

Shear viscosity to entropy density ratio of QCD below the deconfinement temperature

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Abstract

Using chiral perturbation theory we investigate the QCD shear viscosity (η) to entropy density (s) ratio below the deconfinement temperature (~ 170 MeV) with zero baryon number density. It is found that η/s of QCD is monotonically decreasing in temperature (T) and reaches 0.6 with estimated $\sim 50\%$ uncertainty at $T = 120$ MeV. A naive extrapolation of the leading order result shows that η/s reaches the $1/4\pi$ minimum bound proposed by Kovtun, Son, and Starinets using string theory methods at $T \sim 200$ MeV. This suggests a phase transition or cross over might occur at $T \lesssim 200$ MeV in order for the bound to remain valid. Also, it is natural for η/s to stay close to the minimum bound around the phase transition temperature as was recently found in heavy ion collisions.

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1. Introduction

Shear viscosity η characterizes how strongly particles interact and move collectively in a many body system. In general, strongly interacting systems have smaller η than the weakly interacting ones. This is because η is proportional to $\epsilon\tau_{\text{mft}}$, where ϵ is the energy density and τ_{mft} is the mean free time, which is inversely proportional to particle scattering cross section. Recently a universal minimum bound for the ratio of η to entropy density s was proposed by Kovtun, Son, and Starinets [1]. The bound,

$$\frac{\eta}{s} \geq \frac{1}{4\pi}, \quad (1)$$

is found to be saturated for a large class of strongly interacting quantum field theories whose dual descriptions in string theory involve black holes in anti-de Sitter space [2–5].

Recently, η/s close to the minimum bound were found in relativistic heavy ion collisions (RHIC) [6–8]. This discovery came as a surprise. Traditionally, quark gluon plasma (QGP)—the phase of QCD above the deconfinement temperature T_c

(~ 170 MeV at zero baryon density [9])—was thought to be weakly interacting. Partly because lattice QCD simulations of the QGP equation of state above $2T_c$ were not inconsistent with that of an ideal gas of massless particles, $e = 3p$, where e is the energy density and p is the pressure of the system [9]. However, recent analyses of the elliptic flow generated by non-central collisions in RHIC [7,8] and lattice simulations of a gluon plasma [10] yielded η/s close to the minimum bound at just above T_c . This suggests QGP is strongly interacting at this temperature.¹ (However, see Ref. [14] for a different interpretation.)

Given this situation, one naturally wonders if η/s of QCD was already close to the minimum bound at just above T_c , what would happen if we keep reducing the temperature such that the coupling constant of QCD gets even stronger? Will the η/s minimum bound hold up below T_c ? If the bound does hold up, what is the mechanism? Is the change of degrees of freedom through a phase transition or cross over sufficient to save the bound? If the bound does not hold up, what is the implication to string theory?

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¹ See also [11–13]. For discussions of the possible microscopic structure of such a state, see [15–19].

To explore these issues, we use chiral perturbation theory (χ PT) and the linearized Boltzmann equation to perform a model independent calculation to the η/s of QCD in the confinement phase. Earlier attempts to compute meson matter viscosity using the Boltzmann equation and phenomenological phase shifts in the context of RHIC hydrodynamical evolution after freeze out can be found in Refs. [24–26]. In the deconfinement phase, state of the art perturbative QCD calculations of η can be found in Refs. [27,28].

2. Linearized Boltzmann equation for low energy QCD

In the hadronic phase of QCD with zero baryon-number density, the dominant degrees of freedom are the lightest hadrons—the pions. The pion mass $m_\pi = 139$ MeV is much lighter than the mass of the next lightest hadron—the kaon whose mass is 495 MeV. Given that T_c is only ~ 170 MeV, it is sufficient to just consider the pions in the calculation of thermodynamical quantities and transport coefficients for $T \ll T_c$.

The interaction between pions can be described by chiral perturbation theory (χ PT) in a systematic expansion in energy and quark (u and d quark) masses [20–22]. χ PT is a low energy effective field theory of QCD. It describes pions as Nambu–Goldstone bosons of the spontaneously broken chiral symmetry. At $T \ll T_c$, the temperature dependence in $\pi\pi$ scattering can be calculated systematically. At $T = T_c$, however, the theory breaks down due to the restoration of chiral symmetry.²

The shear viscosity η of the pion gas can be calculated either using the Boltzmann equation or the Kubo formula. Since the Boltzmann equation requires semi-classical descriptions of particles with definite position, energy and momentum except during brief collisions, the mean free path is required to be much greater than the range of interaction. Thus the Boltzmann equation is usually limited to low temperature systems. The Kubo formula does not have this restriction. In this approach η can be calculated through the linearized response function

$$\eta = -\frac{1}{5} \int_{-\infty}^0 dt' \int_{-\infty}^{t'} dt \int dx^3 \langle [T^{ij}(0), T^{ij}(\mathbf{x}, t)] \rangle \quad (2)$$

with T^{ij} the spacial part of the off-diagonal energy momentum tensor. One might think a perturbative calculation of the above two point function will give the answer for η . But this cannot be true if $\eta \propto \tau_{\text{mft}}$, as mentioned above, for $\tau_{\text{mft}} \rightarrow \infty$ in the free case. Indeed, the Kubo formula involves an infinite number of diagrams at the leading order (LO) [23]. However, in a weak coupling ϕ^4 theory, it is proven that the summation of LO diagrams is equivalent to solving the linearized Boltzmann equation with temperature dependent particle masses and scattering amplitudes [23]. This proof extended the applicable range of the Boltzmann equation to higher temperature but is restricted to weak coupling theories. In the case we are inter-

ested (QCD with $T \lesssim 120$ MeV), the range of interaction is set by the pion mass which is ~ 1 fm. The mean free path is ~ 410 fm at $T = 60$ MeV and ~ 11 fm at $T = 120$ MeV. Thus, we argue that below $T = 120$ MeV, the interaction is still sufficiently weak that the use of the Boltzmann equation is justified.

The Boltzmann equation describes the evolution of the isospin averaged pion distribution function $f = f(\mathbf{x}, \mathbf{p}, t) \equiv f_p(x)$ (a function of space, time and momentum) as

$$\frac{p^\mu}{E_p} \partial_\mu f_p(x) = \frac{g_\pi}{2} \int_{123} d\Gamma_{12;3p} \{ f_1 f_2 (1 + f_3) (1 + f_p) - (1 + f_1) (1 + f_2) f_3 f_p \}, \quad (3)$$

where $E_p = \sqrt{\mathbf{p}^2 + m_\pi^2}$ and $g_\pi = 3$ is the degeneracy factor for three pions,

$$d\Gamma_{12;3p} \equiv \frac{1}{2E_p} |\mathcal{T}|^2 \prod_{i=1}^3 \frac{d^3 \mathbf{k}_i}{(2\pi)^3 (2E_i)} \times (2\pi)^4 \delta^4(k_1 + k_2 - k_3 - p), \quad (4)$$

and where \mathcal{T} is the scattering amplitude for particles with momenta $1, 2 \rightarrow 3, p$. In χ PT, the LO isospin averaged $\pi\pi$ scattering amplitude in terms of Mandelstam variables (s, t , and u) is

$$|\mathcal{T}|^2 = \frac{1}{9} \sum_{I=0,1,2} (2I+1) |\mathcal{T}^{(I)}|^2 = \frac{1}{9f_\pi^4} \{ 21m_\pi^4 + 9s^2 - 24M_\pi^2 s + 3(t-u)^2 \}. \quad (5)$$

The temperature dependence in pion mass and pion scattering amplitudes can be treated as higher order corrections.

In local thermal equilibrium, the distribution function $f_p^{(0)}(x) = (e^{\beta(x)V_\mu(x)p^\mu} - 1)^{-1}$ with $\beta(x)$ the inverse temperature and $V^\mu(x)$ the four velocity at the space–time point x . A small deviation of f_p from local equilibrium is parametrized as

$$f_p(x) = f_p^{(0)}(x) [1 - \{1 + f_p^{(0)}(x)\} \chi_p(x)], \quad (6)$$

while the energy momentum tensor is

$$T_{\mu\nu}(x) = g_\pi \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p_\mu p_\nu}{E_p} f_p(x). \quad (7)$$

We will choose the $\mathbf{V}(x) = 0$ frame for the point x . This implies $\partial_\nu V^0 = 0$ after taking a derivative on $V_\mu(x)V^\mu(x) = 1$. Furthermore, the conservation law at equilibrium $\partial_\mu T^{\mu\nu}|_{\chi_p=0} = 0$ allows us to replace $\partial_t \beta(x)$ and $\partial_t \mathbf{V}(x)$ by terms proportional to $\nabla \cdot \mathbf{V}(x)$ and $\nabla \beta(x)$. Thus, to the first order in a derivative expansion, $\chi_p(x)$ can be parametrized as

$$\chi_p(x) = \beta(x) A(p) \nabla \cdot \mathbf{V}(x) + \beta(x) B(p) \left(\hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij} \right) \times \left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{V}(x) \right), \quad (8)$$

² The QCD chiral restoration temperature and the deconfinement temperature happen to be close to each other at zero baryon density. We do not distinguish the two in this Letter.

where i and j are spacial indexes.³ A and B are functions of x and p . But we have suppressed the x dependence.

Substituting (8) into the Boltzmann equation, one obtains a linearized equation for B

$$\begin{aligned} & \left(p_i p_j - \frac{1}{3} \delta_{ij} \mathbf{p}^2 \right) \\ &= \frac{g_\pi E_p}{2} \int_{123} d\Gamma_{12;3p} (1+n_1)(1+n_2)n_3(1+n_p)^{-1} \\ & \quad \times [B_{ij}(p) + B_{ij}(k_3) - B_{ij}(k_2) - B_{ij}(k_1)] \\ & \equiv g_\pi \hat{F}_{ij}[B], \\ B_{ij}(p) & \equiv B(p) \left(\hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij} \right), \end{aligned} \quad (9)$$

where we have dropped the factor $(\nabla_i V_j(x) + \nabla_j V_i(x) - \text{trace})$ contracting both sides of the equation and written $f_i^{(0)}(x)$ at this point as $n_i = (e^{\beta E_i} - 1)^{-1}$. There is another integral equation associated with $\nabla \cdot \mathbf{V}(x)$ which is related to the bulk viscosity ζ that will not be discussed in this Letter. The $\nabla \cdot \beta$ and $\partial_t \mathbf{V}$ terms in $p^\mu \partial_\mu f_p^{(0)}$ will cancel each other by the energy momentum conservation in equilibrium mentioned above.

In equilibrium the energy momentum tensor depends on pressure $\mathcal{P}(x)$ and energy density $\epsilon(x)$ as $T_{\mu\nu}^{(0)}(x) = \{\mathcal{P}(x) + \epsilon(x)\} V_\mu(x) V_\nu(x) - \mathcal{P}(x) \delta_{\mu\nu}$. A small deviation away from equilibrium gives additional contribution to $T_{\mu\nu}$ whose spacial components define the shear and bulk viscosity

$$\begin{aligned} \delta T_{ij} &= -2\eta \left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{V}(x) \right) \\ & \quad + \zeta \delta_{ij} \nabla \cdot \mathbf{V}(x). \end{aligned} \quad (10)$$

After putting everything together we obtain

$$\begin{aligned} \eta &= \frac{g_\pi \beta}{10} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{E_p} n_p (1+n_p) B_{ij}(p) \left(p_i p_j - \frac{1}{3} \delta_{ij} \mathbf{p}^2 \right) \\ &= \frac{g_\pi^2 \beta}{10} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{E_p} n_p (1+n_p) B_{ij}(p) \hat{F}_{ij}[B] \\ & \equiv g_\pi^2 \langle B | \hat{F}[B] \rangle. \end{aligned} \quad (11)$$

Here we see immediately that if the scattering cross section is scaled by a factor α ,

$$d\Gamma_{12;3p} \rightarrow \alpha (d\Gamma_{12;3p}), \quad (12)$$

then Eqs. (9) and (11) imply the following scaling

$$\begin{aligned} B_{ij}(p) & \rightarrow \alpha^{-1} B_{ij}(p), \\ \eta & \rightarrow \alpha^{-1} \eta, \end{aligned} \quad (13)$$

with η proportional to the inverse of scattering cross section. This non-perturbative result is a general feature for the linearized Boltzmann equation with two-body elastic scattering.

³ A non-derivative term is not allowed since f_p should be reduced to $f_p^{(0)}$ when β and V^μ become independent of x . There is no term with single spacial derivative on $\beta(x)$ either. The only possible term $(\mathbf{V} \cdot \nabla) \beta(x)$ vanishes in the $\mathbf{V}(x) = 0$ frame.

To find a solution for $B(p)$, one can just solve Eq. (9). But here we follow the approach outlined in Refs. [25,26] to assume that $B(p)$ is a smooth function which can be expanded using a specific set of orthogonal polynomials

$$B(p) = g_\pi^{-1} \sum_{r=0}^{\infty} b_r B^{(r)}(z(p)), \quad (14)$$

where $B^{(r)}(z)$ is a polynomial up to z^r and b_r is its coefficient. The orthogonality condition

$$\int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}^2}{E_p} n_p (1+n_p) B^{(r)}(z) B^{(s)}(z) \propto \delta_{r,s} \quad (15)$$

can be used to construct the $B^{(r)}(z)$ polynomials up to normalization constants. For simplicity, we will choose

$$B^{(0)}(z) = 1.$$

With this expansion, the consistency condition for $B(p)$ in Eq. (11) yields

$$\eta = \sum_r b_r L^{(r)} = \sum_{r,s} b_r \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s, \quad (16)$$

where

$$L^{(r)} = \frac{\beta}{15} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}^2}{E_p} n_p (1+n_p) B^{(r)}(p) \propto \delta_{0,r}. \quad (17)$$

Since b_r is a function of m_π , f_π and T , the b_r 's in Eq. (16) are in general independent functions, such that $L^{(r)} = \sum_s \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s$ [one can show that this solution of b_s gives a unique solution of η], or equivalently

$$\delta_{0,r} L^{(0)} = \sum_s \langle B^{(r)} | \hat{F}[B^{(s)}] \rangle b_s. \quad (18)$$

This will allow us to solve for the b_s and obtain the shear viscosity

$$\eta = b_0 L^{(0)}. \quad (19)$$

In the next section, we will show that this expansion converges well, such that one does not need to keep many terms on the right-hand side of Eq. (18). If only the $s = 0$ term was kept, then

$$\eta \simeq \frac{(L^{(0)})^2}{\langle B^{(0)} | \hat{F}[B^{(0)}] \rangle}. \quad (20)$$

The calculation of the entropy density s is more straightforward since s , unlike η , does not diverge in a free theory. In χ PT, the interaction contributions are all higher order in our LO calculation. Thus we just compute the s for a free pion gas:

$$s = -g_\pi \beta^2 \frac{\partial}{\partial \beta} \frac{\log Z}{\beta}, \quad (21)$$

where the partition function Z for free pions is

$$\frac{\log Z}{\beta} = -\frac{1}{\beta} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \log \{ 1 - e^{-\beta E(p)} \}, \quad (22)$$

up to temperature independent terms.

3. Numerical results

In this section we present the results for η and η/s of QCD up to $T = 120$ MeV at zero baryon number density. In Fig. 1 the LO χ PT result of η using the linearized Boltzmann equation is shown. The lines with circles, squares and triangles correspond to keeping the first one, two and three polynomials on the right-hand side of Eq. (18), respectively. We have used $z(p) = |\mathbf{p}|$ to construct the polynomials. The figure shows the expansion converges rapidly. As a test of the calculation, we also reproduce the shear viscosity result of Ref. [23] for ϕ^4 theory by setting the scattering amplitude $\mathcal{T} = \lambda$ to be a constant. In ϕ^4 theory, η is monotonically increasing in T . If $T \gg m_\phi$, $\eta \propto T^3/\lambda^2$ with T^3 given by dimensional analysis and λ^{-2} by the scaling of coupling shown in Eqs. (12) and (13). In χ PT, however, η is not monotonic in T . At $T \ll m_\pi$, the scattering amplitude is close to a constant, thus χ PT behaves like a ϕ^4 theory. But at $T \gg m_\pi$, $\mathcal{T} \propto T^2/f_\pi^2$ and $\eta \propto f_\pi^4/T$. At what temperature the transition from $\eta \propto T^3$ to $\eta \propto 1/T$ takes place depends on the detail of dynamics. In χ PT, this temperature is around 20 MeV.

The radius of convergence in momentum for χ PT is typically $4\pi f_\pi \sim 1$ GeV. To translate this radius of convergence into temperature, we compute the averaged center of mass momentum $\langle |\mathbf{p}| \rangle = \sqrt{\langle B|\mathbf{p}^2|\hat{F}[B]\rangle/\langle B|\hat{F}[B]\rangle}$. We found that for $T = 120$ and 140 MeV, $\langle |\mathbf{p}| \rangle \simeq 460$ and 530 MeV $< 4\pi f_\pi$. However, χ PT is expected to break down at the chiral restoration temperature (~ 170 MeV). Thus our LO χ PT result can only be trusted up to $T \sim 120$ MeV. At the next-to-leading order (NLO), it is known that the isoscalar $\pi\pi$ scattering length will be increased by $\sim 40\%$ [22]. This will increase the cross section by $\sim 100\%$ and reduce η by $\sim 50\%$ near threshold. This is an unusually large NLO correction since a typical NLO correction at threshold is $\lesssim 20\%$. The large chiral corrections do not persist at the higher order. At the next-to-next-to-leading order (NNLO), the correction is $\sim 10\%$ [22]. Thus, to compute η to 10% accuracy, an NLO χ PT calculation is needed.

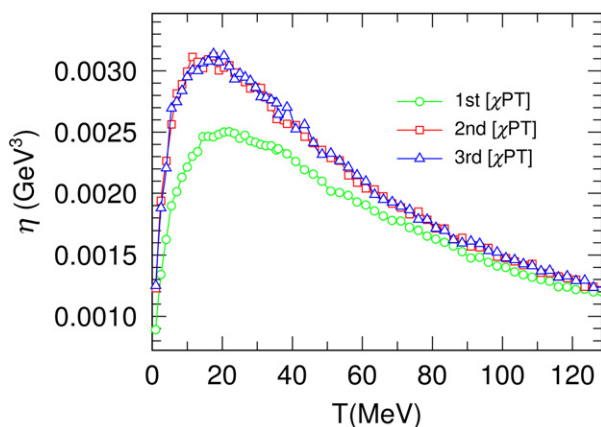


Fig. 1. (Color online.) Shear viscosity as a function of temperature in LO χ PT. 1st, 2nd and 3rd are the results of keeping the first one, two and three polynomials on the right-hand side of Eq. (18), respectively.

The LO χ PT result for η/s is shown in Fig. 2 (line with rectangles). The error is estimated to be $\sim 50\%$ up to 120 MeV. η/s is monotonically decreasing and reaches 0.6 at $T = 120$ MeV. This is similar to the behavior in the $m_\pi = 0$ case (shown as the line with rectangles) where $\eta/s \propto f_\pi^4/T^4$ with $s \propto T^3$ from dimensional analysis and $f_\pi = 87$ MeV in the chiral limit [21,22].

For comparison, we also show the result using phenomenological $\pi\pi$ phase shifts [29] for η but free pions for s . (Our result for η is in good agreement with that of [26] for T between 60 and 120 MeV. For an earlier calculation using the Chapman–Enskog approximation, see Ref. [30].) This amounts to take into account part of the NLO $\pi\pi$ scattering effects but ignore its temperature dependence and the interaction in s . Since not all the NLO effects are accounted for, this η/s is not necessarily more accurate than the one using LO χ PT. The comparison, however, gives us some feeling of the size of error for the LO result we present here. Thus, an error of $\sim 100\%$ at $T = 120$ MeV for the LO result might be more realistic.

Naive extrapolations of the three η/s curves show that the $1/4\pi = 0.08$ minimum bound conjectured from string theory might never be reached as in phase shift result (the first scenario), or more interestingly, be reached at $T \sim 200$ MeV, as in the LO χ PT result (the second scenario). In both scenarios, we see no sign of violation of the universal minimum bound for η/s below T_c . But to really make sure the bound is valid from 120 MeV to T_c , a lattice computation as was performed to gluon plasma above T_c [10] is needed. In the second scenario, assuming the bound is valid for QCD, then either a phase transition or cross over should occur before the minimum bound is reached at $T \sim 200$ MeV. Also, in this scenario, it seems natural for η/s to stay close to the minimum bound around T_c as was recently found in heavy ion collisions.

In the second scenario, one might argue that the existence of phase transition is already known, otherwise we will not have spontaneous symmetry breaking and the corresponding Nambu–Goldstone boson theory at low temperature in the first

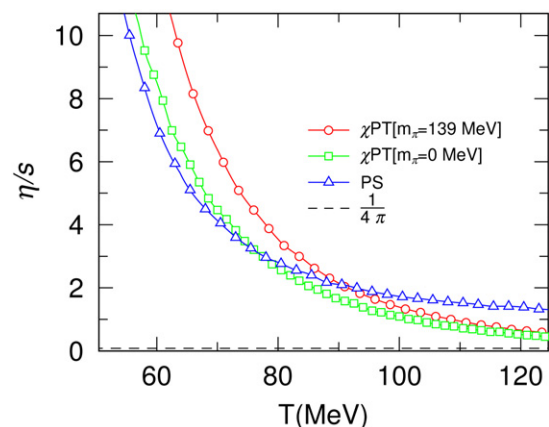


Fig. 2. (Color online.) Shear viscosity to entropy density ratios as functions of temperature. Line with circles (rectangles) is the LO χ PT result with $m_\pi = 139(0)$ MeV and $f_\pi = 93(87)$ MeV. Line with triangles is the result using $\pi\pi$ phase shifts (PS). Dashed line is the conjectured minimum bound $1/4\pi \simeq 0.08$.

place. Indeed, it is true in the case of QCD. However, if the η/s bound is really set by nature, then a phase transition is inevitable in the vicinity of the temperature where the bound is reached. For a spontaneous symmetry breaking theory, the general feature of η/s we see here seems generic. At very high T , collective motion is weak, thus η/s gets smaller at lower T . At very low T in the symmetry breaking phase, the Nambu–Goldstone bosons are weakly interaction at low temperature, thus η/s gets smaller at higher T . A phase transition should occur before the extrapolated η/s curve coming from high T reaches the bound at T_1 . Similarly, a phase transition should occur before the extrapolated η/s curve coming from low T reaches the bound at T_2 . Thus the range of phase transition is $T_1 \leq T_c \leq T_2$. However, it is also possible that the first scenario takes place and η/s bounces back to higher values without a phase transition. In this case, it is less clear what makes η/s non-monotonic and it certainly deserves further study.

It is interesting to note that the degeneracy factor g_π drops out of η while the entropy s is proportional to g_π as in Eqs. (16) and (21), respectively. This suggests the η/s bound might be violated if a system has a large particle degeneracy factor.⁴ For QCD, large g_π can be obtained by having a large number of quark flavors N_f with $g_\pi \sim N_f^2$. However, the existence of confinement demands that the number of colors N_c should be of order N_f to have a negative QCD beta function. After using $f_\pi \propto \sqrt{N_c}$, the combined N_c and N_f scaling of η/s is

$$\frac{\eta}{s} \propto \frac{f_\pi^4}{g_\pi T^4} \propto \frac{N_c^2}{N_f^2}, \quad (23)$$

which is of order one. Thus QCD with large N_c and N_f can still be consistent with the η/s bound below T_c .

4. Conclusion

We have explored whether the conjectured η/s minimum bound will hold up below the QCD deconfinement temperature. Using chiral perturbation theory and the linearized Boltzmann equation we have computed the QCD η/s ratio at zero baryon number density and for $T \leq 120$ MeV. It is found that η/s is monotonic decreasing in T and it reaches 0.6 with estimated 50% uncertainty at $T = 120$ MeV. Naive extrapolations have shown that η/s met the $1/4\pi$ minimum bound conjectured from string theory at $T \sim 200$ MeV as in the LO χ PT case, or η/s stayed above the bound as in the phenomenological phase shift case. In the former case, in order for the η/s lower bound to remain valid at higher temperature, a phase transition or cross over should occur at $T \lesssim 200$ MeV before the bound is reached. We argued that this might be a general feature for spontaneous symmetry breaking theories that the extrapolation of the low (high) temperature η/s curve sets an upper (lower) bound on T_c . Our result also suggests that it is natural for η/s to stay close to the lower bound around the phase transition temperature as was recently found in heavy ion collisions.

As this Letter was being finished, Ref. [31] appeared. In that Letter, some of the relations between T_c and the η/s bound are also discussed.

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