
Electron-electron interactions in highly doped heterojunction

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Abstract

We report results from calculations of temperature-dependent intra and intersubband electron-electron scattering rates in two subbands in a two-dimensional (2D) quantum structure in Random Phase Approximations (RPA). Electron-electron interactions in a single highly doped heterojunction are considered taking into account both intra- and intersubband transitions. Expressions are derived for the time of electron-electron interaction, matrix elements of the full screening potential and dynamic dielectric function in a 2D electron system with the fine structure of the energy spectrum, and for the electron density spatial distribution. The theoretical dependences provide a good description of the experimental times of Landau levels collisional broadening.

Keywords: electron-electron interactions; random-phase approximations; screening potential

1. Introduction

Starting from the pioneering works by Pines and Nozieres (1966) and up to the present, Kadushkin et al. (2004), Kadushkin et al. (2003), Dubois et al. (2013), Yatke et al. (2010), Glazov et al. (2010), Hatke et al. (2009), Raichev (2010), Sica et al. (2001), Goh et al. (2008), Averkiev et al. (2003), Dubois (2010), Ambartsumyan et al. (2014), the

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electron-electron (e-e) interactions are the subject of ever growing interest because of their fundamental role in kinetic phenomena. Among others, one should note the hot electrons effects, quantum corrections to the conductivity, and damping (destruction) of Landau quantization in bulk and two-dimensional semiconductors with degenerate electrons. Also known the anomalies in the low-temperature magnetotransport arising when 2D electrons fill several size-quantized subbands. When the doping level of the AlGaAs/GaAs heterojunction is high enough for \( n_s \) to reach \( 8 \times 10^{11} \text{ cm}^{-2} \), the quantum well contains two size-quantized subbands with energies \( E_m \) and \( E_p \), electron densities \( n_m \) and \( n_p \), respectively.

In the present paper we report the results of the study of the e-e relaxation processes in a system of highly degenerate 2D electrons with the fine structure of the energy spectrum and the electron density spatial distribution. The expressions for the times of electron-electron intra- (\( \tau_{\text{ee, intra}} \)) and intersubband (\( \tau_{\text{ee, inter}} \)) interaction are derived and the matrix elements of the full screening potential \( \omega V_{\text{tot}}(q) \) and the dielectric function \( \chi(\omega,q) \) in the approximation far from the long wave limit are calculated. It is shown that the oscillations in \( \tau_{\text{ee, intra}}(T,n_s) = \tau_{\text{ee, inter}}(T,n_s) \) are related to the excitation of plasmons in the components of 2D electron system and the plasmon spectrum is studied.

2. Electron-electron interactions

Our analysis of e-e interaction is based on the calculation of the conduction band energy structure \( E_c(z) \). The bound state wave function \( \psi(z) \) is determined by the one-dimensional Schrodinger equation:

\[
\left(-\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V_{\text{ext}}(z)\right) \psi(z) = E_m(z) \psi(z)
\]

with boundary conditions \( \psi_j(0) = 0 \), \( \psi_j(\infty) = 0 \).

We assume that the external potential \( V_{\text{ext}}(z) \) appearing in Schrodinger equation satisfies Poisson equation:

\[
\frac{d^2 V_{\text{ext}}(z)}{dz^2} = -4\pi \rho(z) \quad \text{with} \quad V(z = \infty) = 0, \quad eV(z = 0) = E_F,
\]

where \( E_F \) – Fermi energy, charge density:

\[
\begin{cases}
\rho(z) = eN_0^{3/2} \left(1 - \frac{z}{D}\right), & z < D, \\
\rho(z) = -A(z-D) \exp\left(-\frac{(z-D)}{a}\right), & z > D.
\end{cases}
\]

So that \( \rho(z = D) = 0 \).

The constant \( A \) is determined by the condition that local charge was zero:

\[
Q = \int_0^\infty \rho(z) dz = 0.
\]

Similarly to Kadushkin et al. (2004), we approximate the potential well of the heterojunction by a triangular profile with the sharp bends at the size-quantized levels \( E_c = E_c(d_m) \) and \( E_c = E_c(d_p) \). We show, Kadushkin et al. (2003), Ambartsumyan et al. (2014), that intrasubband and intersubband e-e interactions are mediated by intrasubband interactions and the perturbation is extended to the entire 2D system. To within the second order in the external potential \( V_{\text{ext}}(z) \) in the perturbation theory expansion, the time required for the e-e interaction to change the state \( \langle k | p \rangle \) into \( \langle k + q | p - q \rangle \) is given by a well-known expression

\[
\frac{1}{\tau_{\text{ee, intra}}} = \int d\omega \sum_{k,m,q} \left| \omega V_{\text{tot}}^{(k\leftrightarrow q)}(\omega) \right|^2 \delta (E_i(k+q) + E_i(p-q) - E(k) - E_i(p)) f_k f_p \left(1 - f_{k+q}\right) \left(1 - f_{p-q}\right)
\]

We derive that in our heterostructure

\[
\frac{1}{\tau_{\text{ee, inter}}} = \int \frac{d\omega}{\pi \cosh^2(\hbar \omega/2k_B T)} \sum_{k,m,q} \left| \omega V_{\text{tot}}^{(k\leftrightarrow q)}(\omega) \right|^2 \chi_{ik}(\omega) \chi^*_{jk}(\omega),
\]

where \( \chi_{ik}(\omega) \) is a dielectric function, and
\[ V_{\text{tot}}^{\text{ijkl}}(q, \omega) = \lim_{\delta \to 0} \sum_{\Delta I} \frac{f_{k+q} - f_k}{E(k+q) - E(k) + \hbar \omega + i\delta} \]

is matrix element of Coulomb screening potential in random-phase approximation. In the form convenient for calculations the relaxation times for the intrasubband transitions are written as

\[ \frac{1}{\tau_{\text{intra}}} = \frac{E_j^2 m^2 S}{16 \pi^2 d_j^2 \hbar^2 n_j} P_{-n}(T) \zeta(T, n_i, n_j, d_i, d_j), \tag{3} \]

while for the intersubband transitions

\[ \frac{1}{\tau_{\text{inter}}} = \frac{m^2 S}{(d_i - d_m)^2 \hbar^4} \left\{ \left( \frac{E_i - E_m}{\sqrt{8\pi}} \right) Q_{-n}(T) \zeta(T) - \frac{(E_i - E_m)(E_i d_m + E_m d_i)}{\sqrt{8\pi}} W_{-n}(T) \zeta(T) \right\}, \tag{4} \]

for \( i \neq l \). The polynomials in (3) and (4) are

\[ P_{-n}(T) = \frac{B_0}{T^2} + \frac{B_2}{T^4} + \ldots, Q_{-n}(T) = \frac{B_1}{T} - \frac{B_3}{T^3} - \ldots, W_{-n}(T) = \frac{B_3}{T^3} + \frac{B_5}{T^5} + \ldots \]

with the coefficients B defined by the Riemann zeta-function \( \zeta(T, n_i, n_j, d_i, d_j) \). A non-monotonous behavior of \( \tau_{\text{inter}}^i(T) \) is determined by the uniformly converging sums \( P_{-n}(T), Q_{-n}(T) \) and \( W_{-n}(T) \) multiplied by the zeta- and gamma-functions, Dubois et al. (2013)

\[ \zeta(T, n_i, n_j, d_i, d_j) = \frac{d_m}{\Gamma(d_m + d_p)} \int_0^{\frac{d_m}{\hbar^2 \pi \tau_m}} e^{-\frac{d_m}{\hbar^2 \pi \tau_m} \left( \frac{n_i n_j}{n_m + n_i + n_j} \right)^{1/2}} \frac{1 - e^{-t}}{1 - e^{-t}} \, dt. \]

Figures 1a and 1b show the experimental and calculated curves for the time of destruction of Landau quantization for two heterostructures with electron density sufficient for filling of two size-quantized subbands (see Kadushkin et al. (20034), (2004) for the details of the analysis of the experiment).

The products of \( P_{-n}(T), Q_{-n}(T) \) and \( W_{-n}(T) \) with \( \zeta(T, n_i, n_j, d_i, d_j) \) are rather sensitive to the electron concentration in the size-quantized subbands. For example, for \( n_m > 8 \times 10^{11} \text{cm}^{-2} \) (\( n_i = 0.1 n_m, n_j = 0.1 n_m, d_i / d_m = 3.5 \)) the factor \( \zeta(T, n_i, n_j, d_i, d_j) \) in (10) results only in some smoothening of the non-monotonous behavior while at \( n_m > 8 \times 10^{11} \text{cm}^{-2} \) the curve \( \tau_{\text{inter}}^i(T) \) does not contain any non-monotonous parts at all.

The calculations of \( \tau_{\text{intra}}^i \) were performed within the outlined schematic model of the Landau quantization destruction taking into account the paths corresponding to the channels (1-3) including both intra- and intersubband transitions according to (8) and (9) employing the Matthiessen rule \( \tau_{\text{intra}}^i = \sum_i \tau_{\text{inter}}^i \), where the summation is performed over all intra- and intersubband components.

![Fig. 1. Comparison of the experimental \( \tau_{\text{exp}}^i(T) \), Kadushkin et al. (2004), (a) and calculated \( \tau_{\text{calc}}^i(T) \) (b) curves for different values of the \( m \) subband electron density: 1 - 9.1; 2 - 10.0.](image-url)
Bearing in mind the oscillations in $\tau^{\text{exp}}_e$ and $\tau^{\text{th}}_e$, it is natural to expect the resonant response of the components of a complex 2D electron system to the external perturbation $V_{\text{ext}}(\mathbf{q}, \omega)$ at the plasma oscillations frequency. The 2D system responds to the spectrum of $V_{\text{ext}}(\mathbf{q}, \omega)$ by one of its $n_m$, $n_n$, $n_d$ components (or their combination), and time $\tau_v(\tau^{\text{exp}}_v, \tau^{\text{th}}_v)$ the perturbation extends to the entire system resulting in the destruction of quantum states (cyclotron orbits) which is experimentally observed as the reduction of the $\delta(1/B)_T$ oscillations amplitude. The latter is formally an equivalent to the rise of temperature $T$ at which the measurements are taken. Therefore the resonant response featuring the Landau quantization destruction corresponds to a minimum in the curves $\tau^{\text{exp}}_e$ and $\tau^{\text{th}}_e$. We have performed a spectral analysis of the dispersion equations for $\chi(\mathbf{q}, \omega)$ various channels of e-e interactions according to the scheme shown in Fig. 2 and various relative values of the densities $n_m$, $n_n$, $n_d$ in the situation where two size-quantized subbands are occupied ($n_m > 8 \cdot 10^4 \text{cm}^{-2}$). The plasma oscillation frequencies $\omega_v$ are found from the dispersion equation $\chi(\mathbf{q}, \omega) = 0$, the minima in $\tau_v(T)$ corresponding to the minima in $\text{Re} \chi(\mathbf{q}, \omega)$ and $\text{Im} \chi(\mathbf{q}, \omega) = 0$ while the maxima in $\tau_v(T)$ correspond to maxima in $\text{Re} \chi(\mathbf{q}, \omega)$ and $\text{Im} \chi(\mathbf{q}, \omega)$.

To calculate $\text{Re} \chi(\mathbf{q}, \omega)$ and $\text{Im} \chi(\mathbf{q}, \omega)$, the expression should be cast into the form of function $\chi(\omega)$:

$$
\chi_a(\omega) = \frac{1}{\gamma} \sum_q \chi_a(q, \omega).
$$

(5)

\begin{align*}
\text{Fig. 2. Frequency dependence of the dielectric function for the interaction between the main size-quantized } m - \text{subband electrons with the } n - \text{satellite of the } p - \text{subband. } n_m, 10^2 \text{ cm}^{-2}: (1) - 8.5, (2) - 10, (3) - 11.5; d_p / d_m = 3.5.
\end{align*}

Taking into account the characteristic scales $q \in \{d_m^{-1}, d_p^{-1}\}$ of the 2D electron system the summation over $q$ results in the expression

$$
\chi_a(\omega) = f(n_1, \omega) + f(n_2, \omega),
$$

(6)

where

$$
f_i(n_i, \omega) = L \sqrt{n_i} \left\{ \sqrt{2\pi} - \left( \frac{\text{ms} \omega L}{h \sqrt{n_i \pi}} - \ln \left( \frac{\text{ms} \omega L}{h \sqrt{n_i \pi}} \right) + C - C_i \left( \frac{\text{ms} \omega L}{h \sqrt{n_i \pi}} \right) \right)^{1/2} \right\} + \text{c.c.}
$$

(7)

where $C$ is the Euler constant, $f(n_i, \omega)$ is given by (7) after the substitution of $n_i$ by $n_1$. The alternating sums over $k$ in (7) prove to be rapidly converging.

Figure 2 illustrate the partial contributions of various mechanisms to the Landau quantization destruction and the density-dependent singularities. For example, plotted in Fig. 2 are the frequency dependences $\text{Re} \chi(\mathbf{q}, \omega)$ and $\text{Im} \chi(\mathbf{q}, \omega)$ for $n_m = 10^2 \text{ cm}^{-2}$ for three intersubband transition channels. It is seen that the $n_m - n_n$ and $n_n - n_d$ interactions are dominating. Moreover, the resonant frequency is determined by the $n$-satellite density. Shown in Fig. 2 is the influence of the second size-quantized subband filling factor. An increase in the density $n_m$, (and hence $n_n$ and $n_d$) the results in the resonant frequency shift to higher values while the discontinuity in $\text{Re} \chi(\mathbf{q}, \omega)$ and
Im $\chi(q, \omega)$ is reduced which is consistent with the scenario of the schematic model presented by Ambartsumyan et al. (2014).

3. Conclusions

In conclusion, it should be noted that a similar problem for 2D electrons seems to have been first considered by Dubois (2010) and since then numerous attempts have been undertaken, Yatke et al. (2010), Glazov et al. (2010), Hatke et al. (2009), Raichev (2010), Sica et al. (2001), Goh et al. (2008), to study this problem for 2D electron system where several size-quantized subbands are filled at $T \neq 0$ in the long wavelength limit. However, the plasma oscillation spectrum has not been obtained in any of these works.

At present we derive the correct expression for the dynamic, temperature-dependent RPA dielectric function derived in the Boltzmann limit. For interactions involving intersubband transitions, we show nonmonotonic temperature-dependence of electron-electronic interactions.

Characteristic features of 2D electron systems, such as an amplitude-frequency modulation, beatings, and sharp bends in the oscillation amplitude magnetic field dependence make the description of Landau quantization damping in terms of the Dingle temperature rather problematic. Another point to be emphasized out is the fact that in the magnetic field range where a strong amplitude-frequency modulation takes place $p$ – subband electrons are in the state close to the quantum limit and one can only speak of the oscillations period in a rather limited sense.

Also by varying parameters one can obtain a satisfactory agreement with the experiment. This technique offers the possibility of recovering the actual potential profile from the superposition of the curves $\tau^\text{exp}_q$ and $\tau^\text{th}_m$ measured for the samples with different doping levels and therefore showing different variations of the form-factors $d_q / d_m$ and $E_q / E_m$. However, this matching of the calculated curves $\tau^\text{exp}_q$ with the experimental ones $\tau^\text{exp}_q$ is limited by a certain arbitrariness in the adjustable parameters $d_q / d_m$ and $E_q / E_m$ (the potential well form-factors) since the curve $E_s(z)$ cannot be derived with sufficient accuracy because of the uncertainties in $N_p$, $N_A$ and the band discontinuity $\Delta E_s / \Delta E_g$ for the AlGaAs/GaAs heterostructure, Kadushkin et al. (2003), (2004).

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References