Physics Letters B 663 (2008) 132–135

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Odd scalar curvature in anti-Poisson geometry

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ARTICLE INFO

*Article history:*
Received 2 January 2008
Accepted 31 March 2008
Available online 7 April 2008
Editor: M. Cvetič

*PACS:*
02.40.-k
03.65.Ca
04.60.Gw
11.10.-z
11.10.Ef
11.15.Bt

*Keywords:*
BV field–antifield formalism
Odd Laplacian
Anti-Poisson geometry
Semidensity
Connection
Odd scalar curvature

ABSTRACT

Recent works have revealed that the recipe for field–antifield quantization of Lagrangian gauge theories can be considerably relaxed when it comes to choosing a path integral measure $\rho$ if a zero-order term $\nu_\rho$ is added to the $\Delta$ operator. The effects of this odd scalar term $\nu_\rho$ become relevant at two-loop order. We prove that $\nu_\rho$ is essentially the odd scalar curvature of an arbitrary torsion-free connection that is compatible with both the anti-Poisson structure $E$ and the density $\rho$. This extends a previous result for non-degenerate antisymplectic manifolds to degenerate anti-Poisson manifolds that admit a compatible two-form.



## 1. Introduction

The main purpose of this Letter is to report on new geometric insights into the field–antifield formalism. In general, the field–antifield formalism [1–3] is a recipe for constructing Feynman rules for Lagrangian field theories with gauge symmetries. The field–antifield formalism is in principle able to handle the most general gauge algebra, i.e. open gauge algebras of reducible type. The input is usually a local relativistic field theory, formulated via a classical action principle in a geometric configuration space. In the field–antifield scheme, the original field variables are extended with various stages of ghosts, antighosts and Lagrange multipliers—all of which are then further extended with corresponding antifields; the gauge symmetries are encoded in a nilpotent Fermionic BRST symmetry [4,5]; and the original action is deformed into a BRST-invariant master action, whose Hessian has the maximal allowed rank. The full quantum master action

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n \tag{1.1}$$

is determined recursively order by order in $\hbar$ from a consistent set of quantum master equations

$$(S, S) = 0, \tag{1.2}$$

$$(M_1, S) = i(\Delta_\rho S), \tag{1.3}$$

$$(M_2, S) = i(\Delta_\rho M_1) + \nu_\rho - \frac{1}{2}(M_1, M_1), \tag{1.4}$$

$$(M_n, S) = i(\Delta_\rho M_{n-1}) - \frac{1}{2}\sum_{r=1}^{n-1}(M_r, M_{n-r}), \quad n \geqslant 3. \tag{1.5}$$

Here $(\cdot,\cdot)$ is the antibracket (or anti-Poisson structure), $\Delta_\rho$ is the odd Laplacian and $\nu_\rho$ is an odd scalar, which become relevant in perturbation theory at loop order 0, 1, and 2, respectively. It has only recently been realized that the field–antifield formalism can consistently accommodate a non-zero $\nu_\rho$ term, thereby providing a more flexible framework for field–antifield quantization [6–8].

The classical master equation (1.2) is a generalization of Zinn-Justin's equation [9], which allows to set up consistent renormalization (if the field theory is renormalizable). If the theory is not anomalous at the one-loop level, there will exist a local solution $M_1$ to the next Eq. (1.3), and so forth. Although the field–antifield formalism in its basic form is only a formal scheme—i.e. particularly, it assumes that results from finite-dimensional analysis are

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0370-2693/$ – see front matter 
doi:10.1016/j.physletb.2008.03.066

directly applicable to field theory, which has infinitely many degrees of freedom—it has nevertheless been successfully applied to a large variety of physical models. It has mainly been used in a truncated form of the full set of quantum master Eqs. (1.2)–(1.5), where all the following quantities

$$(S,S),(\Delta_\rho S),\nu_\rho,M_1,M_2,M_3,\ldots, \tag{1.6}$$

are set identically equal to zero. One can for instance mention the AKSZ paradigm [10,11] as a broad example that uses the truncated field–antifield formalism (1.6) to quantize supersymmetric topological field theories [12–15]. Currently, very few scientific works describe solutions with non-zero $M_n$'s, primarily due to the singular nature of the odd Laplacian $\Delta_\rho$ in field theory (again because of the infinitely many degrees of freedom). Nevertheless, it should be fruitful to study generic solutions of the full quantum master equation. See the original paper [1] for an interesting solution with $M_1\neq 0$. Finally, it has in many cases been explicitly checked that the field–antifield formalism produces the same result as the Hamiltonian formulation [16–18]. The formalism has also influenced work in closed string field theory [19] and several branches of mathematics. The geometry behind the field–antifield formalism was further clarified in Refs. [20–23].

In this Letter we shall only explicitly consider the case of finitely many variables. Our main result concerns the odd scalar $\nu_\rho$, which is a certain function of the anti-Poisson structure $E^{AB}$ and the density $\rho$, cf. Eq. (6.1) below. It turns out that $\nu_\rho$ has a geometric interpretation as (minus $1/8$ times) the odd scalar curvature $R$ of any connection $\nabla$ that satisfies three conditions; namely that $\nabla$ is (1) anti-Poisson, (2) torsion-free and (3) $\rho$-compatible. This is a rather robust conclusion as we shall prove in this Letter that it even holds for degenerate antibrackets. (Degenerate anti-Poisson structures appear naturally from for instance the Dirac antibracket construction for antisymplectic second-class constraints [7,21,24,25].)

## 2. Anti-Poisson structure $E^{AB}$

An *anti-Poisson* structure is by definition a possibly degenerate $(2,0)$ tensor field $E^{AB}$ with upper indices that is Grassmann-odd

$$\varepsilon\big(E^{AB}\big)=\varepsilon_A+\varepsilon_B+1, \tag{2.1}$$

that is skewsymmetric

$$E^{AB}=-(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{BA}, \tag{2.2}$$

and that satisfies the Jacobi identity

$$\sum_{\text{cycl.}A,B,C}(-1)^{(\varepsilon_A+1)(\varepsilon_C+1)}E^{AD}\big(\overrightarrow{\partial^l_D}E^{BC}\big)=0. \tag{2.3}$$

## 3. Compatible two-form $E_{AB}$

In general, an anti-Poisson manifold could have singular points where the rank of $E^{AB}$ jumps, and it is necessary to impose a regularity criterion to proceed. We shall here assume that the anti-Poisson structure $E^{AB}$ admits a compatible two-form field $E_{AB}$, i.e. that there exists a two-form field $E_{AB}$ with lower indices that is Grassmann-odd

$$\varepsilon(E_{AB})=\varepsilon_A+\varepsilon_B+1, \tag{3.1}$$

that is skewsymmetric

$$E_{AB}=-(-1)^{\varepsilon_A\varepsilon_B}E_{BA}, \tag{3.2}$$

and that is *compatible* with the anti-Poisson structure in the sense that

$$E^{AB}E_{BC}E^{CD}=E^{AD}, \tag{3.3}$$

$$E_{AB}E^{BC}E_{CD}=E_{AD}. \tag{3.4}$$

This is a relatively mild requirement, which is always automatically satisfied for a Dirac antibracket on antisymplectic manifolds with antisymplectic second-class constraints [7,21,24,25]. Note that the two-form $E_{AB}$ is neither unique nor necessarily closed. One can define a $(1,1)$ tensor field as

$$P^A{}_C\equiv E^{AB}E_{BC}, \tag{3.5}$$

or equivalently,

$$P_A{}^C\equiv E_{AB}E^{BC}=(-1)^{\varepsilon_A(\varepsilon_C+1)}P^C{}_A. \tag{3.6}$$

It then follows from either of the compatibility relations (3.3) and (3.4) that $P^A{}_B$ is an idempotent

$$P^A{}_BP^B{}_C=P^A{}_C. \tag{3.7}$$

## 4. The $\Delta_E$ operator

An anti-Poisson structure with a compatible two-form field $E_{AB}$ gives rise to a Grassmann-odd, second-order $\Delta_E$ operator that takes semidensities to semidensities. It is defined in arbitrary coordinates as [7]

$$\Delta_E\equiv\Delta_1+\frac{\nu^{(1)}}{8}-\frac{\nu^{(2)}}{8}-\frac{\nu^{(3)}}{24}+\frac{\nu^{(4)}}{24}+\frac{\nu^{(5)}}{12}, \tag{4.1}$$

where $\Delta_1$ is the odd Laplacian

$$\Delta_\rho\equiv\frac{(-1)^{\varepsilon_A}}{2\rho}\overrightarrow{\partial^l_A}\rho E^{AB}\overrightarrow{\partial^l_B}, \tag{4.2}$$

with $\rho=1$, and where

$$\nu^{(1)}\equiv(-1)^{\varepsilon_A}\big(\overrightarrow{\partial^l_B}\overrightarrow{\partial^l_A}E^{AB}\big), \tag{4.3}$$

$$\nu^{(2)}\equiv(-1)^{\varepsilon_A\varepsilon_C}\big(\overrightarrow{\partial^l_D}E^{AB}\big)E_{BC}\big(\overrightarrow{\partial^l_A}E^{CD}\big), \tag{4.4}$$

$$\nu^{(3)}\equiv(-1)^{\varepsilon_B}\big(\overrightarrow{\partial^l_A}E_{BC}\big)E^{CD}\big(\overrightarrow{\partial^l_D}E^{BA}\big), \tag{4.5}$$

$$\nu^{(4)}\equiv(-1)^{\varepsilon_B}\big(\overrightarrow{\partial^l_A}E_{BC}\big)E^{CD}\big(\overrightarrow{\partial^l_D}E^{BF}\big)P_F{}^A, \tag{4.6}$$

$$\begin{aligned}\nu^{(5)}&\equiv(-1)^{\varepsilon_A\varepsilon_C}\big(\overrightarrow{\partial^l_D}E^{AB}\big)E_{BC}\big(\overrightarrow{\partial^l_A}E^{CF}\big)P_F{}^D\\&=(-1)^{(\varepsilon_A+1)\varepsilon_B}E^{AD}\big(\overrightarrow{\partial^l_D}E^{BC}\big)\big(\overrightarrow{\partial^l_C}E_{AF}\big)P^F{}_B.\end{aligned} \tag{4.7}$$

It is shown in Ref. [7] that the $\Delta_E$ operator defined in Eq. (4.1) does not depend on the choice of local coordinates, it does not depend on the choice of compatible two-form field $E_{AB}$, and it does map semidensities into semidensities. Moreover, the Jacobi identity (2.3) precisely ensures that $\Delta_E$ is nilpotent

$$\Delta_E^2=\frac{1}{2}[\Delta_E,\Delta_E]=0. \tag{4.8}$$

Earlier works on the $\Delta_E$ operator include Refs. [6,25–29].

## 5. The $\Delta$ operator

Classically, the field–antifield formalism is governed by the anti-Poisson structure $E^{AB}$, or equivalently, the antibracket

$$(f,g)\equiv\big(f\overleftarrow{\partial^r_A}\big)E^{AB}\big(\overrightarrow{\partial^l_B}g\big)=-(-1)^{(\varepsilon_f+1)(\varepsilon_g+1)}(g,f). \tag{5.1}$$

Quantum mechanically, the field–antifield recipe instructs one to choose an arbitrary path integral measure $\rho$, and to use it to build a nilpotent, Grassmann-odd, second-order $\Delta$ operator that takes scalar functions into scalar functions. It is natural to build the $\Delta$ operator by conjugating the $\Delta_E$ operator (4.1) with appropriate square roots of the density $\rho$ as follows:

$$\Delta\equiv\frac{1}{\sqrt{\rho}}\Delta_E\sqrt{\rho}. \tag{5.2}$$

In this way the $\Delta$ operator trivially inherits the nilpotency property from the $\Delta_E$ operator,

$$\Delta^2 = \frac{1}{\sqrt{\rho}}\Delta_E^2\sqrt{\rho} = 0. \tag{5.3}$$

In physical applications the nilpotency (5.3) of $\Delta$ is important for the underlying BRST symmetry of the theory.

## 6. The odd scalar $\nu_\rho$

The odd scalar function $\nu_\rho$ is defined as

$$\begin{aligned}\nu_\rho &\equiv (\Delta 1) = \frac{1}{\sqrt{\rho}}(\Delta_E\sqrt{\rho}) \\ &= \nu_\rho^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{24} + \frac{\nu^{(4)}}{24} + \frac{\nu^{(5)}}{12},\end{aligned} \tag{6.1}$$

where $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$, $\nu^{(4)}$, $\nu^{(5)}$ are given in Eqs. (4.3)–(4.7), and the quantity $\nu_\rho^{(0)}$ is given as

$$\nu_\rho^{(0)} \equiv \frac{1}{\sqrt{\rho}}(\Delta_1\sqrt{\rho}). \tag{6.2}$$

The second-order $\Delta$ operator (5.2) decomposes as

$$\Delta = \Delta_\rho + \nu_\rho, \tag{6.3}$$

where $\Delta_\rho$ is the odd Laplacian (4.2). The nilpotency of $\Delta$ implies that

$$\Delta_\rho^2 = (\nu_\rho, \cdot), \tag{6.4}$$

$$(\Delta_\rho \nu_\rho) = 0. \tag{6.5}$$

The possibility of a non-trivial $\nu_\rho$ has only recently been observed, cf. Refs. [6–8]. In the past, the odd scalar term $\nu_\rho$ was not present due to a certain compatibility relation between $E$ and $\rho$, which was unnecessarily imposed, and which (using our new terminology) made $\nu_\rho$ vanish. In terms of the quantum master equation

$$\Delta e^{\frac{i}{\hbar}W} = 0, \tag{6.6}$$

the odd scalar $\nu_\rho$ enters at the two-loop order $\mathcal{O}(\hbar^2)$

$$\frac{1}{2}(W, W) = i\hbar\Delta_\rho W + \hbar^2\nu_\rho, \tag{6.7}$$

which in turn leads to the set of Eqs. (1.2)–(1.5).

## 7. Connection

In the next two Sections 7 and 8 we will briefly state our sign conventions and definitions for the covariant derivative and the curvature in the presence of Fermionic degrees of freedom. A more complete treatment can be found in Refs. [8,30]. Other references include Ref. [31]. Our convention for the left covariant derivative $(\nabla_A X)^B$ of a left vector field $X^A$ is [30]

$$\begin{aligned}(\nabla_A X)^B &\equiv \big(\overrightarrow{\partial^l_A} X^B\big) + (-1)^{\varepsilon_X(\varepsilon_B+\varepsilon_C)}\Gamma_A{}^B{}_C X^C, \\ \varepsilon\big(X^A\big) &= \varepsilon_X + \varepsilon_A.\end{aligned} \tag{7.1}$$

A connection $\Gamma_A{}^B{}_C$ is called *anti-Poisson* if it preserves the anti-Poisson structure $E^{AB}$, i.e.

$$\begin{aligned}0 &= (\nabla_A E)^{BC} \\ &\equiv \big(\overrightarrow{\partial^l_A} E^{BC}\big) + \big(\Gamma_A{}^B{}_D E^{DC} - (-1)^{(\varepsilon_B+1)(\varepsilon_C+1)}(B \leftrightarrow C)\big).\end{aligned} \tag{7.2}$$

It is useful to define a reordered Christoffel symbol $\Gamma^A{}_{BC}$ as

$$\Gamma^A{}_{BC} \equiv (-1)^{\varepsilon_A\varepsilon_B}\Gamma_B{}^A{}_C. \tag{7.3}$$

A *torsion-free* connection $\Gamma^A{}_{BC}$ has the following symmetry in the lower indices:

$$\Gamma^A{}_{BC} = -(-1)^{(\varepsilon_B+1)(\varepsilon_C+1)}\Gamma^A{}_{CB}. \tag{7.4}$$

A connection $\Gamma^A{}_{BC}$ is called *$\rho$-compatible* if

$$\Gamma^B{}_{BA} = \big(\ln\rho\overleftarrow{\partial^r_A}\big). \tag{7.5}$$

There are in principle two definitions for the divergence $\operatorname{div} X$ of a bosonic vector field $X$ with $\varepsilon_X = 0$. The first divergence definition depends on the density $\rho$

$$\operatorname{div}_\rho X \equiv \frac{(-1)^{\varepsilon_A}}{\rho}\overrightarrow{\partial^l_A}\big(\rho X^A\big), \tag{7.6}$$

while the second definition depends on the connection $\nabla$

$$\begin{aligned}\operatorname{div}_\nabla X &\equiv \operatorname{str}(\nabla X) \equiv (-1)^{\varepsilon_A}(\nabla_A X)^A \\ &= \big((-1)^{\varepsilon_A}\overrightarrow{\partial^l_A} + \Gamma^B{}_{BA}\big)X^A.\end{aligned} \tag{7.7}$$

The $\rho$-compatibility condition (7.5) precisely ensures that the two definitions (7.6) and (7.7) coincide, and hence that there is a unique notion of volume [32]. We shall only consider torsion-free connections $\nabla$ that are anti-Poisson and $\rho$-compatible, i.e. connections that satisfy the above three conditions (7.2), (7.4) and (7.5). Then the odd Laplacian $\Delta_\rho$ can be written on a manifestly covariant form

$$\Delta_\rho = \frac{(-1)^{\varepsilon_A}}{2}\nabla_A E^{AB}\nabla_B = \frac{(-1)^{\varepsilon_B}}{2}E^{BA}\nabla_A\nabla_B. \tag{7.8}$$

## 8. Curvature

The Riemann curvature tensor is

$$\begin{aligned}R^A{}_{BCD} &\equiv (-1)^{\varepsilon_A\varepsilon_B}\big(\overrightarrow{\partial^l_B}\Gamma^A{}_{CD}\big) + \Gamma^A{}_{BE}\Gamma^E{}_{CD} \\ &\quad - (-1)^{\varepsilon_B\varepsilon_C}(B \leftrightarrow C).\end{aligned} \tag{8.1}$$

(Note that the ordering of indices on the Riemann curvature tensor is slightly non-standard to minimize appearances of sign factors.) The Ricci tensor is

$$\begin{aligned}R_{AB} &\equiv R^C{}_{CAB} \\ &= \frac{(-1)^{\varepsilon_C}}{\rho}\big(\overrightarrow{\partial^l_C}\rho\Gamma^C{}_{AB}\big) - \big(\overrightarrow{\partial^l_A}\ln\rho\overleftarrow{\partial^r_B}\big) - \Gamma_A{}^C{}_D\Gamma^D{}_{CB} \\ &= -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}R_{BA}.\end{aligned} \tag{8.2}$$

## 9. Odd scalar curvature

The odd scalar curvature $R$ is defined as the Ricci tensor $R_{AB}$ contracted with the anti-Poisson tensor $E^{AB}$,

$$R \equiv R_{AB}E^{BA} = E^{AB}R_{BA}, \qquad \varepsilon(R) = 1. \tag{9.1}$$

We now assert that the odd scalar curvature

$$R = -8\nu_\rho \tag{9.2}$$

of an arbitrary connection $\nabla$ that is anti-Poisson, torsion-free and $\rho$-compatible, is equal to (minus eight times) the odd scalar $\nu_\rho$. In particular one sees that the odd scalar curvature $R$ carries no information about the connection $\nabla$ used, and it depends only on $E$ and $\rho$. Eq. (9.2) was proven for the non-degenerated case in Ref. [8]. The degenerated case is proven in Appendix A.

## Acknowledgements

We would like to thank P.H. Damgaard for discussions. K.B. thanks the Lebedev Physics Institute and the Niels Bohr Institute for warm hospitality. The work of I.A.B. is supported by grants RFBR 05-01-00996, RFBR 05-02-17217 and LSS-4401.2006.2. The work of K.B. is supported by the Ministry of Education of the Czech Republic under the project MSM 0021622409.

## Appendix A. Proof of the Main Eq. (9.2)

Eq. (C.9) in Ref. [8] yields that the odd scalar curvature $R$ can be written as

$$R = -8\nu_\rho^{(0)} - \nu^{(1)} - \frac{1}{2}R_I, \tag{A.1}$$

where $\nu_\rho^{(0)}$, $\nu^{(1)}$ and $R_I$ are defined in Eqs. (6.2), (4.3) and (A.2), respectively. Since the expression (A.2) below for $R_I$ only depends on the torsion-free part of the connection, one does in principle not need the torsion-free condition (7.4) from now on. The heart of the proof consists of the following ten "one-line calculations":

$$R_{\mathrm{I}} \equiv \Gamma^A{}_{BC}\big(E^{CB}\overleftarrow{\partial}{}^r_A\big) = \Gamma^A{}_{BC}\big(\big(E^{CD}E_{DF}E^{FB}\big)\overleftarrow{\partial}{}^r_A\big) = 2R_{\mathrm{II}} + R_{\mathrm{III}}, \tag{A.2}$$

$$R_{\mathrm{II}} \equiv \Gamma^A{}_{BC}P^C{}_D\big(E^{DB}\overleftarrow{\partial}{}^r_A\big) = -R_{\mathrm{IV}} - \nu^{(2)}, \tag{A.3}$$

$$R_{\mathrm{III}} \equiv (-1)^{\varepsilon_A(\varepsilon_C+1)}\Gamma_F{}^A{}_B E^{BC}\big(\overrightarrow{\partial}{}^l_A E_{CD}\big)E^{DF} = 2R_{\mathrm{III}} + R_{\mathrm{V}}, \tag{A.4}$$

$$R_{\mathrm{IV}} \equiv \Gamma^A{}_{BC}E^{CD}\big(\overrightarrow{\partial}{}^l_D E^{BF}\big)E_{FA} = R_{\mathrm{VI}} - R_{\mathrm{IV}}, \tag{A.5}$$

$$R_{\mathrm{V}} \equiv (-1)^{\varepsilon_A\varepsilon_C}\Gamma_F{}^A{}_B P^B{}_C\big(\overrightarrow{\partial}{}^l_A E^{CD}\big)P_D{}^F = R_{\mathrm{VII}} - \nu^{(5)}, \tag{A.6}$$

$$R_{\mathrm{VI}} \equiv \Gamma^A{}_{BC}\big(E^{CB}\overleftarrow{\partial}{}^r_D\big)P^D{}_A = 2R_{\mathrm{VIII}} + R_{\mathrm{IX}}, \tag{A.7}$$

$$\begin{aligned} R_{\mathrm{VII}} &\equiv (-1)^{(\varepsilon_A+1)(\varepsilon_C+1)}E_{AB}\Gamma^B{}_{CD}E^{DF}\big(\overrightarrow{\partial}{}^l_F E^{AG}\big)P_G{}^C \\ &= R_{\mathrm{IV}} - R_{\mathrm{VIII}}, \end{aligned} \tag{A.8}$$

$$R_{\mathrm{VIII}} \equiv \Gamma^A{}_{BC}P^C{}_D\big(E^{DB}\overleftarrow{\partial}{}^r_F\big)P^F{}_A = -R_{\mathrm{IV}} - \nu^{(5)}, \tag{A.9}$$

$$R_{\mathrm{IX}} \equiv (-1)^{\varepsilon_A(\varepsilon_C+1)}\Gamma_G{}^A{}_B E^{BC}P_A{}^D\big(\overrightarrow{\partial}{}^l_D E_{CF}\big)E^{FG} = -R_{\mathrm{X}} - \nu^{(4)}, \tag{A.10}$$

$$R_{\mathrm{X}} \equiv (-1)^{\varepsilon_A}\Gamma_F{}^A{}_B E^{BC}\big(\overrightarrow{\partial}{}^l_C E_{AD}\big)E^{DF} = -R_{\mathrm{III}} - \nu^{(3)}. \tag{A.11}$$

Here we have used the upper compatibility relation (3.3) for the two-form $E_{AB}$ in the second equality of Eqs. (A.2), (A.7), (A.8), (A.9) and (A.10); the lower compatibility relation (3.4) for the two-form $E_{AB}$ in the second equality of Eq. (A.4); the anti-Poisson property (7.2) for the connection $\nabla$ in the second equality of Eqs. (A.3), (A.6), (A.9), (A.10) and (A.11); and the Jacobi identity (2.3) in the second equality of Eqs. (A.5) and (A.8). From these ten relations (A.2)–(A.11), the quantity $R_{\mathrm{III}}$ can be determined as follows:

$$\begin{aligned} -R_{\mathrm{III}} &= R_{\mathrm{V}} = R_{\mathrm{VII}} - \nu^{(5)} = (R_{\mathrm{IV}} - R_{\mathrm{VIII}}) + (R_{\mathrm{IV}} + R_{\mathrm{VIII}}) = 2R_{\mathrm{IV}} \\ &= R_{\mathrm{VI}} = 2R_{\mathrm{VIII}} + R_{\mathrm{IX}} = -2\big(R_{\mathrm{IV}} + \nu^{(5)}\big) + \big(R_{\mathrm{III}} + \nu^{(3)} - \nu^{(4)}\big) \\ &= 2R_{\mathrm{III}} + \big(\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}\big), \end{aligned} \tag{A.12}$$

so that

$$R_{\mathrm{III}} = \frac{1}{3}\big(-\nu^{(3)} + \nu^{(4)} + 2\nu^{(5)}\big). \tag{A.13}$$

Next, $R_{\mathrm{I}}$ can be expressed in terms of $R_{\mathrm{III}}$:

$$\frac{1}{2}R_{\mathrm{I}} = R_{\mathrm{II}} + \frac{1}{2}R_{\mathrm{III}} = -\big(R_{\mathrm{IV}} + \nu^{(2)}\big) + \frac{1}{2}R_{\mathrm{III}} = R_{\mathrm{III}} - \nu^{(2)}. \tag{A.14}$$

Inserting Eqs. (A.13) and (A.14) into Eq. (A.1) yields the main Eq. (9.2):

$$\begin{aligned} R &= -8\nu_\rho^{(0)} - \nu^{(1)} - \frac{1}{2}R_{\mathrm{I}} \\ &= -8\nu_\rho^{(0)} - \nu^{(1)} + \nu^{(2)} + \frac{1}{3}\big(\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}\big) \\ &= -8\nu_\rho. \end{aligned} \tag{A.15}$$

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