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Asymptotic safety of gravity and the Higgs boson mass

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ABSTRACT

There are indications that gravity is asymptotically safe. The Standard Model (SM) plus gravity could be valid up to arbitrarily high energies. Supposing that this is indeed the case and assuming that there are no intermediate energy scales between the Fermi and Planck scales we address the question of whether the mass of the Higgs boson m_H can be predicted. For a positive gravity induced anomalous dimension $A_\lambda > 0$ the running of the quartic scalar self interaction λ at scales beyond the Planck mass is determined by a fixed point at zero. This results in $m_H = m_{\min} = 126$ GeV, with only a few GeV uncertainty. This prediction is independent of the details of the short distance running and holds for a wide class of extensions of the SM as well. For $A_\lambda < 0$ one finds m_H in the interval $m_{\min} < m_H < m_{\max} \simeq 174$ GeV, now sensitive to A_λ and other properties of the short distance running. The case $A_\lambda > 0$ is favored by explicit computations existing in the literature.

Though gravity is non-renormalizable by perturbative methods, it may exist as a field theory non-perturbatively [1], exhibiting a non-trivial ultraviolet fixed point (FP) of the functional renormalization group flow [2–4]. In [5] such a fixed point was indeed found in the so-called Einstein–Hilbert truncation. Many works (for a recent review see [6]), based on the exact functional renormalization group equation (FRGE) of [4] (for a review see [7]), produced further evidence in favor of this conjecture. The non-perturbative FP of [5] stays in place when higher order operators are added to Einstein–Hilbert action, when the form of the infrared cutoff is changed, etc. A similar picture arises in lattice formulations of quantum gravity [8] (for a recent review see [9]). Yet another indication comes from perturbative computations [10].

The "flowing action" or "effective average action" Γ_k includes all quantum fluctuations with momenta larger than an infrared cutoff scale. For $k \to \infty$ no fluctuations are included and $\Gamma_{k\to\infty}$ coincides with the classical or microscopic action, while for $k \to 0$ the flowing action includes all quantum fluctuations and becomes the generating functional of the one-particle irreducible Green's functions. The scale dependence of Γ_k obeys an exact functional renormalization group equation [4]. It is of a simple one loop type, but nevertheless can be solved only approximately by suitable non-perturbative truncations of its most general functional form.

From the studies of the functional renormalization group for Γ_k one infers a characteristic scale dependence of the gravitational constant or Planck mass,

$$M_P^2(k) = M_P^2 + 2\xi_0 k^2, \tag{1}$$

where $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV is the low energy Planck mass, and ξ_0 is a pure number, the exact value of which is not essential for our considerations. From investigations of simple truncations of pure gravity one finds $\xi_0 \approx 0.024$ from a numerical solution of FRGE [5,11,12]. For scattering with large momentum transfer q the effective infrared cutoff k^2 is replaced by q^2. Thus for $q^2 \gg M_p^2$ the effective gravitational constant $G_N(q^2)$ scales as $\frac{1}{16\pi\xi_0 q^2}$, ensuring the regular behavior of high energy scattering amplitudes.

We can distinguish two qualitatively different regimes, separated by a transition scale

$$k_{tr} = \frac{M_P}{\sqrt{2\xi_0}} \approx 10^{19}\ \text{GeV}. \tag{2}$$

For the "high energy regime" $k \gtrsim k_{tr}$ we observe scaling behavior $M_P^2(k)/k^2 \approx 2\xi_0$, characteristic for an ultraviolet fixed point. In contrast, for the "low energy regime" the effects of graviton loops are effectively switched off and $M_P^2(k)$ becomes a scale independent constant. Eq. (1) describes the typical behavior for a "relevant parameter" characterizing a deviation from an exact fixed point. We observe that the high energy regime is essentially determined by canonical dimensional scaling in absence of mass scales. We expect this form to hold for a wide class of theories with an ultraviolet fixed point, where the high energy regime may involve additional fields or even higher dimensions. The numerical value of ξ_0 will then depend on the precise model. (In the presence of an

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anomalous dimension for the graviton Eq. (1) holds for an appropriate renormalized coupling $M_p^2(k)$.) In the "low energy regime" $k \lesssim k_{tr}$ the running of the gravitational couplings is essentially stopped.

For $k \gtrsim k_{tr}$ the running of the dimensionless couplings of the Standard Model is modified by gravitational contributions. We may denote these couplings by x_j for the gauge couplings g_1, g_2, g_3 of $U(1)$, $SU(2)$, $SU(3)$, h for the top Yukawa coupling and λ for the self interaction of the Higgs scalar. The gravitational contribution to the beta-functions β_j^{grav} takes typically the form[1]

$$\beta_j^{grav} = \frac{a_j}{8\pi}\frac{k^2}{M_p^2(k)}x_j. \tag{3}$$

For the high energy regime this amounts to effective anomalous dimensions

$$A_j = \frac{a_j}{16\pi\xi_0}. \tag{4}$$

For small x_j Eq. (3) describes the leading contribution, such that

$$x_j(k) \sim k^{A_j}. \tag{5}$$

The general form (3) is again dictated by simple scaling arguments. Explicit computations confirm these expectations [11,12,14–19]. In general, the constants a_j will depend on the precise model which describes the high energy regime. We emphasize that for $a_j < 0$ the running of x_j is asymptotically free, at least for small enough values of the coupling. For the low energy regime $k^2 \lesssim k_{tr}^2$ the gravitational contributions become negligible.

Within this setting a very economical description of all interactions in Nature may be possible. One can assume that there is no new physics associated with any intermediate energy scale (such as Grand Unified scale or low energy supersymmetry) between the weak scale and k_{tr}. All confirmed observational signals in favor of physics beyond the Standard Model as neutrino masses and oscillations, dark matter and dark energy, baryon asymmetry of the Universe and inflation can be associated with new physics *below* the electroweak scale, for reviews see [20,21] and references therein. The minimal model—νMSM, contains, in addition to the SM particles, 3 relatively light singlet Majorana fermions and the dilaton. These fermions could be responsible for neutrino masses, dark matter and baryon asymmetry of the Universe. The dilaton may lead to dynamical dark energy [22,23] and realizes spontaneously broken scale invariance which either emerges from the cosmological approach to a fixed point [22,24] or is an exact quantum symmetry [25,26]. Inflation can take place either due to the SM Higgs [27] or due to the asymptotically safe character of gravity [28]. Yet another part of new physics, related, for example, to the strong CP problem or to the flavor problem, may be associated with the Planck energy. In this Letter we show that this scenario leads to a prediction of the Higgs mass, which can be tested at the LHC.

A convenient language for understanding the origin of this prediction is the concept of infrared intervals [29]. Consider first the low energy regime where graviton loops can be neglected and the x_j follow the perturbative renormalization group equations of the SM, $k\partial x_j/\partial_k = \beta_j^{\mathrm{SM}}$, with one loop expressions

$$\beta_1^{\mathrm{SM}} = \frac{41}{96\pi^2}g_1^3, \qquad \beta_2^{\mathrm{SM}} = -\frac{19}{96\pi^2}g_2^3,$$

$$\beta_3^{\mathrm{SM}} = -\frac{7}{16\pi^2}g_3^3, \tag{6}$$

$$\beta_h^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[\frac{9}{2}h^3 - 8g_3^2h - \frac{9}{4}g_2^2h - \frac{17}{12}g_1^2h\right], \tag{7}$$

$$\beta_\lambda^{\mathrm{SM}} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda h^2 - 9\lambda\left(g_2^2 + \frac{1}{3}g_1^2\right) - 6h^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2g_1^2\right]. \tag{8}$$

The ratio λ/h^2 has a partial infrared fixed point [29,30] (up to small modifications due to the gauge couplings). Since the running within the low energy regime extends only over a finite range between k_{tr} and the Fermi scale k_F, this fixed point needs not to be approached arbitrarily close. Instead, arbitrary couplings at the scale k_{tr} in the allowed range $0 \leqslant h^2(k_{tr}) < \infty, 0 \leqslant \lambda(k_{tr}) \leqslant \infty$ are mapped by the RG-flow to an infrared interval of allowed couplings at the Fermi scale k_F. For known top quark mass or fixed $h(k_F)$ the infrared interval for $\lambda(k_F)/h^2(k_F)$, centered around the partial fixed point, determines the allowed values of the mass of the Higgs doublet. The upper limit $\lambda_{\max}(k_F)$ corresponds to the "triviality bound". Numerically, it coincides essentially with the requirement that for $k < k_{tr}$ the SM-coupling should remain within the perturbative range [31,32], but its validity extends beyond perturbation theory [33]. The lower limit $\lambda_{\min}(k_F)$ arises from the observation that even for $\lambda(k_{tr}) = 0$ a nonzero $\lambda(k_F)$ is generated due to the term $\sim h^4$ in β_λ. An extended range of large $\lambda(k_{tr})$ is mapped to $\lambda(k_F)$ close to $\lambda_{\max}(k_F)$, while a large range of small $\lambda(k_{tr})$ is mapped to values of $\lambda(k_F)$ close to the lower bound $\lambda_{\min}(k_F)$ [29]. This observation will be crucial for our prediction.

We next discuss the running in the high energy regime. The allowed values of $x_j(k_{tr})$ correspond now to the infrared interval for the first stage of the running. Since we want this running to hold for arbitrarily large k, the infrared intervals are completely determined by the possible fixed points. If some coupling or ratio of couplings has only one infrared stable fixed point, the value at k_{tr} must be given by the fixed point value and becomes predictable. In case of an ultraviolet fixed point only, the value of the coupling at the transition scale k_{tr} remains undetermined, since arbitrary values of $x_j(k_{tr})$ run to the ultraviolet fixed point for $k \to \infty$. Finally, we consider the case where a coupling or combination of couplings x has an infrared stable fixed point at x_{IR} and a second ultraviolet stable fixed point at x_{UV}. For $x_{IR} < x_{UV}$ the infrared interval for x is given by $x(k_{tr}) \geqslant x_{IR}$, while for $x_{IR} > x_{UV}$ one finds $x(k_{tr}) \leqslant x_{IR}$.

The most interesting situation for a prediction of the mass of the Higgs scalar arises if h^2 has an ultraviolet fixed point $h_{UV}^2 = 0$, while λ has an infrared fixed point $\lambda_{IR} = 0$ in the limit where h and g_i vanish for $k \gg k_{tr}$. This setting is realized for $a_h < 0$, $a_\lambda > 0$. In this case $\lambda(k_{tr})$ is predicted very close to zero, such that $\lambda(k_F)$ will be very close to the lower bound $\lambda_{\min}(k_F)$. This results in a Higgs-scalar mass $m_H \approx 126$ GeV, see below. We emphasize that only inequalities for a a_h and a_λ are needed for this prediction, while the precise value of these constants does not matter. It is also essential that the sign of the gravity contribution to the running of all gauge couplings is negative, $a_i < 0$.

To substantiate the general discussion given above, consider the pure SM coupled to gravity, with running couplings given by

$$k\frac{dx_j}{dk} = \beta_j^{\mathrm{SM}} + \beta_j^{grav}. \tag{9}$$

As for the gauge couplings, we will fix their values at small energies to the experimental ones, but will leave λ and h undetermined for the time being.

[1] We would like to stress that the definition of the running couplings here is based on the gauge-invariant high energy physical scattering amplitudes [1], rather than on the minimal subtraction (MS) scheme of the dimensional regularization. In the MS scheme perturbative Einstein gravity does not contribute to the β functions of the Standard Model couplings [13].

First, let us look at the gauge sector. Assume for simplicity that $a_1 = a_2 = a_3 = a_g$, which is true for one-loop computations, performed up to now, due to the universality of the gravitational interactions. For $a_g < 0$ all gauge couplings are asymptotically free. Indeed, the computations of [15,14] yield a *negative* sign for a_g, with $|a_g| \sim 1$. In this case the gauge coupling constants g_2 and g_3 cannot be predicted. The gauge coupling g_1 has two fixed points

$$g^2_{1,UV} = 0, \qquad g^2_{1,IR} = \frac{6\pi |a_g|}{41\xi_0}, \tag{10}$$

such that $g_1^2(k_{tr}) \leqslant g^2_{1,IR}$. For a realistic gauge coupling one needs $g_1(k_{tr}) \approx 0.5$, and this requires $a_g < a_g^{\rm crit} \approx -0.013$. Then the Landau pole problem for the $U(1)$-coupling is solved due to the presence of the fixed point. We will assume in the following $a_g < a_g^{\rm crit}$ and take for definiteness the value $|a_g| \sim 1$. For large enough $k \gg k_{tr}$ the gauge couplings can be neglected for the running of h and λ.

Consider now the top Yukawa coupling h. For a positive anomalous dimension $a_h > 0$ one finds only an *IR*-stable fixed point at $h^2_{IR} = 0$. This would predict $h^2(k_{tr}) = 0$, and therefore a vanishing top quark mass. Clearly, this case is rejected by experiment. For the interesting case $a_h < 0$ there are two fixed points

$$h^2_{UV} = 0, \qquad h^2_{IR} = \frac{2\pi |a_h|}{9\xi_0}, \tag{11}$$

implying $h(k_{tr}) \leqslant h_{\max}(k_{tr}) \approx h_{IR}$. (There are small numerical modifications of the second relation due to the presence of the gauge couplings.) We may compute numerically the value of $h(k_{tr})$ which corresponds to the (central) experimental value of the top quark mass, $m_t = 171.3$ GeV [34]. Since this has to be smaller than $h_{\max}(k_{tr})$ a realistic setting requires $a_h < a_h^{\rm crit} \approx -0.005$. An interesting scenario would arise if $h(k)$ gets close to the fixed point value h_{IR} for $k \gg k_{tr}$. In this case the top quark mass becomes predictable, and a realistic value requires $a_h = a_h^{\rm crit}, h_{IR} \approx 0.38$.

At present, the value of a_h is not known reliably. For example, in [35] it was shown that gravity contributions make the Yukawa coupling asymptotically free in quantum R^2 gravity with matter. Ref. [19] studied the gravitational running of Yukawa couplings in the FRGE approach for the Einstein–Hilbert type of truncation and found different signs for a_h in different gauges. In this work the wave function renormalization for the fermions and scalars was not included and the sensitivity to the truncation type was not investigated. In what follows we will simply assume that $a_h < a_h^{\rm crit}$ which is the only realistic case for observations.

Let us turn now to the behavior of the scalar self-coupling λ. The gravitational corrections can only promote the SM to the rank of fundamental theory if the running of λ does not lead to any pathologies up to the Planck scale. In other words, the Landau pole must be absent for $k \lesssim k_{tr}$ [31,32,36], and λ must be positive for all momenta up to k_{tr} [37–39]. There is a large parameter space available on the plane m_H, m_t, where both conditions are satisfied. Close to the experimental value of the top mass, it is described by the infrared interval for $\lambda(k_F)$, corresponding to $m_{\min} < m_H < m_{\max}$. Here

$$m_{\min} = \left[126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5\right] \text{GeV}, \tag{12}$$

and

$$m_{\max} = \left[173.5 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.118}{0.002} \times 0.1\right] \text{GeV}, \tag{13}$$

where α_s is the strong coupling at the Z-mass, with theoretical uncertainty in $m_{\min}$ equal to ± 2.2 GeV. These numbers are taken from the recent two-loop analysis of [40] (see also [42,41] and earlier computations in [43–46]). The value of $m_{\max}$ corresponds to the (somewhat arbitrary) criterion $\lambda(k_{tr}) < 6$, but changes only very little for arbitrarily large $\lambda(k_{tr})$. The admitted region contains also very small top and Higgs masses, excluded experimentally.

As we have already said, a specific prediction of the Higgs boson mass can be given if a_λ is positive. In fact, the evidence that this is indeed the case for the SM coupled to gravity comes from computations of [11,12], giving

$$a_\lambda \approx 3.1, \qquad A_\lambda \simeq 2.6. \tag{14}$$

A contribution with the same sign and similar magnitude was found previously in [47].

Let us elucidate the structure of the solution to the RG equation for λ in this case. For $a_h < a_h^{\rm crit}$, $a_g < a_g^{\rm crit}$, asymptotic freedom of the gauge and Yukawa couplings implies that they can be neglected for $k \gg k_{tr}$. (We assume here for simplicity negative values of a_h, a_g of the order one, such that this regime is reached for scales only moderately above k_{tr}.) The remaining terms in β_λ drive then λ quickly to an approximate infrared fixed point $\lambda_{IR} = 0$. This is the only fixed point such that $\lambda(k_{tr})$ becomes predictable. The actual value $\lambda(k_{tr})$ differs from zero only due to the presence of nonzero gauge and Yukawa couplings. For large enough h the term $\sim -h^4$ in β_λ dominates over the terms $\sim g^4$, such that λ is driven to a small positive value. For realistic values of $h(k_{tr})$ and $g_i(k_{tr})$ the effects driving λ away from zero are small, however, and they act only in a small region $k \gtrsim k_{tr}$ before $h(k)$ and $g_i(k)$ drop to very small value for larger k. As a result, $\lambda(k_{tr})$ remains very small, such that $\lambda(k_F)$ is predicted very close to the lower bound of the infrared interval. This yields a robust prediction $m_H = m_{\min}$, independently of the precise values of the constants a_j!

Indeed, for this scenario the trajectory of $\lambda(k)$ is given by the special solution

$$\lambda(k) = -\int_k^\infty \frac{dk'}{k'} \left(\frac{1 + 2\xi_0 k^2/M_P^2}{1 + 2\xi_0 k'^2/M_P^2}\right)^{\frac{a_\lambda}{32\pi\xi_0}} \times \beta_\lambda^{\rm SM}\big(x_j(k')\big). \tag{15}$$

For $a_\lambda > 0$ only a small range of k' in the vicinity of k contributes to the integral in Eq. (15). We infer

$$\lambda(k_{tr}) = -C\beta_\lambda^{\rm SM}\big(h(k_{tr}), g_i(k_{tr})\big), \tag{16}$$

where C is positive and of order one. Since $\lambda(k_{tr})$ is small we can omit the terms involving λ in $\beta_\lambda^{\rm SM}$, such that $\lambda(k_{tr})$ is fixed in terms of $h(k_{tr})$ and $g_i(k_{tr})$.

While the effects of the terms $\sim h^4$, g^4 in β_λ are numerically small, they impose nevertheless an important lower bound for the allowed values of the top quark mass. For too small values of h the positive contributions $\sim g^4$ will dominate over the negative values $\sim -h^4$ in β_λ. Since the terms $\sim \lambda a_\lambda$ and $\sim \lambda^2$ drive $\lambda(k)$ quickly towards zero as k is lowered, a remaining positive β_λ for $\lambda = 0$ would induce $\lambda(k)$ to run to negative values. Such a behavior can be associated with radiatively induced spontaneous symmetry breaking of the Coleman–Weinberg [48] type, at a scale close to k_{tr} or above. A realistic scenario of electroweak symmetry breaking, supplemented by cosmological considerations such as Higgs-inflation [27] or asymptotically safe inflation [28], has to exclude this case, therefore requiring that $\lambda(k)$ remains positive for all values of k. From numerical solutions of the RG-equations we infer a lower bound for the top quark mass

$$m_t \geqslant m_t^{\min}, \tag{17}$$

where $m_t^{\min} \simeq 170$ GeV, slightly depending on the values of anomalous dimensions (for example, $a_g = a_h = -1, a_\lambda = 3$ gives

$m_t^{\min} \simeq 173.4$ GeV, whereas $a_g = -1, a_h = -0.5, a_\lambda = 3$ leads to $m_t^{\min} \simeq 169.3$ GeV). It is interesting that the experimental value $m_t = 171.3$ GeV is very close to this lower limit. This implies that both λ and β_λ are close to zero at the transition scale k_{tr},

$$\lambda(k_{tr}) \approx 0, \qquad \beta_\lambda(k_{tr}) \approx 0. \tag{18}$$

This suggests that the fundamental theory may be characterized by a fixed point at $\lambda = 0$ also for nonzero h and g, thereby predicting m_t to be close to 170 GeV. Furthermore, the requirement that the Yukawa contribution to β_λ continues to dominate over the gauge boson contributions for very large k imposes a constraint

$$a_g \leqslant a_h \leqslant a_h^{\text{crit}}. \tag{19}$$

If this condition holds, we find a range of k (larger than k_{tr}) for which the running of λ can be approximated by the simplified equation

$$\beta_\lambda = \frac{a_\lambda}{16\pi\xi_0}\lambda + \frac{1}{16\pi^2}\left(24\lambda^2 + 12\lambda h^2 - 6h^4\right). \tag{20}$$

For a fixed point behavior of the Yukawa coupling $a_h = a_h^{\text{crit}}, h = h_* = h_{IR}$ (11) this yields a fixed point for λ obeying

$$24\lambda_*^2 + 12\lambda h_*^2 - 6h_*^4 + \frac{\pi a_\lambda \lambda_*}{\xi_0} = 0. \tag{21}$$

These findings can be verified by an explicit numerical solution of the RG-equations (9). For better accuracy, in the numerical computations we used for β_j^{SM} the two-loop RG equations and one-loop pole matching of the physical parameters, see [49,42] and also [40]. We run the normalization flow towards the ultraviolet by increasing k, starting at the Fermi scale k_F with the gauge couplings as inferred from experiment and with a given fixed m_t. For $a_g < 0, a_g \leqslant a_h \leqslant a_h^{\text{crit}}, a_\lambda > 0$ we find that indeed only a single value of $\lambda(k_F)$ can be extrapolated to arbitrarily large k, while larger values diverge and smaller values turn negative. This corresponds to the prediction that the infrared interval consists only of one point, corresponding to the approximate fixed point $\lambda_{IR} = 0$ for sufficiently large k where h^2 and g^2 can be neglected, or to the value λ_* in Eq. (21) if $a_h = a_h^{\text{crit}}$. This solution exists only provided the bound for m_t (17) is obeyed. For example, for $a_g = -1, a_h = -0.5$, and $a_\lambda = 3$ the admitted RG trajectories exist for a large variety of top masses: $m_t = 171.3$ GeV leads to $m_H \simeq 126.5$ GeV, whereas $m_t = 230$ GeV requires $m_H \simeq 233$ GeV.

Let us now choose the experimental value for the top quark mass and determine the Higgs boson mass which corresponds to the allowed value of $\lambda(k_F)$. As expected, the prediction is quite insensitive to the specific values of a_g, a_h and a_λ and reads

$$m_H = m_{\min}. \tag{22}$$

The value of m_H can only increase if the top Yukawa coupling is close to its non-Gaussian fixed point, h_{IR}, realized for $a_h = a_h^{\text{crit}}$, which leads to the existence of the non-trivial fixed point in λ (21). Taking, as an example, $a_g = -1$, $a_h^{\text{crit}} \simeq -0.005$, one gets from (21) $\lambda_* = 0.043$. This shifts up the prediction of the Higgs mass by not more than 8 GeV. Taking smaller $|a_g|$ decreases this shift.[2]

The prediction (22) can be tested at the LHC.[3] Given the fact that the accuracy in the Higgs mass measurements at the LHC can reach 200 MeV, the reduction of theoretical uncertainty and of experimental errors in the determination of the top quark mass and of the strong coupling constant are highly desirable. As was discussed in [40], the theoretical error can go down from 2.2 GeV to 0.4 GeV if one upgrades the one-loop pole matching at the electroweak scale and two loop running up to the Planck scale to the two-loop matching and 3-loop running. Note that 3-loop beta-functions for the SM are not known by now, and that the two-loop pole matching has never been carried out. Clearly, computations of signs and magnitudes of gravitational anomalous dimensions A_i are needed to remove yet another source of uncertainties.

The prediction $m_H \approx m_{\min}$ does not only hold for the hypothesis that the SM plus gravity describes all the physics relevant for the running of couplings. It generalizes to many extensions of the SM and gravity, including possibly even higher dimensional theories. Of course, the precision of the prediction gets weaker if a much larger class of models is considered. Nevertheless, only two crucial ingredients are necessary for predicting $m_H \approx m_{\min}$: (i) Above a transition scale k_{tr} the running should drive the quartic scalar coupling rapidly to an approximate fixed point at $\lambda = 0$, only perturbed by small contributions to β_λ from Yukawa and gauge couplings. This is generically the case for a large enough anomalous dimension $A_\lambda > 0$. (ii) Around k_{tr} there should be a transition to the SM-running in the low energy regime. This transition may actually involve a certain splitting of scales as "threshold effects", for example by extending the SM to a Grand Unified theory at a scale near k_{tr}. It is sufficient that these threshold effects do not lead to a rapid increase of λ in the threshold region. This will be the case if the λ-independent contributions to β_λ only involve perturbatively small couplings in a threshold region extending over only a few orders of magnitude.

A possible alternative to the above prediction appears if we have a negative anomalous dimension $A_\lambda < 0$. In this case the approximate IR-fixed point λ_{IR} for vanishing h and g is shifted away from the "Gaussian fixed point" $\lambda = 0$. From Eqs. (3), (8) one finds for $h = g = 0$

$$\lambda_{IR} = \frac{\pi|a_\lambda|}{24\xi_0} = \frac{2\pi^2}{3}|A_\lambda|. \tag{23}$$

In this case the IR-interval becomes $0 \leqslant \lambda(k_{tr}) \leqslant \lambda_{IR}$. Again, nonzero h, g will slightly shift the infrared interval. However, the value of $\lambda(k_{tr})$ depends now strongly on the details of the running in the high energy regime, in particular on the value of λ_{IR} (or A_λ). Without a precise knowledge of this running this alternative only predicts m_H to be in the IR-interval $m_{\min} < m_H < m_{\max}$, with $m_{\min}$ and $m_{\max}$ given by Eqs. (12), (13).

Finally, we turn to the running of the mass parameter in the Higgs-potential $\mu^2(k)$. So far, we have implicitly assumed that the Fermi scale is fixed to its experimental value. For $g_i = 0, h = 0$, a vanishing Fermi scale corresponds to a second order phase transition between a phase with spontaneous electroweak symmetry breaking and a phase with unbroken (global) $SU(2) \times U(1)$ symmetry. If we choose $\mu^2(k_{tr})$ to correspond precisely to the phase transition the Fermi scale will vanish. A second order phase transition corresponds to an exact fixed point, for which an effective dilatation symmetry of the low energy theory becomes realized [29,54]. (The scale transformations of these "low energy dilata-

[2] The values of the Higgs mass we found are consistent with a possibility of inflation due to the SM Higgs boson [27]. The Higgs-inflation requires the consistency of the SM up to the lower, than M_P energy scale $k \sim \frac{M_P}{\xi}$, where $\xi = 700 - 10^5$ is the value of the non-minimal coupling of the Higgs field to the curvature Ricci scalar [50,40] (see also [51,52]), the smaller ξ correspond to smaller Higgs masses.

[3] The fact that the SM scalar self-coupling is equal to zero *together* with its β-function at the Planck scale for the particular values of the top-quark and Higgs masses was first (to the best of our knowledge) noticed in [53]. These authors put forward the hypothesis of a "multiple point principle", stating that the effective potential for the Higgs field must have two minima, the one corresponding to our vacuum, whereas another one must occur at the Planck scale. Our reasoning is completely different. Though the sense of the "multiple point principle" remains unclear to us, we would like to note that the prediction of the Higgs mass from it coincides with ours (the specific numbers in [53] are different, as they were based on one-loop computation).

tions" keep the Planck mass fixed and are therefore different from a possible fundamental dilatation symmetry.) At the phase transition, the running of μ^2 is given by a critical trajectory $\mu_*^2(k)$.

Deviations from the critical trajectory, $\delta\mu^2(k) = \mu^2(k) - \mu_*^2(k)$, behave as a relevant parameter for the low energy running. The running of $\delta\mu^2$ is governed by an anomalous dimension

$$k\frac{\partial}{\partial k}\delta\mu^2 = A_\mu \delta\mu^2, \tag{24}$$

such that a small $\delta\mu^2$ remains small during the flow. The small parameter $\delta\mu^2$ is natural in a technical sense—it is associated to a small deviation from an (almost) exact symmetry, i.e. the low energy dilatations [54]. An appropriate renormalization group improved perturbation theory requires no fine tuning order by order [54], the anomalous dimension A_μ can be computed in a perturbative series in the couplings.

An important question concerns the allowed "*IR*-interval" for $\delta\mu^2(k_{tr})$. This will depend on the size and sign of A_μ in the high energy regime. For a large positive A_μ one infers that $\delta\mu^2(k_{tr})$ should be very close to zero. In particular, for $A_\mu > 2$ the dimensionless ratio $\delta\mu^2/k^2$ is attracted to zero, corresponding to "self organized criticality" [30,55]. This could help to understand the small ratio between the Fermi and Planck scales. From presently published results [11,12] for the scalar theory coupled to gravity one infers $A_\mu = 1.83$; what happens in the full SM is unknown.

In conclusion, we discussed the possibility that the SM, supplemented by the asymptotically safe gravity plays the role of a fundamental, rather than effective field theory. We found that this may be the case if the gravity contributions to the running of the Yukawa and Higgs coupling have appropriate signs. The mass of the Higgs scalar is predicted $m_H = m_{\min} \simeq 126$ GeV with a few GeV uncertainty if all the couplings of the Standard Model, with the exception of the Higgs self-interaction λ, are asymptotically free, while λ is strongly attracted to an approximate fixed point $\lambda = 0$ (in the limit of vanishing Yukawa and gauge couplings) by the flow in the high energy regime. This can be achieved by a positive gravity induced anomalous dimension for the running of λ. A similar prediction remains valid for extensions of the SM as grand unified theories, provided the split between the unification and Planck-scales remains moderate and all relevant couplings are perturbatively small in the transition region. Detecting the Higgs scalar with mass around 126 GeV at the LHC could give a strong hit for the absence of new physics influencing the running of the SM couplings between the Fermi and Planck/unification scales.

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References

[1] S. Weinberg, in: S.W. Hawking, W. Israel (Eds.), General Relativity: An Einstein Centenary Survey, Cambridge University Press, 1979, p. 790, chapter 16.
[2] K.G. Wilson, J.B. Kogut, Phys. Rep. 12 (1974) 75.
[3] J. Polchinski, Nucl. Phys. B 231 (1984) 269.
[4] C. Wetterich, Phys. Lett. B 301 (1993) 90.
[5] M. Reuter, Phys. Rev. D 57 (1998) 971.
[6] A. Codello, R. Percacci, C. Rahmede, Ann. Phys. 324 (2009) 414.
[7] J. Berges, N. Tetradis, C. Wetterich, Phys. Rep. 363 (2002) 223.
[8] J. Ambjorn, J. Jurkiewicz, R. Loll, Phys. Rev. Lett. 93 (2004) 131301.
[9] J. Ambjorn, J. Jurkiewicz, R. Loll, arXiv:0906.3947 [gr-qc].
[10] M.R. Niedermaier, Phys. Rev. Lett. 103 (2009) 101303.
[11] R. Percacci, D. Perini, Phys. Rev. D 68 (2003) 044018.
[12] G. Narain, R. Percacci, arXiv:0911.0386 [hep-th].
[13] D. Ebert, J. Plefka, A. Rodigast, Phys. Lett. B 660 (2008) 579.
[14] J.E. Daum, U. Harst, M. Reuter, arXiv:0910.4938 [hep-th].
[15] S.P. Robinson, F. Wilczek, Phys. Rev. Lett. 96 (2006) 231601.
[16] A.R. Pietrykowski, Phys. Rev. Lett. 98 (2007) 061801.
[17] D.J. Toms, Phys. Rev. D 76 (2007) 045015.
[18] D.J. Toms, Phys. Rev. Lett. 101 (2008) 131301.
[19] O. Zanusso, L. Zambelli, G.P. Vacca, R. Percacci, arXiv:0904.0938 [hep-th].
[20] M. Shaposhnikov, arXiv:0708.3550 [hep-th].
[21] A. Boyarsky, O. Ruchayskiy, M. Shaposhnikov, Annu. Rev. Nucl. Part. Sci. 59 (2009) 191.
[22] C. Wetterich, Nucl. Phys. B 302 (1988) 668.
[23] C. Wetterich, Nucl. Phys. B 302 (1988) 645.
[24] C. Wetterich, Phys. Rev. Lett. 90 (2003) 231302;
C. Wetterich, Phys. Rev. Lett. 102 (2009) 141303;
arXiv:0911.1063 [hep-th].
[25] M. Shaposhnikov, D. Zenhausern, Phys. Lett. B 671 (2009) 187.
[26] M. Shaposhnikov, D. Zenhausern, Phys. Lett. B 671 (2009) 162.
[27] F.L. Bezrukov, M. Shaposhnikov, Phys. Lett. B 659 (2008) 703.
[28] S. Weinberg, arXiv:0911.3165 [hep-th].
[29] C. Wetterich, The mass of the Higgs particle, in: G. Furlan, et al. (Eds.), Superstrings, Unified Theories, Cosmology 1987, World Scientific, 1988, p. 403.
[30] C. Wetterich, Phys. Lett. B 104 (1981) 269.
[31] L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B 136 (1978) 115.
[32] N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. B 158 (1979) 295.
[33] M. Lüscher, P. Weisz, Nucl. Phys. B 290 (FS20) (1987) 25;
T. Papenbrock, C. Wetterich, Z. Phys. C 65 (1995) 519.
[34] C. Amsler, et al., Particle Data Group, Phys. Lett. B 667 (2008) 1.
[35] I.L. Shapiro, Class. Quantum. Grav. 6 (1989) 1197.
[36] M. Lindner, Z. Phys. C 31 (1986) 295.
[37] N.V. Krasnikov, Yad. Fiz. 28 (1978) 549.
[38] P.Q. Hung, Phys. Rev. Lett. 42 (1979) 873.
[39] H.D. Politzer, S. Wolfram, Phys. Lett. B 82 (1979) 242;
H.D. Politzer, S. Wolfram, Phys. Lett. B 83 (1979) 421, Erratum.
[40] F. Bezrukov, M. Shaposhnikov, JHEP 0907 (2009) 089.
[41] J. Ellis, J.R. Espinosa, G.F. Giudice, A. Hoecker, A. Riotto, Phys. Lett. B 679 (2009) 369.
[42] J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002.
[43] G. Altarelli, G. Isidori, Phys. Lett. B 337 (1994) 141.
[44] J.A. Casas, J.R. Espinosa, M. Quiros, Phys. Lett. B 342 (1995) 171.
[45] J.A. Casas, J.R. Espinosa, M. Quiros, Phys. Lett. B 382 (1996) 374.
[46] T. Hambye, K. Riesselmann, Phys. Rev. D 55 (1997) 7255.
[47] L. Griguolo, R. Percacci, Phys. Rev. D 52 (1995) 5787.
[48] S. Coleman, E. Weinberg, Phys. Rev. D 7 (1973) 1888.
[49] M.X. Luo, Y. Xiao, Phys. Rev. Lett. 90 (2003) 011601.
[50] F.L. Bezrukov, A. Magnin, M. Shaposhnikov, Phys. Lett. B 675 (2009) 88.
[51] A. De Simone, M.P. Hertzberg, F. Wilczek, Phys. Lett. B 678 (2009) 1.
[52] A.O. Barvinsky, A.Y. Kamenshchik, C. Kiefer, A.A. Starobinsky, C. Steinwachs, arXiv:0904.1698 [hep-ph].
[53] C.D. Froggatt, H.B. Nielsen, Phys. Lett. B 368 (1996) 96.
[54] C. Wetterich, Phys. Lett. B 140 (1984) 215.
[55] S. Bornholdt, C. Wetterich, Phys. Lett. B 282 (1992) 399.