Alexandria Engineering Journal (2016) 55, 2893–2899

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ORIGINAL ARTICLE

A study of quintic B-spline based differential quadrature method for a class of semi-linear Fisher-Kolmogorov equations

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Received 4 April 2016; revised 22 June 2016; accepted 27 June 2016
Available online 20 July 2016

KEYWORDS

Quintic B-spline;
Extended Fisher-Kolmogorov equation;
Differential quadrature method;
Stability;
SSP-RK43 method

Abstract In the present manuscript, Fisher-Kolmogorov equation is solved numerically by adopting a differential quadrature technique that uses quintic B-spline as the basis functions for space integration. The derivatives are approximated using differential quadrature method. The weighting coefficients are obtained by semi-explicit algorithm. Five-band Thomas algorithm has been employed to solve the resultant algebraic system that can be reduced into a penta-diagonal matrix. Stability analysis of method has also been done. The accuracy of the proposed scheme is demonstrated by applying on three test problems. Theoretical attributes such as existence, uniqueness and regularity of Fisher-Kolmogorov equations are also conferred. The outcomes are depicted graphically to confirm accuracy of the findings and performance of this method and a comparative study is done with results available in the literature. The computed results are found to be in good agreement with the analytical solutions.

1. Introduction

In this paper, we have adopted a quintic B-spline differential quadrature approach to solve a extended Fisher-Kolmogorov equations of order four, that can be written as

$$v_t + \gamma v_{xxxx} - v_{xx} + g(v) = 0, \quad x \in [p, q], \; t \in [0, \tau] \tag{1}$$

where $v \in [p, q] \times [0, \tau]$ is a real valued function, $g(v) = v^3 - v$ and $\gamma > 0$ with the boundary conditions

$$v(p, t) = v_0, \qquad v(q, t) = v_1 \tag{2}$$

$$v_{xx}(p, t) = 0, \qquad v_{xx}(q, t) = 0 \tag{3}$$

and initial condition

$$v(x, 0) = v_i, \qquad x \in [p, q] \tag{4}$$

Coullet et al. [1], Dee and Van saarlos [2–4] introduced the extended Fisher-Kolmogorov equation. For $\gamma = 0$, the equation reduces to Fisher-Kolmogorov equation in standard form.

The extended Fisher-Kolmogorov equation appears in a variety of applications such as pattern formation [3] and

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.06.019
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spatiotemporal chaos [1] in bi-stable systems, phase transition near a Lifshitz point [5,6]. It has been derived as an amplitude equation at the onset of instabilities near certain degenerate points [7]. Recently, attention has been given to steady state equation in Eq. (1). The stationary problem displays periodic, homoclinic or heteroclinic solutions, depending on γ. Peletier and Troy [8,9] studied the solution of extended Fisher-Kolmogorov equation in steady state by adopting shooting method. Peletier et al. [8,9], Bengurai et al. [10] and Dhanmjaya et al. [11] discussed the existence of solution, its uniqueness and regularity for Eq. (1). Aghamohamadi et al. [12] worked on nonlinear Fisher's equation using tension spline method. These papers have differentiated and analyzed two different cases depending on value of γ, for $\gamma \leqslant 1/8$ and $\gamma > 1/8$, where the behavior of solutions is variable. It has been shown that for $\gamma > 1/8$ monotonicity is lost but, there exists unique solution for $\gamma \leqslant 1/8$. In this work, a different numerical approach based on differential quadrature method and B-spline functions is proposed for the solution of extended Fisher-Kolmogorov Eq. (1).

Bellman et al. [14] initially introduced differential quadrature method for solving PDEs. Spline based differential quadrature method is used in [15,16]. Cheng et al. [17] have used Hermite polynomial to compute the weighting coefficients of differential quadrature method. Striz et al. [18] and Bonzani [19] used harmonic functions and Sinc functions respectively as basis functions. Korkmaz [20], Korkmaz and Daĝ [21–23], Saka et al. [24], Mittal and Dahiya [33,34] and Arora [35] used various differential quadrature methods which are found to produce accurate solutions with ease. The idea of iterative differential quadrature method has been introduced by Tomasiello [25,26]. In her work, iterative differential quadrature method and the stability of DQ solutions have been discussed. An article totally dedicated to the study of DQ based methods has been published [27]. A new method based on least square DQ method has also been introduced to deal with the buckling of structures with elastic support [28].

In this work, we reduce the given problem into a system of ordinary differential equations by expanding the derivatives of the unknown function as the function of fifth order B-spline with unknown coefficients. The weighting coefficients of the differential quadrature method are found by using a five-banded Thomas algorithm for penta-diagonal systems. The differential quadrature method has great advantages over the traditional methods, specially it does prevent perturbation in order to find better solutions to given nonlinear equations. Its advantages over different bases are rendered to its simple implementation, wide applicability and conceptual simplicity.

The organization of this paper is as follows. Section 2 gives a description of the quintic B-spline differential quadrature method. In Section 3, the method is implemented on the extended Fisher-Kolmogorov equation with suitable handling of conditions on boundary. In Section 4, stability analysis of the method is carried out. Section 5 presents some test examples of the extended Fisher-Kolmogorov equation. A conclusive section is provided at the end in Section 6.

2. Quintic B-spline differential quadrature method

The domain under consideration i.e., $p \leqslant x \leqslant q$ is discretized into a grid of equal length $h = x_{i+1} - x_i$, by the knots x_i where $i = 0, 1, 2, \ldots, N$ such that $p = x_1 < x_2, \ldots, x_{N-1} < x_N = q$. Let $Q_m(x)$, be the quintic B-splines with knot points x_i. The B-splines $Q_{-1}, Q_0, \ldots, Q_{N+2}$ act as a basis for functions defined over the domain $[p, q]$. The quintic B-splines are defined as [29]

$$Q_m(x) = \frac{1}{h^5}\begin{cases} (x - x_{m-3})^5 & x \in [x_{m-3}, x_{m-2}) \\ (x - x_{m-3})^5 - 6(x - x_{m-2})^5 & x \in [x_{m-2}, x_{m-1}) \\ (x - x_{m-3})^5 - 6(x - x_{m-2})^5 + 15(x - x_{m-1})^5 & x \in [x_{m-1}, x_m) \\ (x_{m+3} - x)^5 - 6(x_{m+2} - x)^5 + 15(x_{m+1} - x)^5 & x \in [x_m, x_{m+1}) \\ (x_{m+3} - x)^5 - 6(x_{m+2} - x)^5 & x \in [x_{m+1}, x_{m+2}) \\ (x_{m+3} - x)^5 & x \in [x_{m+2}, x_{m+3}) \\ 0 & \text{otherwise}, \end{cases} \qquad m = -1, 0, 1, \ldots, N+1, N+2 \tag{5}$$

In a fair share, six knots at a time are enclosed by each quintic B-spline so that a knot in its turn is concealed by six quintic B-splines. Table 1 comprises the nonzero values of functions given by $Q_m(x)$ and its derivatives at given knot points.

Differential quadrature method approximates derivative of a given function by writing it as the linear combination of its values at specific discrete nodal points over the solution domain of a problem. Given a function $v(x)$, that is smooth enough over the solution domain then its derivative with respect to x at any node can be approximated as

$$v_x(x_i, t) = \sum_{j=1}^{N} a_{ij}^{(1)} v(x_j), \quad i = 1, 2, \ldots, N \tag{6}$$

where $a_{ij}^{(1)}$ are the unknown weighting coefficients of the first order partial derivatives with respect to x.

The derivatives of order two or higher can be calculated by the Shu's recurrence formula [30]

$$a_{ij}^{(r)} = r\left[a_{ij}^{(1)} a_{ii}^{(r-1)} - \frac{a_{ij}^{(r-1)}}{x_i - x_j}\right], \quad \textit{for } i \neq j \\ i,j = 1, 2, \ldots, N; \qquad r = 2, 3, \ldots, N-1 \tag{7}$$

$$a_{ii}^{(r)} = -\sum_{j=1, j\neq i}^{N} a_{ij}^{(r)}, \qquad \textit{for } i = j \tag{8}$$

Here, $a_{ij}^{(r-1)}$ and $a_{ij}^{(r)}$ denote the weighting coefficients of partial derivatives of order $(r-1)$ and (r) in the direction of x-axis.

From Eq. (6), it is clear that the fundamental problem for approximating the derivatives of the unknown function with differential quadrature method is to determine the weighting coefficients a_{ij} by means of a set of base functions that span the domain under consideration. The weighting coefficients can be computed by employing different basis. Here we are using quintic B-spline basis given in Eq. (5) for our calculations. Bashan et al. [31] used quintic B-splines for solving KdVB equation, whereas Korkmaz et al. [32] used both quartic and quintic B-splines for numerical approximation of advection–diffusion equations.

Employing quintic B-splines as test functions in fundamental differential quadrature method Eq. (6) gives

$$\frac{\partial Q_m(x_i)}{\partial x} = \sum_{j=m-2}^{m+2} a_{ij}^{(1)} Q_m(x_j), \quad m = -1, 0, \ldots, N+2, \quad i = 1, 2, \ldots, N \tag{9}$$

An arbitrary choice of i leads to an algebraic equation system

$$\begin{bmatrix} Q_{-1,-3} & Q_{-1,-2} & Q_{-1,-1} & Q_{-1,0} & Q_{-1,1} & & & & & \\ & Q_{0,-2} & Q_{0,-1} & Q_{0,0} & Q_{0,1} & Q_{0,2} & & & & \\ & & \cdots & \cdots & \cdots & \cdots & \cdots & & & \\ & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & & & Q_{N+1,N+2} & Q_{N+1,N} & Q_{N+1,N+1} & Q_{N+1,N+2} & Q_{N+1,N+3} & \\ & & & & & Q_{N+2,N} & Q_{N+2,N+1} & Q_{N+2,N+2} & Q_{N+2,N+3} & Q_{N+2,N+4} \end{bmatrix} A_1 = \phi_1 \tag{10}$$

where $Q_{i,j}$ denotes $Q_i(x_j)$, $A_1 = [a_{i,-3}\ a_{i,-2}\ \ldots\ a_{i,N+3}\ a_{i,N+4}]$ and $\phi_1 = \left[\frac{\partial Q_{-1}(x_i)}{\partial x}\ \frac{\partial Q_0(x_i)}{\partial x} \ldots \frac{\partial Q_{N+1}(x_i)}{\partial x}\ \frac{\partial Q_{N+2}(x_i)}{\partial x}\right]^T$. The weighting coefficients a_{ij} corresponding to i-th grid point can be determined by solving algebraic system of Eqs. (12). This system has $(N+8)$ unknowns in $(N+4)$ equations. We require four additional equations for a unique solution. Adding equations

$$\begin{aligned} \frac{\partial^2 Q_{-1}(x_i)}{\partial x^2} &= \sum_{j=-3}^{1} a_{ij}^{(1)} Q_{-1}(x_j) \\ \frac{\partial^2 Q_0(x_i)}{\partial x^2} &= \sum_{j=-2}^{2} a_{ij}^{(1)} Q_0(x_j) \\ \frac{\partial^2 Q_{N+1}(x_i)}{\partial x^2} &= \sum_{j=N-1}^{N+3} a_{ij}^{(1)} Q_{N+1}(x_j) \\ \frac{\partial^2 Q_{N+2}(x_i)}{\partial x^2} &= \sum_{j=N}^{N+4} a_{ij}^{(1)} Q_{N+2}(x_j) \end{aligned} \tag{11}$$

to the system (12) becomes $L_1 A_1 = \phi_2$, where

$$L_1 = \begin{bmatrix} Q_{-1,-3} & Q_{-1,-2} & Q_{-1,-1} & Q_{-1,0} & Q_{-1,1} & & & & & \\ Q'_{-1,-3} & Q'_{-1,-2} & Q'_{-1,-1} & Q'_{-1,0} & Q'_{-1,1} & & & & & \\ & Q_{0,-2} & Q_{0,-1} & Q_{0,0} & Q_{0,1} & Q_{0,2} & & & & \\ & Q'_{0,-2} & Q'_{0,-1} & Q'_{0,0} & Q'_{0,1} & Q'_{0,2} & & & & \\ & & \cdots & \cdots & \cdots & \cdots & \cdots & & & \\ & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & & & Q_{N+1,N+2} & Q_{N+1,N} & Q_{N+1,N+1} & Q_{N+1,N+2} & Q_{N+1,N+3} & \\ & & & & Q'_{N+1,N+2} & Q'_{N+1,N} & Q'_{N+1,N+1} & Q'_{N+1,N+2} & Q'_{N+1,N+3} & \\ & & & & & Q_{N+2,N} & Q_{N+2,N+1} & Q_{N+2,N+2} & Q_{N+2,N+3} & Q_{N+2,N+4} \\ & & & & & Q'_{N+2,N} & Q'_{N+2,N+1} & Q'_{N+2,N+2} & Q'_{N+2,N+3} & Q'_{N+2,N+4} \end{bmatrix} \tag{12}$$

where $A_1 = \left[a_{i,-3}^{(1)} a_{i,-2}^{(1)} \ldots a_{i,N+3}^{(1)} a_{i,N+4}^{(1)}\right]$ and $\phi_2 = \left[\frac{\partial Q_{-1}(x_i)}{\partial x} \frac{\partial^2 Q_{-1}(x_i)}{\partial x^2} \frac{\partial Q_0(x_i)}{\partial x} \frac{\partial^2 Q_0(x_i)}{\partial x^2} \ldots \frac{\partial Q_{N+1}(x_i)}{\partial x} \frac{\partial^2 Q_{N+1}(x_i)}{\partial x^2} \frac{\partial Q_{N+2}(x_i)}{\partial x} \frac{\partial^2 Q_{N+2}(x_i)}{\partial x^2}\right]^T$ using the values of quintic B-splines and its derivatives at the nodal points and eliminating $a_{i,-3}^{(1)}, a_{i,-2}^{(1)}, a_{i,N+3}^{(1)}$, and $a_{i,N+4}^{(1)}$, we obtain an algebraic system having a 5-banded coefficient matrix of the form $L_2 A_2 = \phi_3$ where

$$L_2 = \begin{bmatrix} 37 & 82 & 21 & & & & & \\ 8 & 33 & 18 & 1 & & & & \\ 1 & 26 & 66 & 26 & 1 & & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ & & 1 & 26 & 66 & 26 & 1 & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & & 1 & 18 & 33 & 8 \\ & & & & & 21 & 82 & 37 \end{bmatrix} \tag{13}$$

$\phi_3 = [\varphi_{-1}\varphi_0, \ldots, \varphi_{N+1}\varphi_{N+2}]$ where

$$\varphi_{-1}(x_i) = \frac{1}{30}\left[-5Q_{-1}^{(1)}(x_i) + hQ_{-1}^{(2)}(x_i) + 40Q_0^{(1)}(x_i) + 8hQ_0^{(2)}(x_i)\right],$$

$$\varphi_0(x_i) = \frac{1}{10}\left[5Q_0^{(1)}(x_i) - hQ_0^{(2)}(x_i)\right],$$

$$\varphi_j(x_i) = Q_j^{(1)}(x_i), \quad for \quad j = 2, 3, \ldots, N-1, N,$$

$$\varphi_{N+1}(x_i) = \frac{1}{10}\left[5Q_{N+1}^{(1)}(x_i) + hQ_{N+1}^{(2)}(x_i)\right]$$

$$\varphi_{N+2}(x_i) = \frac{-1}{30}\left[-40Q_{N+1}^{(1)}(x_i) + 8hQ_{N+1}^{(2)}(x_i) + 5Q_{N+2}^{(1)}(x_i) + hQ_{N+2}^{(2)}(x_i)\right],$$

The system $L_2 A_2 = \phi_3$ is solved by penta-diagonal Thomas algorithm. Solving for $a_{i,-1}^{(1)}, a_{i,0}^{(1)}, \ldots, a_{i,N+1}^{(1)}, a_{i,N+2}^{(1)}$ and putting

Table 1 $Q_m(x)$ and derivatives on nodes.

x	x_{m-3}	x_{m-2}	x_{m-1}	x_m	x_{m+1}	x_{m+2}	x_{m+3}
$Q_m(x)$	0	1	26	66	26	1	0
$Q'_m(x)$	0	$5/h$	50/h	0	-50/h	$5/h$	0
$Q''_m(x)$	0	$20/h^2$	$40/h^2$	$-120/h^2$	$40/h^2$	$20/h^2$	0
$Q'''_m(x)$	0	$60/h^3$	$-120/h^3$	0	$120/h^3$	$-60/h^3$	0
$Q''''_m(x)$	0	$120/h^4$	$-480/h^4$	$720/h^4$	$-480/h^4$	$120/h^4$	0
$Q'''''_m(x)$	0	$120/h^5$	$-600/h^5$	$1200/h^5$	$600/h^5$	$-120/h^5$	0

these values in Eq. (6), we get the approximation for partial derivatives of first order at i-th grid point, for $i = 1, \ldots, N$.

Using Shu's recurrence formula to calculate $a_{ij}^{(2)}, a_{ij}^{(3)}$ and $a_{ij}^{(4)}$ and using them to approximate partial derivatives of higher order.

3. Discretization and solution of extended Fisher-Kolmogorov equation

The extended Fisher-Kolmogorov equation is given by

$$v_t = -\gamma v_{xxxx} + v_{xx} - (v^3 - v), x \in [p, q], \quad t \in [0, \tau], \tag{14}$$

with the boundary conditions

$$v(p, t) = v_0, \qquad v(q, t) = v_1 \tag{15}$$

$$v_{xx}(p, t) = 0, \qquad v_{xx}(q, t) = 0 \tag{16}$$

and initial condition

$$v(x, 0) = v_i, \qquad x \in [p, q] \tag{17}$$

Then the derivative approximations provided in Eq. (6) have been used in Eq. (14). The application of boundary condition results in

$$\frac{dv(x_i)}{dt} = -\gamma \sum_{j=1}^{N} a_{ij}^{(4)} v(x_j, t) + \sum_{j=1}^{N} a_{ij}^{(2)} v(x_j, t) - v(x_i, t)^3 + v(x_i, t),$$
$$i = 1, 2, \ldots, N \tag{18}$$

with boundary conditions

$$v(x_1, t) = v_0, \qquad v(x_N, t) = v_1$$

$$v_{xx}(x_1, t) = 0, \qquad v_{xx}(x_N, t) = 0 \tag{19}$$

and initial condition

$$v(x, 0) = v_i, \qquad x \in [p, q] \tag{20}$$

Then this system of ODEs in Eq. (18) is integrated in time employing an appropriate method. Here, we have used strong stability preserving fourth order Runge–Kutta method of stage three (SSP-RK43) for its advantages such as correctness, memory requirements and stability.

4. Stability analysis

Von Neumann stability analysis can be easily done when one is working with the classical finite difference. As with the case here, Von Neumann stability criterion cannot be used with differential quadrature method discretized systems. For this purpose, matrix stability or energy stability methods have been extensively discussed in the literature [25,36,37]. Let us consider a problem that is time-dependent along with a set of suitable initial and boundary conditions, given as

$$\frac{\partial V}{\partial t} = l(V) \tag{21}$$

Here, l is the operator in space. Discretizing Eq. (21), we attain a system of ODEs:

$$\frac{dv}{dt} = [D_c]v + \bar{b} \tag{22}$$

where v represents unknown functional at knots except at boundaries, D_c denotes the coefficient matrix and $\bar{b}$ is the non-homogeneous part. Stability conditions of the computational method for solving Eq. (21) are dependent on the stability of ODEs in Eq. (22). The matrix D_c plays an inevitable part as its eigenvalues directly determine the exact solutions. Therefore, the whole scenario can be well described using the eigenvalues of the coefficient matrix. When the real part of all the eigenvalues of D_c, $R(\lambda_i) \leqslant 0$, it shows the stable exact solutions as $t \to \infty$. The matrix D_c can be determined as

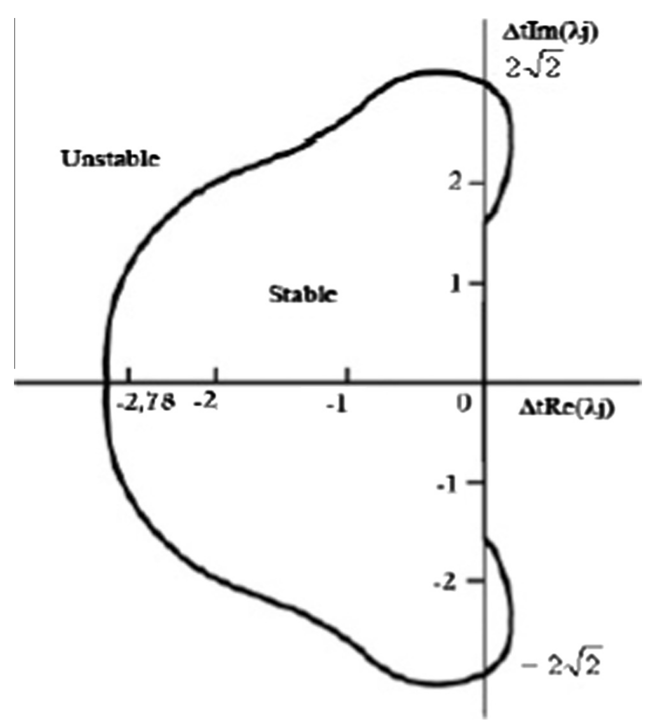


Figure 1 Stability region.

$[D_{ij}] = -\gamma a_{ij}^{(4)} + a_{ij}^{(2)} + 1$. The stable solution for t tending to infinity requires

1. For real eigenvalues, $-2.78 < \Delta t \cdot \lambda_i < 0$.
2. For eigenvalues having complex part only, $-2\sqrt{2} < \Delta t \cdot \lambda_i < 2\sqrt{2}$.
3. For complex eigenvalues, $\Delta t \cdot \lambda_i$ should lie in the sector shown in Fig. 1.

For complex eigenvalues, real parts can be small enough positive numbers that lie within a prescribed tolerance limit. [38]. For linearization, we have assumed $\alpha_i = v(x_i), \alpha_i$ constant; therefore, the eigenvalues need to be calculated for every problems. As it will induce changes in the entries of the coefficient matrix D. Eigenvalues have been calculated for different grid sizes and have been found in region in Fig. 1.

The convergence rate of the method adopted above is to be calculated. For this purpose, the formula given below has been used to compute the order of convergence

$$Order = \frac{log(E(N_2)/E(N_1))}{log(N_1/N_2)}$$

where $E(N_1)$ is the error and N_1 is the count of partitions. The order of convergence has been calculated with both error norms L_2, L_∞.

$$L_2(u) = \sqrt{\sum_{i=1}^{i=N} |u(x_i,t) - U(x_i,t)|^2}$$

$$L_\infty = \max_m |u_m^{exact} - u_m^{num}|$$

To handle non-availability of exact solutions, the computed solution is compared with solutions obtained with $N = 160$ at different intervals taking it as definite solution.

5. Numerical illustrations

To verify the correctness of the presented scheme, three numerical problems are presented.

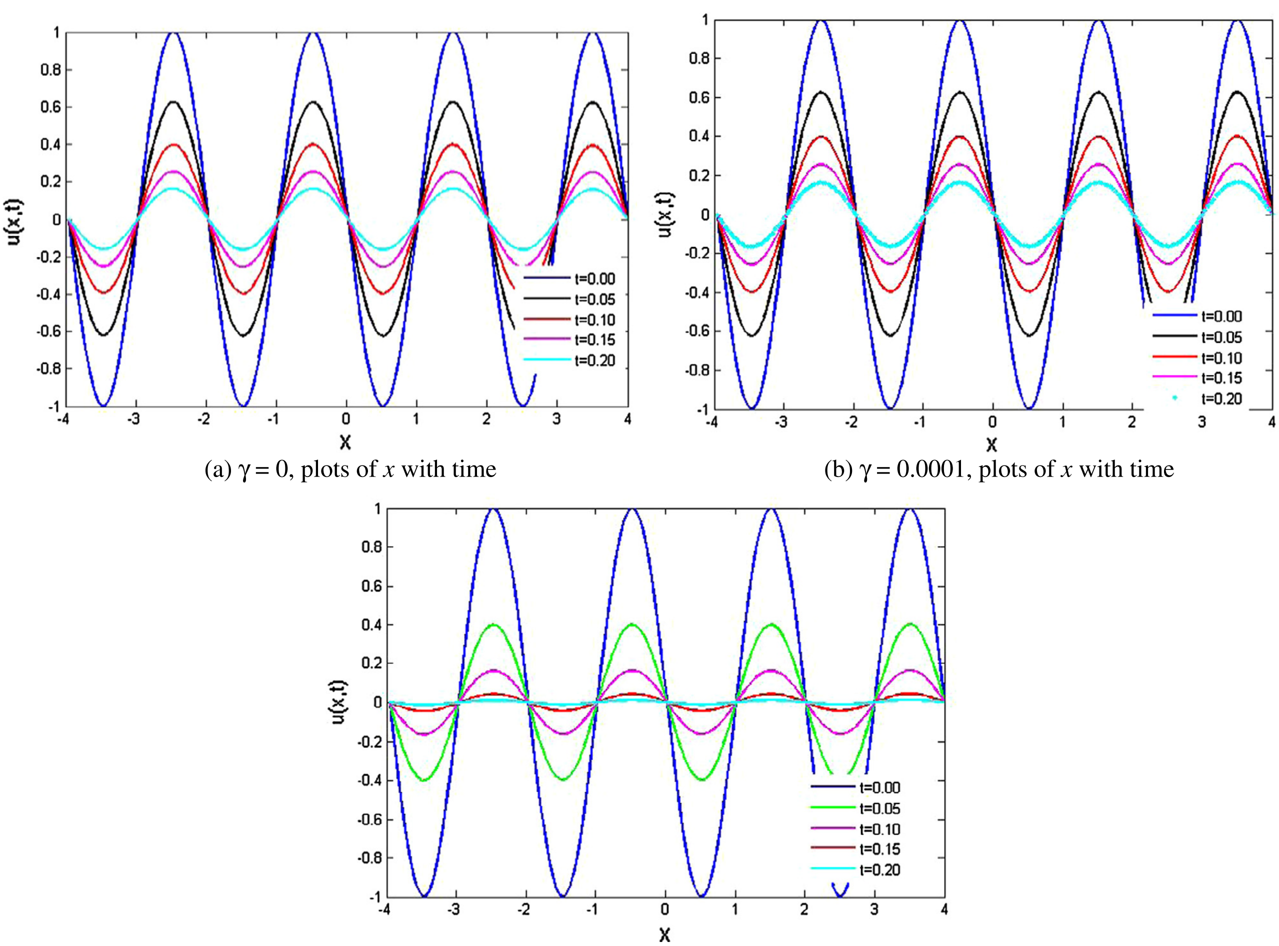
Problem 1. [11,13] Consider extended Fisher-Kolmogorov Eq. (1) in the domain [−4, 4] with initial conditions given as

$$v(x,0) = -\sin \pi x, \qquad x \in [p,q] \tag{23}$$

condition on the boundaries are given as

$$v(-4,t) = 0, \qquad v(4,t) = 0,$$

$$v_{xx}(-4,t) = 0, \qquad v_{xx}(4,t) = 0, \tag{24}$$



(a) $\gamma = 0$, plots of x with time

(b) $\gamma = 0.0001$, plots of x with time

(c) $\gamma = 0.1$, plots of x with time

Figure 2 Plots for u for varying values of γ.

Fig. 2 depicts numerical solutions at various time levels with $\Delta t = 1/1000$, for $\gamma = 0, \gamma = 0.0001$ and $\gamma = 0.1$ respectively. Uniform grid has been taken with $N = 80$. For $\gamma = 0$ and $\gamma = 0.0001$, the nature of solution is almost likewise. For $\gamma = 0.1$, solutions decay to 0 very quickly which ensures the stabilizing nature of the extended Fisher-Kolmogorov equation. The numerical solutions imitate the qualitative behavior of extended Fisher-Kolmogorov equation very well enough. Table 2 shows the L_2 and L_∞ norms and the convergence rate at $t = 0.2$. γ is taken to be 0.1, with $N = 20, 40$, and 80. The Table 3 clarifies the third order convergence of the scheme.

Problem 2. [11,13] Consider extended Fisher-Kolmogorov equation in Eq. (1) in the domain $[a, b] = [-4, 4]$ with boundary conditions

$$v(-4,t) = 1, \qquad v(4,t) = 1,$$

$$v_{xx}(-4,t) = 0, \qquad v_{xx}(4,t) = 0, \tag{25}$$

with initial conditions

$$v(x,0) = 10^{-3}\exp(-x^2), \qquad x \in [p,q] \tag{26}$$

Here, the numerical approximations have been calculated with $\gamma = 0.0001$. Fig. 3 depicts numerical solutions at various time levels with $\Delta t = 1/1000$. It is noticed that as time increases, the solution decays and attains a stable state approaching the value 1.

Problem 3. [11,13] The extended Fisher-Kolmogorov Eq. (1) with $\gamma = 0.0001, \Delta t = 0.001$, grid size $N = 100$ and initial conditions

$$v(x,0) = -10^{-3}\exp(-x^2), \quad -4 \leqslant x \leqslant 4, \tag{27}$$

with the conditions on boundary as

$$v(-4,t) = -1, \quad v(4,t) = -1. \tag{28}$$

Fig. 4 depicts the numerical approximation at varying times. As t increases, the solution decays and attains a stable state approaching the value -1.

Table 2 The order of convergence for $u(x,t)$ in Problem 1 at $t = 0.2$.

N	L_2	O	L_∞	O
20	2.1347E−002	–	1.1547E−003	–
40	2.2159E−003	2.91	1.2239E−003	3.01
80	3.12301E−004	2.82	1.5313E−004	2.93

Table 3 The order of convergence for $u(x,t)$ in Problem 1 at $t = 0.2$ as in [13].

N	L_2	O	L_∞	O
20	1.1158E−002	–	5.5097E−003	–
40	2.81459E−003	2.04	1.3387E−003	2.04
80	5.6571E−004	2.31	2.8340E−004	2.23

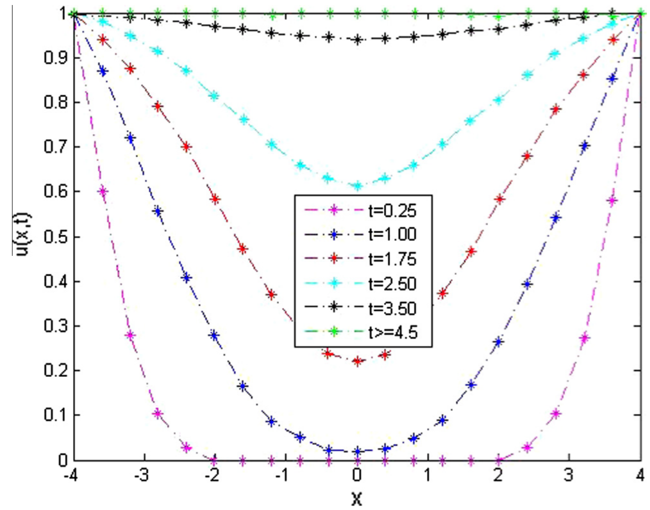


Figure 3 $\gamma = 0.0001$, plots of x with time for Problem 2.

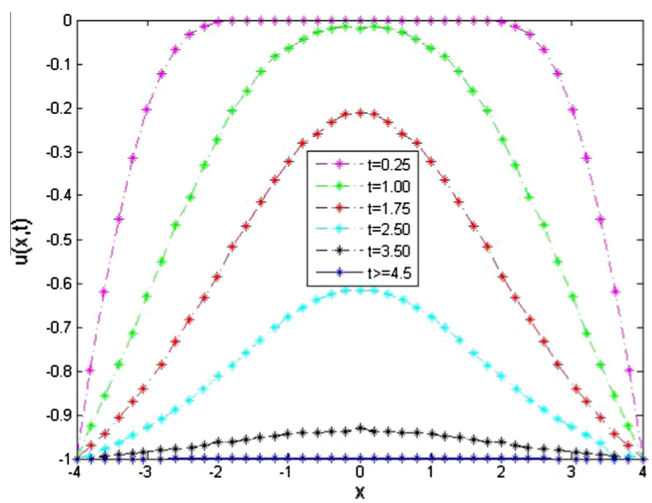


Figure 4 $\gamma = 0.0001$, plots of x with time for Problem 3.

6. Summary

In the present manuscript, a numerical treatment for the semi-linear extended Fisher-Kolmogorov equation is presented using a DQ method taking basis of fifth degree B-spline functions. The applicability of quintic B-spline method employed in the present work is straight forward and very simple. The algorithm can handle a diverse class of linear equations and can also handle nonlinearity of the problem. The solutions attained are depicted graphically at discrete levels of time that present alike features as provided in the literature. Rate of convergence is calculated and compared. L_2 and L_∞ error norms are also calculated for comparison purposes. It is seen that the numerical approximations are in excellent compliance with the solutions available in the literature. The stability analysis using matrix method has also been done. The method is shown to be as unconditionally stable for any step-length.

Acknowledgment

The author Ms. Sumita Dahiya would like to thank Ministry of Human Resource and Development with Grant No. MHR-02-23-200-429 for their funding.

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