Estimating Air Shower Fluctuations from the Monte Carlo Simulation Code CORSIKA

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Abstract

The study of cosmic ray extensive air showers (EAS) offers a unique way to estimate hadronic interaction cross-section at energies beyond the reach of accelerator experiments. Usually the hadron-air cross-section is estimated either by evaluating the absorption of the hadron initiated EAS flux penetrating the atmosphere or from the distribution of shower maximum. Employing a detailed Monte Carlo (MC) simulation code, the present work critically examined the analysis method usually adopted in obtaining cross-section through attenuation of EAS and estimated the degree of air shower fluctuations affecting the measurements of hadron-air cross sections.

1. Introduction

The study of cosmic ray (CR) extensive air showers (EASs) offers a unique opportunity to extract p-air interaction cross-section at energies well beyond the LHC region at CERN [1]. Generally, EAS experiments map the depth of first interaction of the CR particle on an average basis by exploiting its relationship to some air shower observables. The absorption of EAS flux having the same kind of CR particles, the same primary energy and stage of their development at different atmospheric depths is linked with the average mean free path of shower induced particle and this property is often exploited to estimate proton-air cross-section [2]. While estimating p-air cross section from experimental data, the selection of showers of constant energy and stage is usually done by considering simultaneously constant muon size and shower size ranges of small widths at the observation level whereas to ensure the primaries of the selected showers are of the same type, additionally the magnitude of shower size is chosen suitably. This technique is usually called the constant $N_e - N\mu$ method which is being applied in arrays of particle detector experiments. The
stated method of estimating hadronic cross section is, however affected mainly by the intrinsic shower fluctuations in the EAS longitudinal development [3]. Besides, the method is found to depend strongly on the choice of high energy interaction models [3]. To disentangle these fluctuations from those of the first interaction point is not an easy task.

The rate of shower $f(\theta)$ of a given primary energy interval selected through their muon size bin and shower size bin corresponding to the maximum development (choosing a common tail from the frequency distribution of the shower size for different zenith angles) is expected to attenuate with increasing atmospheric depth ($sec \theta$). Utilizing this fact the experimental absorption length $\Lambda_{obs}$, is obtained from the following equation:

$$ f(\theta) = G(\theta)f(0)exp\left[\frac{-X_0}{\Lambda_{obs}}(sec \theta - 1)\right] \tag{1} $$

In equation (1), $X_0$ denotes the vertical atmospheric depth of the location while $G(\theta)$ measures its geometrical acceptance in a particular angular bin. The absorption length of EAS ($\Lambda_{obs}$) and the interaction mean free path ($\lambda_{obs}$) are, however, not the same but Monte Carlo simulation results suggest that they are connected through the relation [4].

$$ \Lambda_{obs} = k \times \lambda_{obs} \tag{2} $$

where $k$ is a constant parameter; the numerical value of $k$ depends on the primary energy range, observation level of the experiment and few other factors and has to be obtained from Monte Carlo simulations for a particular EAS observation. The common perception is that the deviation of $k$ from unity is originated mainly by fluctuations in air shower development. Besides, different features of the hadronic interaction models and the rate of EAS development also could be partly responsible for ($k \neq 1$). The purpose of the present work is to critically examine whether fluctuations in air shower development is the prime cause for the deviation of the value of $k$ from unity or not. For this purpose a detailed Monte Carlo simulation study has been conducted.

2. The simulation

The computer simulations are performed utilizing the CORSiKA, a M C code to simulate air showers [5] with QGSJET 01 v 1c [6] high energy hadronic interaction model. Hadrons with energies below 80 GeV/n are treated with the GHE ISHA 2002d [7] interaction model. The shower size ($N_e$) is obtained directly by adding all $e^+$ and $e^-$ from the produced $\gamma$-rays involving electromagnetic interactions with the EGS4 option [8]. The zenith angle was restricted to events with $\leq 50^\circ$. The EAS events were generated mainly for proton and iron nuclei as primaries. Showers have been simulated with a particular energy of $2 \times 10^{14}$ eV and spectral index $-2.7$. The EAS events were simulated at the geographical position to the experimental site of ARGO-YBJ (latitude $30.11^\circ$N, longitude $90.53^\circ$E and $607gm.cm^{-2}$) [9]. About $2 \times 10^4$ EAS events were generated for each primary particle running the M C simulation code.

3. The analysis method

Most of the observables required for the analysis are obtained directly from the simulated data. The absorption length $\Lambda_{sim}$, evaluated from Eqn.(1) is supposed to be combined the effects of the first interaction length $\Lambda_{sim}$ and fluctuations of the EAS development. In experimental evaluation of cross section by air shower method a fixed primary energy is usually chosen by selecting EAS of same muon size. Since the prime motivation of the work is to judge the effect of air shower fluctuations on measurement of cross sections, a large number of EAS events were generated at a fixed primary energy. While taking a fixed shower size bin one may choose showers of the same depth between the first interaction point and the observation level, under the assumption of no fluctuations in the successive interactions [10]. The shower size bin is selected from the tail portion of the shower size distributions corresponding to different small zenith angle bins ($\Delta \theta = 3^\circ - 7^\circ, 8^\circ - 12^\circ$ etc.) at a particular primary energy. Finally measuring the attenuation of frequencies (event rate) of showers with fixed energy and shower size at different atmospheric depths ($sec \theta$), the absorption length $\Lambda_{sim}$ has been measured. The detection efficiencies at different inclinations have to be taken carefully for estimating $\Lambda_{sim}$. Due to the imposed full detection efficiency for simulated data, the geometrical
acceptance in each individual $sec \theta$ bin is simply given by $\Delta sec \theta = sec70^0 - sec30^0$, $sec120^0 - sec80^0$, ... etc. and has been taken carefully for estimating $\Lambda_{sim}$.

The Monte Carlo simulation directly gives the value of the hadronic mean free path $\lambda_{sim}$ for each air shower event which has been noted and a frequency distribution of directly observed $\lambda_{sim}$ for a common shower size bin is formed. The mean value of $\lambda_{sim}$ is obtained from the Gaussian fit of the $\lambda_{sim}$ distribution. The $k$-parameter is finally obtained by taking the ratio of $\Lambda_{sim}$ to $\lambda_{sim}$.

In order to compare the effects of the intrinsic fluctuations on $k$ with other EAS observables sensitive to air shower fluctuations we have considered the depth of shower maximum and EAS slope parameter i.e. LAP [11].

The depth of shower maximum is estimated for each EAS event from the longitudinal profile of EAS. The mean $LAP$, the depth of shower maximum and the $k$-parameter have been studied at a particular energy for both proton and iron induced showers.

4. Results & Discussion

4.1. The $k$-parameter for Proton & Iron initiated showers

The main objective of the work is to examine critically the physical meaning of the parameter $k$ related with EAS fluctuations. Simulated EAS events for p and Fe at several small zenith angle intervals are generated and study the shower size spectra at a fixed primary energy of $2 \times 10^{14}$ eV. The selection of showers are made in such a way that with selected shower which have $N_e$ between $7.5 \times 10^4$ and $1.15 \times 10^5$ for protons and between $3.5 \times 10^4$ and $5.5 \times 10^4$ for irons respectively. Such selections of $N_e$ are shown in Fig. 1.

Fig. 1. (a)-(b) Distributions of shower size for p and Fe induced showers at different mean zenith angles having energy $2 \times 10^{14}$ eV. The selected showers have $log_{10} N_e=4.87 - 5.06$ and $4.54 - 4.74$ for p and Fe respectively.

The whole selection procedure discussed is now applied to evaluate $\Lambda_{sim}$ and then $k$ for both the proton and iron showers. Event numbers as a function of $sec < \theta >$ after correction for the geometrical acceptance of each angular bin are shown in Fig. 3 for proton and iron.

The parameter $< \theta >$ denotes the mean zenith angle, obtained from each angular bin. It should be noted that for iron showers we have gone up to only $30^0$ for evaluating the attenuation length due to higher rate of absorption of EAS electrons with increasing depth. The mean depths of first hadronic interaction corresponding to $log_{10} N_e = 4.87 - 5.06$ and $4.54 - 4.74$ for p and Fe primaries could be obtained from the fitting of their frequency distributions and are shown in Fig. 2. The absorption lengths and resulting fluctuation parameters are summed up in Table 1. The given uncertainties are the statistical ones.

4.2. EAS fluctuations reflected in composition sensitive observables

The observable parameters the depth of shower maximum ($X_m$) and its fluctuations ($\sigma(X_m)$) were used widely for CR composition analyses in ultra-high energy region. The fluctuations are larger for proton than for iron initiated
Fig. 2. (a) Distributions of depth of first interaction for proton and (b) Same as figure 2a but for iron with different log$_{10}N_e$ bins. Gaussian fit (—fit) are made.

shower, as expected. Since $X_m$ distribution contains the imprint of the effect of first interaction ($\lambda_{int}$) distribution therefore fluctuations of $\lambda_{int}$ should be larger for proton than iron. Since all these effects in the initial stage of shower development have an impact on the later stage of EAS so the fluctuations in the slope of lateral electron distribution (in terms of LAP) would also obey the similar behaviour.

Fig. 3. (a) Distributions of sec$\theta$ for proton and (b) iron initiated showers including the correction for the geometrical acceptance in each angular bin with different log$_{10}N_e$ bin.

Fig. 4 shows the distributions of the event-by-event values of $\lambda_{int}$, $X_m$ and $s_{local}$ in simulated samples of proton and iron primaries at $2 \times 10^{14}$ eV. The fluctuations of proton and iron initiated showers are clearly visible in the above figures and these magnitudes are the expected ones in the concerned primary energy.

Meaningful information about EAS fluctuations can be extracted from the results of fits for the simulated samples of proton and iron initiated showers is shown in Table 2.

The magnitudes of fluctuations obtained from all standard measurements in this section clearly contradict the results obtained in the section A in terms of $k$ listed in Table 1 on EAS fluctuations.

Table 1. Summary of the results obtained from attenuation studies using constant $N_e - N_\mu$ method for proton and iron at energy $2 \times 10^{14}$.

<table>
<thead>
<tr>
<th>Primary</th>
<th>$\log_{10}N_e$</th>
<th>$\Lambda_{sim}$</th>
<th>$\lambda_{sim}$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>4.87 - 5.06</td>
<td>52.1 ± 3.9</td>
<td>39.84 ± 4.3</td>
<td>1.31 ± 0.04</td>
</tr>
<tr>
<td>Fe</td>
<td>4.54 - 4.74</td>
<td>19.1 ± 1.4</td>
<td>8.44 ± 0.87</td>
<td>2.26 ± 0.09</td>
</tr>
</tbody>
</table>
Fig. 4. (a) Distributions of depth of first interaction; (b) Depth of shower maximum and (c) Local age for proton and iron primaries at $2 \times 10^{14}$ eV.

Table 2. The different columns show the results of the fits to the average distributions constructed from the simulated events. The widths in all three parameters are larger for proton than iron showers.

<table>
<thead>
<tr>
<th>Primary</th>
<th>$\Delta \lambda_{\text{int}}$</th>
<th>$\Delta X_{\text{m}}$</th>
<th>$\Delta s_{\text{local}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>42.5</td>
<td>139.0</td>
<td>0.15</td>
</tr>
<tr>
<td>Fe</td>
<td>9.10</td>
<td>60.7</td>
<td>0.12</td>
</tr>
</tbody>
</table>

5. Discussion & conclusions

The absorption lengths of proton and iron induced showers at their maximum development in air are estimated at a fixed energy $2 \times 10^{14}$ eV at the ARGO-YBJ level from Monte Carlo simulation results. The technique involving absorption of CR events for a fixed primary energy has been applied for estimating $\Lambda_{\text{sim}}$. The mean free path $\lambda_{\text{sim}}$ is related to $\Lambda_{\text{sim}}$ via the $k$-factor as given in Eqn. (2) for simulated data. Finally the $k$ parameters for both proton and Fe initiated showers have been evaluated. While measuring cross section by absorption technique muon size is usually considered in EAS experiments to fix the primary energy. In the present study a fixed primary energy was used for simulation. Thereby the results are not affected by the uncertainty of energy estimation.

It is found that the mean value of $k$ for iron induced showers exceeds that for the proton initiated showers. The deviation of $k$ from unity is believed to be mainly due to the fluctuations in cascade development. However, in such a case $k$ for iron induced showers is expected to be smaller than that for proton showers. Note that the fluctuation in the first interaction length is the main source of fluctuation which is higher in proton shower. In the absorption length technique of measuring cross section we essentially study the distribution of first interaction length and hence the technique is unaffected by the fluctuation of the first interaction length. In the development of air shower after the first interaction, the fluctuations of initial few stages are also important. This is because the leading particle carries
about 50% of interaction energy. Hence the effect of fluctuation in proton initiated air showers should be higher than in a Fe induced shower (which has much more secondaries and hence the net effect of fluctuation is reduced when sum of the fluctuations of all secondaries are taken into account) even when cascade development after the first interaction is considered. Such a feature is revealed from the fluctuations of shower age as shown in the last one of Fig. 4. So if fluctuation is the main cause of non-unity of $k$, the parameter should be smaller for Fe shower than proton shower which is in contradiction to our findings. Hence the physical meaning usually attached to the $k$-parameter is questionable.

Usually $k$ is assumed from an EAS model, which biases the final results for all the experimental cross-sections. The characteristics of hadronic interactions may be responsible for the deviation of $k$ from unity. The parameter $k$ strongly depends on the average in-elasticity and in-elasticity-distribution of p and $\pi$ reactions. Particularly on which other important features of hadronic interactions are responsible for the $k$-factor will be a subject of future investigation. In addition, a systematic study of the energy dependence of the $k$-factor is also needed.

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References