetry distribution of the normal to the final state lepton pair relative to the beam direction. A small but non-zero effect is found proportional to the Z boson width. The effect is proportional to the vector coupling of the Z to leptons which is suppressed by the closeness of $\sin^2(\theta_W)$ to $1/4$. This sug-

gests that a significantly larger result might be found in photon plus hadronic jets. Some discussion of such effects in exclusive channels has already appeared [4]. In the photon plus leptons channel, the current calculation provides a good bound on standard model backgrounds to a search for genuine T-violation in J/Ψ decay due, for example, to a possible electric dipole moment of the charm quark for which no experimental bounds have as yet been published.

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