Soils and Foundations 2012;52(1):99–113

The Japanese Geotechnical Society

Soils and Foundations

www.sciencedirect.com
journal homepage: www.elsevier.com/locate/sandf

Spatial-temporal prediction of secondary compression using random field theory

Pongwit Rungbanaphan[a,*], Yusuke Honjo[b], Ikumasa Yoshida[c]

[a]Shimizu Corporation, Civil Engineering Technology Division, Design Department, Seavans South, No.2-3, Shibaura 1-chome, Minato-ku, Tokyo 105-8007, Japan
[b]Gifu University, Gifu, Japan
[c]Tokyo City University, Tokyo, Japan

Available online 7 February 2012

Abstract

A methodology is presented for observation-based settlement predictions by considering the spatial correlation structure of soil. The spatial correlation is introduced among the settlement model parameters, and the settlements at various points are spatially correlated through these geotechnical parameters, which naturally describe the phenomenon. The method is based on Bayesian estimations, considering both prior information, including spatial correlation, and observed settlements, to search for the best estimates of the parameters. Within the Bayesian framework, the optimized selection of the auto-correlation distance, by Akaike's Bayesian Information Criterion (ABIC), and the spatial interpolation of the model parameters, by the kriging method, are also proposed. The application of the proposed approach in secondary compression settlement predictions, based on the linear relationship between settlement and the logarithm of time, is presented in this paper. Several case studies are carried out using both simulated settlement data and actual field observation data. It is concluded that the accuracy of settlement predictions can be improved by taking into account the spatial correlation structure, especially when the spacing of the observation points is shorter than half of the auto-correlation distance, and that the proposed approach produces rational predictions of settlements at any location and at any time with quantified errors.

Keywords: Secondary compression; Settlement prediction; Statistical analysis; Spatial correlation; Random field; Bayesian estimation

1. Introduction

Thus far, all methods for predicting future settlements using past observations have been based solely on the temporal dependence of their quantity. However, the fact that soil properties tend to exhibit a spatial correlation structure has been clearly shown in several past studies, e.g., Lumb (1974), Vanmarcke (1977), DeGroot and Baecher (1993), and Baecher and Christian (2003, 2008). Therefore, it is natural to expect that the accuracy of settlement predictions can be improved by taking into account the spatial correlation of the soil properties, by which the observed settlement data from all the different observation points can be rationally utilized. Furthermore, by introducing the spatial correlation, it is possible to predict ground settlements at any arbitrary point and at any time by considering the spatial–temporal structure. This study is actually an attempt to search for such an approach.

In order to include the spatial correlation in the settlement prediction model, the Bayesian approach is chosen

*Corresponding author. Tel.: +81 90 4213 2085.
E-mail addresses: pongwit@shimz.co.jp, pongwit_n@yahoo.com (P. Rungbanaphan).

Peer review under responsibility of The Japanese Geotechnical Society
doi:10.1016/j.sandf.2012.01.013

Production and hosting by Elsevier

Notation

c_1, c_2	constant parameters
I	total number of unknown parameters
$I_{n,n}$	$n \times n$ identity matrix
K	total number of observation time steps
k	observation time steps
M_k	coefficient matrix relating unknown parameters to observation data at observation time step k
m_0, m_1	model parameters for secondary compression model ($S \sim \log(t)$ method)
m_1^*, m_0^*	mean of m_1 and m_0 for simulation of model parameters
$m_{1,0}^*(x_i)$, $m_{0,0}^*(x_i)$	prior mean at observation point x_i of m_1 and m_0
N_{sim}	total number of simulations
N_x	total number of estimated values
n	total number of observation points
S	secondary compression settlement
$S(\omega_1,\omega_2)$	spectral density function
$S_k(x_i)$	observed secondary compression settlement at observation point x_i at observation time step k
s	spacing of observation points
t	time of compression ($S \sim \log(t)$ method)
t_k	observation time at observation time step k
V_C	auto-covariance matrix
V_ε	covariance matrix of observation error
V_ε'	matrix representing the relative error magnitude of each observation
V_θ	covariance matrix of unknown parameter
w_i	kriging weights attached to the data at each observation point
$X_{est,i}$, $X_{true,i}$	estimated value and true value
$\underline{x}_i$	spatial vector coordinates at observation point x_i
Y	set of all observation data
Y_k	observation vector at observation time step k
$y_k(x_i)$	observation data at an observation point x_i at observation time step k
$z_i^*(x_i)$	estimate of unknown parameter z_i at observation point x_j
δ	vector representing uncertainty of prior mean of unknown parameters
ε	observation error vector
η	auto-correlation distance
θ	unknown parameter vector
θ^*	Bayesian estimator of unknown parameter vector
θ_0	prior mean of unknown parameter vector
μ	Lagrange multiplier
$\rho(\lvert \underline{x}_i - \underline{x}_j \rvert)$	auto-correlation function
$\sigma_{m1}^2, \sigma_{m0}^2$	variance of m_1 and m_0 for simulation of model parameters
$\sigma_{m1,0}^2$, $\sigma_{m0,0}^2$	prior variance of m_1 and m_0
σ_{zi}^2	prior variance of unknown parameters z_i
σ_ε^2	variance of observation error
ϕ_{jk}	random phase angles, uniformly and independently distributed in interval $(0,2\pi)$
ω_1, ω_2	frequency domain
$0_{n,n}$	$n \times n$ zero matrix

for this research due to its ability to systematically combine the subjective information, i.e., prior information, including the spatial correlation, and objective information, namely the observation data. A general formulation is presented based on the Bayesian estimation concept to identify the best estimator of a stochastic Gaussian field of unknown parameters when the observation is made at discrete spatial points and at a discrete time by considering the spatial correlation. However, due to the fact that the spatial correlation structure is controlled by the auto-correlation distance, the estimation of this key parameter is necessary. By considering this as a model selection problem, it will be shown that, within the Bayesian framework, the auto-correlation distance and the observation errors can be appropriately selected based on Akaike's Bayesian Information Criterion (ABIC) (Akaike, 1980; Honjo and Kashiwagi, 1999).

Based on the estimated parameters at discrete observation points and the estimated auto-correlation distance, the kriging technique is adopted for estimating the values of unknown parameters at any unobserved points. This method provides an unbiased and least-error estimator built on the data from a random field for which second-order stationary is assumed (Krige, 1966; Matheron, 1973; Wackernagel, 1998). In fact, it can be proved that the kriging method shares the same conceptual basis with the Bayesian estimation (Hoshiya and Yoshida, 1996). This confirms the conformity of the theory used in this research. The derivation of the ordinary kriging, using the Bayesian formulation, is also presented in the Appendix of this paper.

The application of the proposed approach in spatial-temporal predictions of secondary compression settlements is presented in this paper. This type of settlement is significant in highly organic soil such as peat. The linear relationship between the logarithm of time and the settlement, which is a common empirical relationship found in secondary compression (Bjerrum, 1967; Garlanger, 1972; Mesri et al., 1997, etc.), is adopted in this study for use as an observation-based settlement prediction model.

In order to investigate the performance of the proposed approach in spatial-temporal settlement predictions, both simulated data and field observation data of the secondary compression settlements are used as the input for calculations under several test conditions. For the case of the simulated data, the settlement data are generated based on the assigned spatial correlation structure of the model parameters using the frequency domain technique (Shinozuka, 1971; Shinozuka and Jan, 1972). As for the field observation data, the observed settlements of peat from a ground improvement site in a suburb of Tokyo, Japan, are used. The calculation results from both cases will be presented and discussed later in this paper.

2. Spatial-temporal process

2.1. Bayesian estimation considering spatial correlation structure

In order to improve the accuracy of the estimations and to introduce local estimations, the utilization of the Bayesian estimation, including spatial correlation, is proposed. This approach uses prior information of the unknown parameters (denoted as θ in this paper), which characterize the soil behavior, e.g., model parameters or soil properties, and observation data (denoted as Y in this paper), e.g., observed settlements or movements, from all observation points, to search for the best estimates of the unknown parameters. The formulation consists of two statistical components, namely the observation model and the prior information model. These two models will then be combined by Bayes' theorem to obtain the solution.

2.1.1. Observation model

This model relates the observation data to the unknown parameters. The unknown parameters (e.g., model parameters) are defined in a multivariate stochastic Gaussian field $\theta(X)=[z_1(x),\ z_2(x),\ \ldots,\ z_I(x)]^T$ where X is a spatial vector coordinate and I is the total number of unknown parameters. Let θ denote the best estimator of $\theta(X)$ for a discrete spatial point field, $x_1, x_2, \ldots, x_n$, as follows:

$$\theta = \left[z_1^*,\quad z_2^*,\quad \ldots,\quad z_I^* \right]^T \tag{1}$$

where

$$z_i^* = \left[z_i^*(x_1),\quad z_i^*(x_2),\quad \ldots, z_i^*(x_n) \right]^T,\quad i = 1,2,\ldots,I \tag{2}$$

At a specific time step k, Y_k is defined as the observation data (e.g., observed settlement) at $x_1, x_2, \ldots, x_n$, where

$$Y_k = \left[y_k(x_1),\quad y_k(x_2),\quad \ldots,\quad y_k(x_n) \right]^T \tag{3}$$

It is assumed here that observation Y is expressed as a linear function of θ with an observation error ε as follows:

$$Y_k = M_k\theta + \varepsilon \tag{4}$$

where M_k is the $n \times (nI)$ coefficient matrix and ε is the Gaussian observation error vector, which is assumed to follow N(0, V_ε). V_ε is defined as a covariance matrix of ε, where $V_\varepsilon = \sigma_\varepsilon^2 V_\varepsilon^{'}$. σ_ε^2 is the variance of the observation errors and $V_\varepsilon^{'}$ is an $n \times n$ matrix, the components of which give the relative error magnitude of each observation. However, $V_\varepsilon^{'}$ is assumed to be the identity matrix ($I_{n,n}$) throughout this study, which implies that the observation errors are assumed to be spatially independent.

Given θ and σ_ε^2, the predicted settlement distribution at any time t_k can be represented by the following multivariate normal distribution:

$$p(Y_k|\theta, \sigma_\varepsilon^2) = (2\pi)^{-n/2}|V_\varepsilon|^{-1/2} \exp\left[-\frac{1}{2}(Y_k - M_k\theta)^T V_\varepsilon^{-1}(Y_k - M_k\theta) \right] \tag{5}$$

2.1.2. Prior information model

It is assumed that the prior information on the unknown parameters has the following structure:

$$\theta = \theta_0 + \delta \tag{6}$$

where θ_0 is the prior mean vector (nI dimension), a deterministic vector, and δ is the uncertainty of the prior mean, which is assumed to follow N(0, V_θ), where V_θ is a prior covariance matrix. By introducing the spatial correlation structure in the formulation of V_θ, we have

$$V_\theta = \begin{bmatrix} \sigma_{z1}^2 V_C & & & 0 \\ & \sigma_{z2}^2 V_C & & \\ & & \ddots & \\ 0 & & & \sigma_{zI}^2 V_C \end{bmatrix} \tag{7}$$

where $\sigma_{z1}^2, \sigma_{z2}^2, \ldots, \sigma_{zI}^2$ represent the prior variance of unknown parameters $z_1, z_2, \ldots, z_I$, respectively. V_C is the auto-covariance matrix, which is defined as

$$V_C = \begin{bmatrix} \rho(|\underline{x}_1 - \underline{x}_1|) & \cdots & \rho(|\underline{x}_1 - \underline{x}_n|) \\ \vdots & \ddots & \vdots \\ \rho(|\underline{x}_n - \underline{x}_1|) & \cdots & \rho(|\underline{x}_n - \underline{x}_n|) \end{bmatrix} \tag{8}$$

$\rho(|\underline{x}_i - \underline{x}_j|)$ denotes the auto-correlation function where $\underline{x}_i$ and $\underline{x}_j$ are the spatial vector coordinates. Several analytical expressions for the auto-correlation function have been proposed in past literature, but none of them can claim any fundamental basis (Vanmarcke, 1977). An exponential type of auto-correlation function is chosen for the current study because it has been widely used in geotechnical applications (e.g., Vanmarcke, 1977; Fenton and Griffiths, 2002; Griffiths and Fenton, 2004, etc.). The function is given as

$$\rho(|\underline{x}_i - \underline{x}_j|) = \exp[-|\underline{x}_i - \underline{x}_j|/\eta] \tag{9}$$

where η is the auto-correlation distance.

It should be noted that, for the sake of simplification, there are two important assumptions about the correlation structure for formulating the above covariance matrix. Firstly, $z_1, z_2, \ldots, z_I$ are assumed to be independent of each other. Secondly, the correlation structures of these parameters are identical, meaning that they share the same auto-correlation distance.

Given η, prior means, and prior variances of the unknown parameters, the prior distribution of the model parameters is also a multivariate normal distribution of the following form:

$$p(\theta|\eta) = (2\pi)^{-n}|V_\theta|^{-1/2} \exp\left[-\frac{1}{2}(\theta - \theta_0)^T V_\theta^{-1}(\theta - \theta_0) \right] \tag{10}$$

2.1.3. Bayesian estimation

Suppose that the set of observations Y_k at the discrete time step $k = 1, 2, \ldots, K$ has already been obtained. By employing Bayes' theorem, the posterior distribution of state vector θ can be formulated as

$$p(\theta|Y,\sigma_\varepsilon^2,\eta) \propto p(\theta|\eta)\prod_{k=1}^{K} p(Y_k|\theta,\sigma_\varepsilon^2) \tag{11}$$

where Y denotes the set of all observation data, i.e., $Y=(Y_1, Y_2, \ldots, Y_K)$. By incorporating Eqs. (5) and (10) into the above equation, we have

$$p(\theta|\eta)\prod_{k=1}^{K} p(Y_k|\theta,\sigma_\varepsilon^2) = (2\pi)^{-(2n+Kn)/2}|V_\theta|^{-1/2}|V_\varepsilon|^{-K/2}$$
$$\exp\left\{-\frac{1}{2}\left[(\theta-\theta_0)^T V_\theta^{-1}(\theta-\theta_0)+\sum_{k=1}^{K}(Y_k-M_k\theta)^T V_\varepsilon^{-1}(Y_k-M_k\theta)\right]\right\} \tag{12}$$

The Bayesian estimator of θ, i.e., θ^*, is the one that maximizes the above function. Therefore, it is equivalent to minimizing objective function J as follows:

$$\min[J(\theta|\sigma_\varepsilon^2,\eta)] = J(\theta^*|\sigma_\varepsilon^2,\eta)$$
$$= (\theta^*-\theta_0)^T V_\theta^{-1}(\theta^*-\theta_0)+\sum_{k=1}^{K}(Y_k-M_k\theta^*)^T V_\varepsilon^{-1}(Y_k-M_k\theta^*) \tag{13}$$

It should be noted that σ_ε^2 and η are assumed to be given in this case. The Bayesian method, however, does not offer a rational way to determine these values. In order to choose the most appropriate values for σ_ε^2 and η, based on the information on hand, the use of Akaike's Bayesian Information Criterion (ABIC) is proposed and presented in the next section.

2.2. Akaike's Bayesian Information Criterion

Choosing appropriate values for σ_ε^2 and η can be considered as a model selection problem. Akaike's Bayesian Information Criterion (ABIC), which is used in this study, is developed based on the same information theory principle as Akaike's Information Criterion (AIC), which is a specific approach used for selecting the best model from several alternative models (Akaike, 1980; Honjo and Kashiwagi, 1999). By considering σ_ε^2 and η as hyperparameters, the Baysian likelihood for this formulation is given as

$$L(\sigma_\varepsilon^2,\eta|Y) = \int p(\theta|\eta)\prod_{k=1}^{K} p(Y_k|\theta,\sigma_\varepsilon^2)d\theta \tag{14}$$

Introducing Eq. (12) into Eq. (14), we obtain

$$L(\sigma_\varepsilon^2,\eta|Y) = \int_{-\infty}^{\infty}(2\pi)^{-(2n+Kn)/2}|V_\theta|^{-1/2}|V_\varepsilon|^{-K/2}$$
$$\exp\left\{-\frac{1}{2}\left[(\theta-\theta_0)^T V_\theta^{-1}(\theta-\theta_0)+\sum_{k=1}^{K}(Y_k-M_k\theta)^T V_\varepsilon^{-1}(Y_k-M_k\theta)\right]\right\}d\theta \tag{15}$$

By performing integration of the above equation, we have

$$L(\sigma_\varepsilon^2,\eta|Y) = (2\pi)^{-Kn/2}|V_\theta|^{-1/2}|V_\varepsilon|^{-K/2}\left|\sum_k^K M_k^T V_\varepsilon^{-1}M_k + V_\theta^{-1}\right|^{-1/2}$$
$$\exp\left\{-\frac{1}{2}\left[(\theta^*-\theta_0)^T V_\theta^{-1}(\theta^*-\theta_0)+\sum_{k=1}^{K}(Y_k-M_k\theta^*)^T V_\varepsilon^{-1}(Y_k-M_k\theta^*)\right]\right\} \tag{16}$$

Note that θ^* denotes the Bayesian estimator of θ, obtained by minimizing the objective function, as shown in Eq. (13). The log Bayesian likelihood of (16) is given as

$$l(\sigma_\varepsilon^2,\eta|Y) = -\frac{1}{2}\ln|V_\theta| - \frac{K}{2}\ln|V_\varepsilon| - \frac{1}{2}\ln\left|\sum_k^K M_k^T V_\varepsilon^{-1}M_k\right.$$
$$\left. + V_\theta^{-1}\right| - \frac{1}{2}\left[J(\theta^*|\sigma_\varepsilon^2,\eta)\right] + c_1 \tag{17}$$

where c_1 represents a constant term. The general definition of ABIC (Akaike, 1980) is given as ABIC$=(-2)$ log (maximum Bayesian likelihood)$+2$ (number of hyperparameters). In this case, the number of hyperparameters is fixed at 2, i.e., σ_ε^2 and η. Thus, we have

$$ABIC = \ln|V_\theta| + K\ \ln|V_\varepsilon| + \ln\left|\sum_k^K M_k^T V_\varepsilon^{-1}M_k + V_\theta^{-1}\right|$$
$$+[J(\theta^*|\sigma_\varepsilon^2,\eta)] + c_2 \tag{18}$$

where c_2 denotes a constant term. By trial and error, the values for σ_ε^2, η, and corresponding θ^*, which give the minimum value of ABIC, can be obtained. This can be considered as the optimized selection of these parameters.

2.3. Local estimation by the kriging method

Based on the calculated statistical inferences of the model parameters at the observation points, i.e., θ^*, and the estimated auto-correlation distance, the statistics of the model parameters at any arbitrary location can be determined by the ordinary kriging method (Krige, 1966; Matheron, 1973; Wackernagel, 1998). This method provides an unbiased and least-error estimator built on the data from a random field for which a second-order stationary is assumed. In fact, this method can also be derived based on the aforementioned concept of the Bayesian method. This emphasizes that all of the proposed formulations for the spatial-temporal process are also found in the basic concept of the Bayesian approach. The derivation is summarized and presented in the Appendix.

Based on the unknown parameters at the n observation points, i.e., $[z_i^* = z_i^*(x_i), z_i^*(x_2), \ldots, z_i^*(x_n)]^T$, the values of the parameters, at an arbitrary point x_0, i.e., $z_i^*(x_0)$, can be estimated by the following equations:

$$\begin{bmatrix} z_1^*(x_0) \\ z_2^*(x_0) \\ \vdots \\ z_I^*(x_0) \end{bmatrix} = \begin{bmatrix} z_1^{*T} \\ z_2^{*T} \\ \vdots \\ z_I^{*T} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{19}$$

where

$$\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} \rho(|\underline{x}_1-\underline{x}_1|)\cdots\rho(|\underline{x}_1-\underline{x}_n|) & -1 \\ \vdots \quad \ddots \quad \vdots & \vdots \\ \rho(|\underline{x}_n-\underline{x}_1|)\cdots\rho(|\underline{x}_n-\underline{x}_n|) & -1 \\ 1 \quad \cdots \quad 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \rho(|\underline{x}_1-\underline{x}_0|) \\ \vdots \\ \rho(|\underline{x}_n-\underline{x}_0|) \\ 1 \end{bmatrix} \tag{20}$$

w_i $(i=1, \ldots, n)$ are the weights attached to the data at each of the observation points, μ is the Lagrange multiplier used for minimizing the kriging error, and $\underline{x}_0$ denotes the spatial vector coordinate at x_0. $\rho(|\underline{x}_i-\underline{x}_j|)$ represents the auto-correlation function as defined in Eq. (9). It is emphasized here that it is also possible to calculate the estimation error of the interpolation by kriging, the details of which are presented in several publications (e.g., Wackernagel, 1998).

2.4. Settlement prediction model

The application of the proposed approach for the spatial-temporal prediction of ground settlements is presented in this paper. The basic model chosen in this study is the linear relationship between the settlement (S) and the logarithm of time (t), i.e., $S\sim\log(t)$ method. This model is considered to be rational and practical for the prediction of secondary compression (Bjerrum, 1967; Garlanger, 1972; Mesri et al., 1997, etc.). The equation is given as

$$S = m_0 + m_1 \log(t) \tag{21}$$

where S is the secondary compression settlement, m_0 and m_1 are model parameters, and t is the compression time.

Suppose that the secondary compression settlements at n observation points $x_1, x_2, \ldots, x_n$, have been sequentially observed at discrete time t_k for $k=1, 2, \ldots, K$. In this case, the components of the observation model equations given in Section 2.1.1 can be defined based on the above settlement prediction model as follows:

$$\theta = [m_1^*(x_1), \quad m_1^*(x_2), \quad \ldots, \quad m_1^*(x_n), \quad m_0^*(x_1), \quad m_0^*(x_2), \quad \ldots, \quad m_0^*(x_n)]^T \tag{22}$$

$$Y_k = \left[S_k(x_1), \quad S_k(x_2)^2, \ldots, S_k(x_n)^n \right]^T \tag{23}$$

$$M_k = \begin{bmatrix} \log(t_k) & 0 & \vdots & \\ & \ddots & \vdots & I_{n,n} \\ 0 & \log(t_k) & \vdots & \end{bmatrix} \tag{24}$$

where $I_{n,n}$ denotes the $n\times n$ identity matrix. As stated previously in Section 2.1.1, it is assumed that $V_\varepsilon = \sigma_\varepsilon^2 I_{n,n}$. In the same way, the components of the prior information model equations, given in Section 2.1.2, can be defined as follows:

$$\theta_0 = [m_{1,0}^*(x_1), \quad m_{1,0}^*(x_2), \quad \ldots, \quad m_{1,0}^*(x_n), \quad m_{0,0}^*(x_1), \quad m_{0,0}^*(x_2), \quad \ldots, \quad m_{0,0}^*(x_n)]^T \tag{25}$$

$$V_\theta = \begin{bmatrix} \sigma_{m1,0}^2 V_C & 0_{n,n} \\ 0_{n,n} & \sigma_{m0,0}^2 V_C \end{bmatrix} \tag{26}$$

where $m_{1,0}^*(x_i)$, $m_{0,0}^*(x_i)$ denote the prior mean at observation point x_i of m_1 and m_0, respectively; $\sigma_{m1,0}^2$ and $\sigma_{m0,0}^2$ represent the prior variance of m_1 and m_0, respectively; $0_{n,n}$ denotes an $n\times n$ zero matrix.

From the above definitions, it is clear that the spatial correlation of soil properties is included in the form of the spatial correlation of m_1 and m_0 instead of that of settlements. The authors believe that this is the most suitable way to introduce the spatial correlation structure into the settlement prediction model due to the fact that the physical correlation of the ground settlement actually results from the spatial correlation of the soil properties.

3. Simulation experiments

3.1. Random field generation by frequency-domain technique

To investigate the performance of the proposed approach, a two-dimensional random field of model parameters (m_0, m_1) is generated based on the assumed mean, variance, and auto-correlation distance. The settlement (S), which is now considered as observation data, is then calculated by Eq. (21), using the generated parameters and the assumed variance of the observation error, σ_ε^2 (i.e., $S=m_0+m_1\log(t)+\varepsilon$). Performing the spatial-temporal updating procedure, previously stated in Section 2.1 and based on the generated observation data, the statistical inferences of the model parameters at each observation point can be back-calculated. A comparison of these inferences with the simulated ones, namely the true values, reveals the efficiency of the procedure.

Various techniques have been proposed by several authors for random field generation, e.g., the turning bands method (Matheron, 1973), the frequency domain technique (Shinozuka, 1971; Shinozuka and Jan, 1972), and the local average subdivision method (Fenton and Vanmarcke, 1990). The frequency domain technique is chosen for this study in order to avoid the streaking problem, which is found in the turning bands method, and the difficulties of implementing the local average subdivision method (Fenton, 1994). The frequency domain technique concentrates on the spectral density function (SDF) of the process, which is defined as the Fourier transform of the auto-correlation function. For an exponential type of auto-correlation function, with an auto-correlation distance of η, it can be proved that the SDF is defined as

$$S(\omega_1, \omega_2) = \frac{1}{2\pi\eta((1/\eta^2)+(\omega_1^2+\omega_2^2))^{3/2}} \tag{27}$$

which is a function of the frequency domains, ω_1 and ω_2

Assuming that the power of the employed SDF is negligible outside intervals $[-\omega_{1,0},\omega_{1,0}]$ and $[-\omega_{2,0},\omega_{2,0}]$, the simulated stationary Gaussian random field at any coordinates (x, y) can be expressed as the following series of cosine functions:

$$X(x,y) = \sum_{k=1}^{M_2}\sum_{j=1}^{M_1}\sqrt{2S(\omega_{1j},\omega_{2k})\Delta\omega_1\Delta\omega_2}\ \cos(\omega_{1j}x+\omega_{2k}y+\phi_{jk}) \tag{28}$$

where $\Delta\omega_1 = 2\omega_{1,0}/M_1$, $\Delta\omega_2 = 2\omega_{2,0}/M_2$, $\omega_{1j} = -\omega_{1,0}+(j-1/2)\Delta\omega_1$, $\omega_{2k} = -\omega_{2,0}+(k-1/2)\Delta\omega_2$, and ϕ_{jk} are random phase angles, uniformly and independently distributed in the interval $(0,2\pi)$; M_1 and M_2 are the number of equally divided intervals of ranges $[-\omega_{1,0},\omega_{1,0}]$ and $[-\omega_{2,0},\omega_{2,0}]$, respectively. Care must be taken when selecting these ranges and discretization intervals to ensure that the spectral density function is adequately approximated. Based on the above equations, the realization (r_p) of a random parameter, with mean μ_p and standard deviation σ_p, at any coordinates (x, y), is defined as $r_p = \mu_p + \sigma_p X(x, y)$.

3.2. Improvement of the estimation by considering spatial correlation structure

A series of simulation experiments was performed based on the aforementioned procedure. A decision was made to limit the number of simulations for each experiment to 50 ($N_{sim}=50$) and to choose the sample size of 36 ($n=36$). The observation points are arranged in a square grid pattern with an even spacing of s and a total width of L, as shown in Fig. 1. For the purpose of recognizing the performance of the approach, these selections seem sufficient.

For model parameter generation, it is assumed that the mean and the standard deviation of the random field of these parameters are $m_1^* = 100$ mm, $m_0^* = -100$ mm, and $\sigma_{m1} = \sigma_{m0} = 10$ mm, implying that the coefficient of variation (COV) = 0.1. It is assumed that the settlement is observed 21 times (i.e., total observation time step $K=21$) from the 10th to the 1000th day. Note that, by substituting $m_1^*$and m_0^* into Eq. (21), the estimated settlement on the 1000th day is 200 mm. For the current study, the observation error (σ_ε) is assumed to be 10 mm. By assigning the desired values to the auto-correlation distance (η), the random values of the model parameters together with the observed settlement at each observation point can be generated, as described in Section 3.1. It should be emphasized that the generated data actually represents a set of the settlement data observed from an area for which the true values of the settlement model parameters and the underlying spatial correlation structures are known.

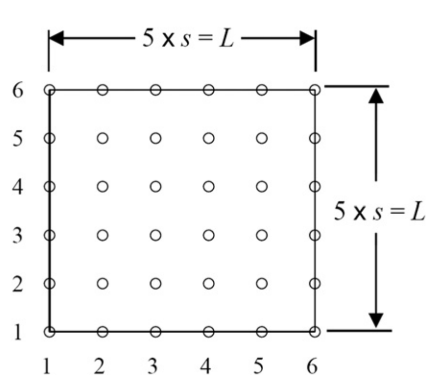

Fig. 1. Layout of observation plans.

Based on the generated observation data, the procedure proposed in Section 2.1 is performed to back-estimate the model parameters. The prior mean of the model parameters, $m_{1,0}^*(x_i)$ and $m_{0,0}^*(x_i)$ (see Eq. (25)), are assumed to be the same at every observation point and equal to m_1^* and m_0^*, respectively. As for $\sigma_{m1,0}$ and $\sigma_{m0,0}$ (see Eq. (26)), COV is assumed to be 0.4, i.e., $\sigma_{m1,0} = \sigma_{m0,0} = 40$ mm. This relatively large value of COV is assumed in order to limit the influence of the prior information, which commonly is not known in practice. The auto-correlation distance and the observation error are also assigned the same values as those used for generating the simulated data, namely, the true values.

In order to examine the advantages of considering the spatial correlation structure, the Bayesian estimation, using the observed settlement of each observation point to estimate the model parameters of that point itself, i.e., ignoring the spatial correlation structure, is also performed based on the same conditions as those mentioned above. This case can be represented by assuming a small value for η, i.e., $\eta/s = 0.001$. The different model parameters are randomly generated 50 times ($N_{sim}=50$) and the estimation errors, as a percent of the true values, are calculated. The estimations based on these two different conditions are compared and presented in Figs. 2 and 3.

The estimation errors are represented by the term 'mean absolute error', which is defined as the mean of the absolute percentage of the observation error taken from all observation points and random simulations, as follows:

$$\text{mean absolute error}(\%) = \frac{\sum_{i=1}^{N_x}\left|((X_{est,i}-X_{true,i})/X_{true,i})\times 100\right|}{N_x} \tag{29}$$

where $X_{est,i}$ and $X_{true,i}$ denote the estimated value and the true value, respectively, of the parameter to be estimated at each observation point for each simulation; N_x is the total number of estimated values, i.e., $N_x = n \times N_{sim}$. It is clear that the true values of the model parameters are known. However, those of the settlement have to be estimated. Eq. (21) is used for calculating both the true values and the estimated values of the settlement at any time t, using the true values and the estimated values of the model parameters, respectively.

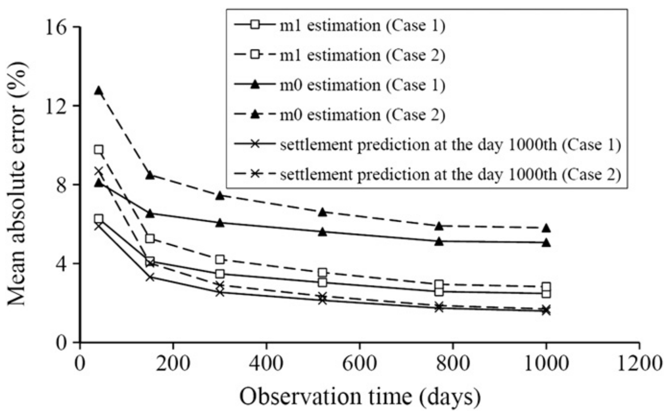

Fig. 2. Estimation errors versus observation times for estimations at observation points, assuming $\eta/s=4$. Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

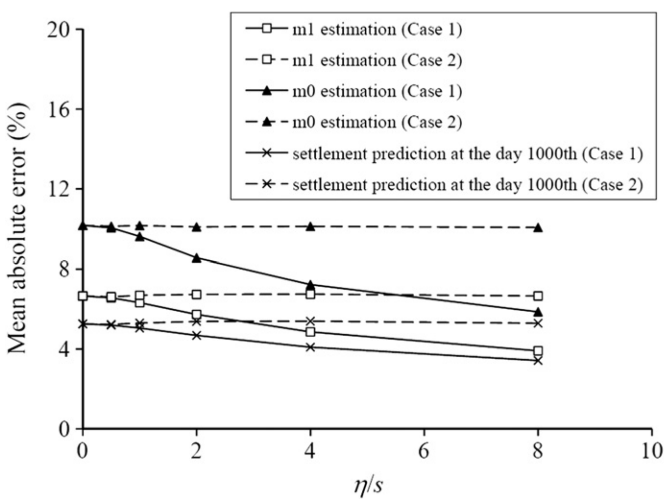

Fig. 3. Estimation errors versus η/s ratios for estimations at observation points, using data from the 10th to the 100th day. Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

Fig. 2 illustrates the plots of the mean absolute errors of the model parameter estimation and the settlement prediction on the 1000th day against the observation time. For emphasis, the observation time mentioned here is defined as the time span over which the data are used in the calculation. It is assumed that the ratios of auto-correlation distance to spacing, η/s, equal 4. Clearly, the mean absolute errors for the cases which consider the spatial correlation structure (Case 1) are lower than those for which the spatial correlation structure is ignored (Case 2), regardless of the observation time. This confirms that estimations can be improved by taking into account the spatial correlation structure. The fact that the difference is larger at the earlier stage of observation emphasizes the advantage of using the proposed method for the estimation at an early time. This trend is the same for both model parameters and settlement estimations.

To investigate the sensitivity of this improvement to changes in the spatial correlation structure, the same calculations at different values for the η/s ratio are performed. Only 11 time steps of the observations from the 10th to the 100th day are selected for use in the calculations. The mean absolute error of the model parameter estimation and the settlement prediction on the 1000th day are determined as illustrated in Fig. 3.

It can be seen from Fig. 3 that the errors in the model parameter and the settlement estimation decrease with the increase in the η/s ratio, when spatial correlation is considered. This leads us to conclude that, by the proposed method, a stronger spatial correlation gives a better estimation. It should be noted that this improvement becomes significant when the observation spacing is shorter than half of the auto-correlation distance, i.e., $\eta/s>2.0$.

3.3. Estimation of settlement at an arbitrary location

As mentioned previously, one of the advantages of the proposed method is its ability to estimate the settlement at any arbitrary location. By applying the kriging method (see Section 2.3), the model parameter at a selected point can be approximated based on the estimated parameters at the observation points. Then, the settlement at that point, at any time, can be predicted using Eq. (21).

To investigate the level of error for this type of estimation, similar calculations to those described in the previous section are performed. However, for each calculation, one of the observation points is removed from consideration. Then, the model parameters and the settlement at the removed observation point will be estimated using only the generated data of the remaining observation points. Due to the fact that the true values of the model parameters at each removed observation point are unknown in this case, the estimation errors are determined by comparing the estimated values with the parameter values which are generated from the corresponding random sampling with those of the other observation points, the data of which are used for the estimation. Figs. 4 and 5 show the plots of these estimation errors against the observation time and the η/s ratio, respectively.

For comparison purposes, the calculations presented in Figs. 4 and 5 are analogous with those shown in Figs. 2 and 3, while $\eta/s=4$ is assumed in Fig. 4 and the data from the 10th to the 100th day is used in Fig. 5. The number of simulations for each trial is also 50 ($N_{sim}=50$). The mean absolute error is calculated according to Eq. (29), averaging the estimation errors at all observation points and all simulations. The values of the other parameters are also the same as those assigned in Section 3.2.

It can be observed from Fig. 4 that, similar to Fig. 2, the estimation error decreases with the observation time. In other words, the more observation data we have, the better estimation we can obtain. Fig. 5 clearly shows that the error is significantly higher if a weaker spatial correlation structure is assumed, especially for the settlement estimation.

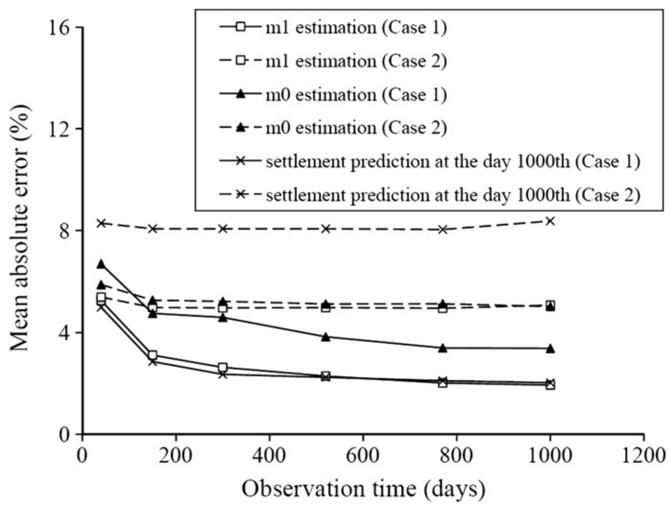

Fig. 4. Estimation errors versus observation times for estimations at removed observation points, assuming $\eta/s=4$. Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

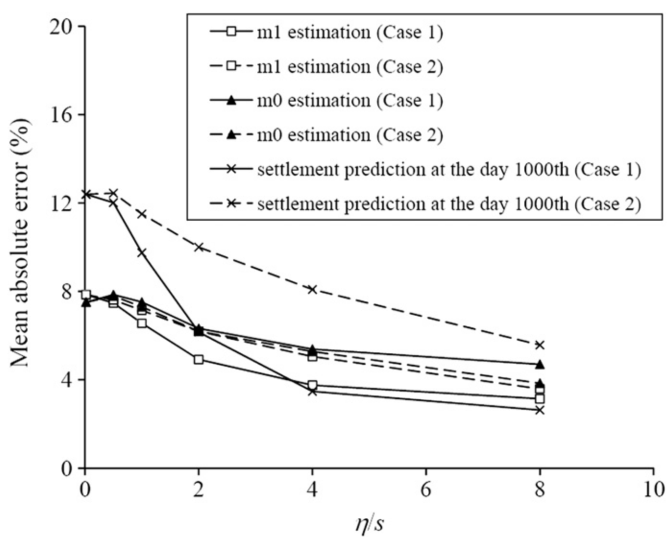

Fig. 5. Estimation errors versus η/s ratios for estimations at removed observation points, using data from the 10th to the 100th day. Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

This emphasizes the advantage of the proposed approach for cases in which a strong spatial correlation structure of the parameters is found. Clearly, cases which consider the spatial correlation structure provide better estimations than cases which do not, namely they yield lower estimation errors especially for the settlement estimations.

4. Case study

4.1. Description of the case

To investigate the performance of the proposed method for practical applications, a case study using the actual field observation data of secondary compression (Ueda et al., 1986), has been performed. The site is a residential land development project located in a suburb of Tokyo, Japan. This area is covered by a thick alluvial deposit, which can be classified as a surface layer of peat followed by a very soft clay layer down to a thickness of about 17 m (Fig. 6). Below these layers, layers of medium dense sand and silt are found, respectively. In order to avoid a large amount of settlement, due to the thick soft soil layer at the surface, the soil conditions are improved by preloading prior to the construction. As shown in Fig. 7, the preloading surcharge was filled up to the maximum thickness of about 6 m during the preloading period of approximately 900 days.

The settlement observations were performed both during the preloading period by the settlement plates and after the removal of the surcharge by measuring the settlement of the boundary stone around the housing lots. After the removal of the surcharge, considered to be the secondary compression and used in this study, the settlement was observed about every 600 m^2 with a total of 42 observation points. The location plan for these observation points and

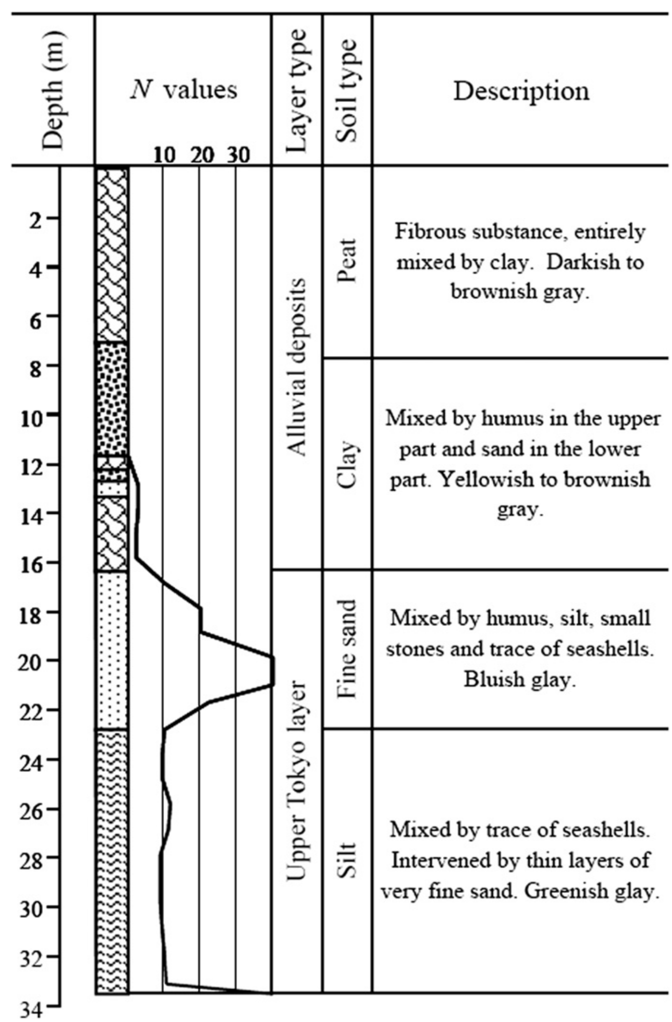

Fig. 6. Soil conditions.

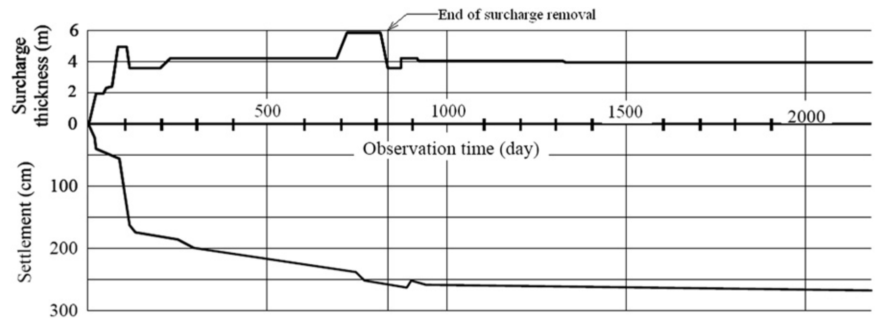

Fig. 7. Surcharge thickness and settlement versus time.

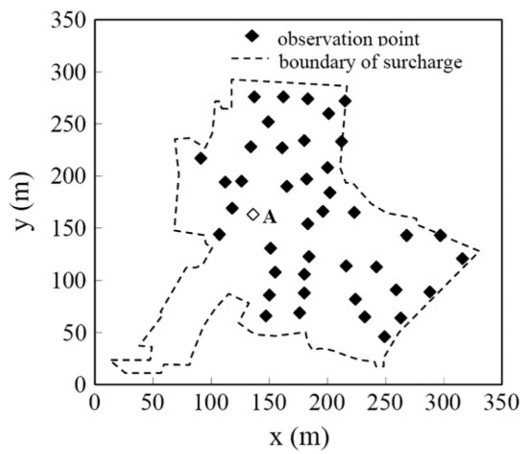

Fig. 8. Location plan of the observation points and the surcharge area.

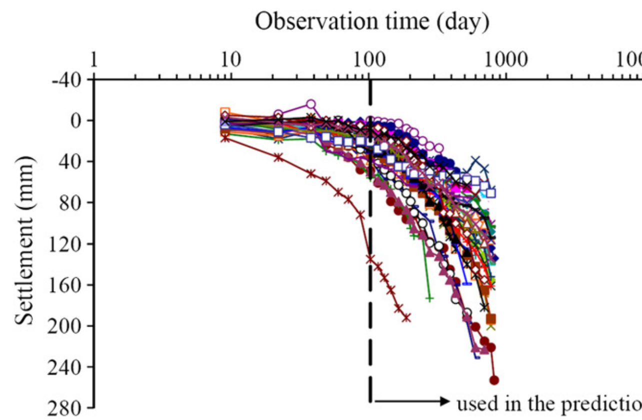

Fig. 9. Observed settlement versus time (after surcharge removal) for all observation points.

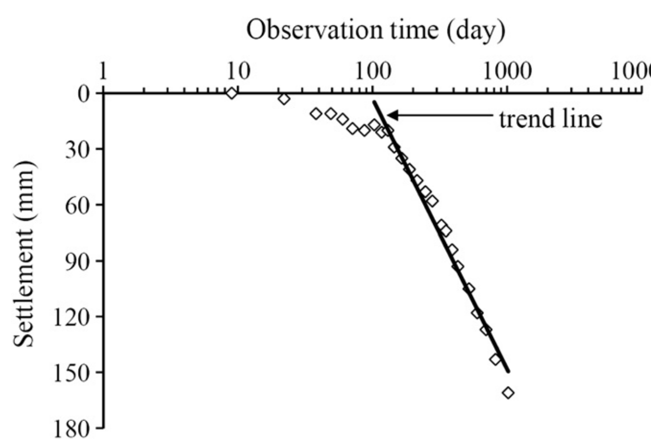

Fig. 10. Observed settlement versus time (after surcharge removal) with trend line at point A (see Fig. 8).

the surcharge area is presented in Fig. 8, while all of the observation data are shown as semi-logarithmic plots of settlement and time in Fig. 9. For emphasis, the observation time in Fig. 9 represents the time after the surcharge removal. This should not be confused with the observation time in Fig. 7, which refers to the time measured from the beginning of preloading.

Various techniques have been proposed by several authors for predicting future settlements with the observed settlements, for example, the hyperbola method (Sridharan et. al., 1987; Tan, 1994), the $S \sim \log(t)$ method (Bjerrum, 1967; Garlanger, 1972; Mesri et al., 1997), and Asaoka's (1978) method. In this study, the $S \sim \log(t)$ method is considered to be the most suitable approach due to the fact that the primary consolidation is expected to be completed before the surcharge removal. Thus, the settlement occurring afterward should result from the secondary compression process. Fig. 10 shows an example of the $S \sim \log(t)$ plot at an observation point. It can be seen that by excluding a part of the data in the early period of observation, within which the secondary compression is considered to be influenced by the rebound effect due to the surcharge removal, this semi-logarithmic relationship fits the observation data quite well. By investigating the settlement data for all the observation points, the data before the 103rd day are excluded from the calculation by judgment.

Choosing the appropriate prior statistics for the unknown parameters (m_1 and m_0) is also an important issue. What has been done in the current research is that the prior means of m_1 and m_0, i.e., $m^*_{1,0}(x_i)$ and $m^*_{0,0}(x_i)$, were assumed to be equal to the values of the slope and the intercept of the trend line resulting from the linear regression analysis of the plots between the settlement and the logarithm of time, considering the data from all of observation points. On the other hand, the prior variances, i.e., $\sigma^2_{m1,0}$ and $\sigma^2_{m0,0}$, were selected by trying several values for prior coefficients of variation (COV) and choosing the one which gave the results that were relatively insensitive to changes in the prior means. Based on this approach, the prior means of m_1 and m_0, which are assumed to be the same for every observation point, are assigned as 109.7 and -204.1 mm, respectively, while the prior COV of 0.4 is chosen for calculating the prior variance of both parameters.

The above method of selecting prior statistics is barely an example in cases where there is no other information about the values of the unknown parameters of concern. In practice, the ranges in values can be based on, for example, past experiences, empirical relationships, or data collected from past projects in the vicinity. One can then choose values for the mean and the COV depending one's level of confidence regarding the source of the information. This is actually one of the advantages of the proposed approach, by which such subjective information can be systematically included in any estimation.

4.2. Estimation of the auto-correlation distance and observation error

In Section 2.2, it was proposed that the auto-correlation distance (η) and the standard deviation of the observation error (σ_ε) can be appropriately selected based on Akaike's Bayesian Information Criterion (ABIC). Considering the observation data, together with prior information on the model parameters, the *ABIC* for each pair of η and σ_ε can be determined by Eq. (18). The values for η and σ_ε that give the minimum value for *ABIC* will serve as the optimized selections of these parameters. We emphasize here that η represents only the horizontal correlation in this study.

Fig. 11 shows an example of the contours of *ABIC* in the η and σ_ε space for cases in which all of the settlement data, until the last step of observation, i.e., the 1017th day, are considered. In this case, the estimated η and σ_ε are 30 m and 7.0 mm, respectively. Obviously, the estimated values for the observation errors are more likely to be insensitive than those for the auto-correlation distance.

In practice, the observation data are collected stepwise for a period of time. Therefore, it is natural to sequentially update the estimation once new sets of observations are provided. Fig. 12 illustrates the plots of the estimated values of η and σ_ε versus the observation time, which again refers to the time span over which the observation data are used. It can be observed that the estimated values for the auto-correlation distance tend to decrease with the observation time, while those for the observation error tend to increase, depending on the characteristics of the observation data. Both of these estimations seem unstable at the early stage of the observation, indicating insufficiency in the observation data for the calculations. However, they become more stable as the observation data accumulates.

Auto-correlation distance η is known to be a pamameter which is difficult to determine with high certainty. Therefore, concern regarding accurary in the selection of η by means of the proposed formulation is required. The discussion about this, however, has been presented in the authors' previous paper (Rungbanaphan et al., 2010). It is found that even though errors in the selection of η may seem significant, the effect of these errors on settlement predictions with spatial interpolation is relatively small. Therefore, it is concluded that the proposed method for selecting η yields results that are accurate enough for spatial-temporal estimations by the proposed approach.

4.3. Settlement estimation and prediction

Based on the procedure proposed in Section 3.2, the settlement estimation at observation points with

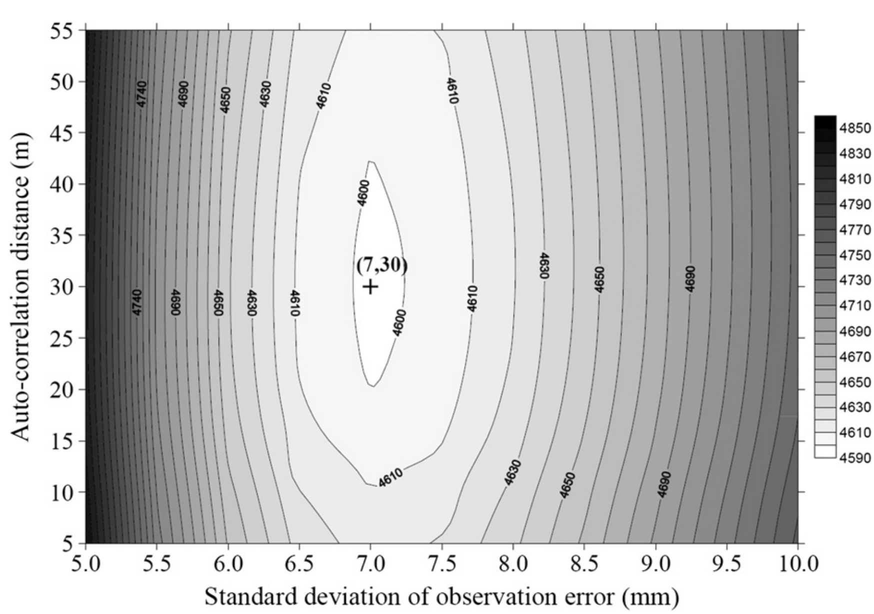

Fig. 11. An example of contour of ABIC for the case which considers the observation data until the last step of the observation (the 1017th day).

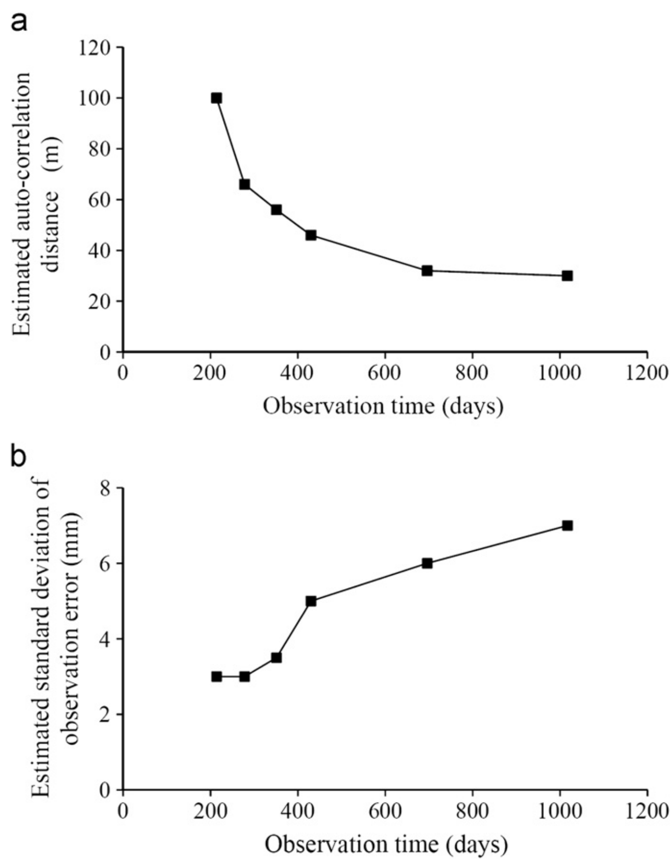

Fig. 12. Estimated values of the auto-correlation distance (η) and the standard deviation of the observation error (σ_ε) at different observation times: for (a) auto-correlation distance and (b) standard deviation of observation error.

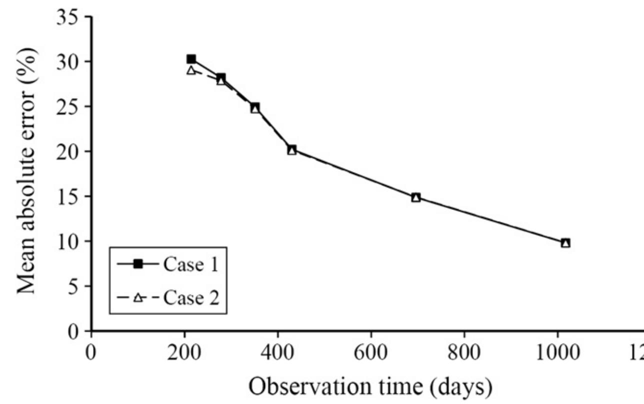

Fig. 13. Estimation error versus observation times of settlement prediction at the last step of the observation (the 1017th day). Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

consideration given to spatial correlation can be performed. To represent the estimation error, the term 'mean absolute error', previously defined in Eq. (29), is also used. However, in the current case, the estimated value is the estimated settlement, while the true value is the observed one. N_x is the total number of observation points, i.e., $N_x = n$.

Fig. 13 shows the plots of the mean absolute errors for the prediction of settlements at the last observation time step (1017th day) versus the observation time. For comparison purposes, both cases, namely the one in which the spatial correlation is considered (Case 1) and the one in which it is ignored (Case 2), are also presented. It should be noted that, for the case which considers spatial correlation, the estimated values for the auto-correlation distance, shown in Fig. 12, are used in the calculations.

Corresponding to the results of the simulation examples shown in Fig. 2, the prediction error decreases with the increase in the available observation data. However, for the current set of observation data, considering the spatial correlation does not significantly improve the estimation in terms of the mean error. This may be due to the fact that the auto-correlation distance (η) is relatively short in comparison to the spacing between the observation points (s) in this case. For this set of the field observations, $\eta \approx 30$ and $s \approx 25$ m, therefore, the ratio $\eta/s \approx 1.0$. Fig. 13 clearly shows that considering the spatial correlation does not bring about a significant improvement at this level of the η/s ratio.

To further investigate the performance of the proposed method in dealing with the space-time problem, the observation data at each selected observation point is removed and the settlement estimations, or predictions, at this point are performed using the rest of the observations. A comparison between the settlement estimated by these parameters and the one actually observed reveals the estimation error. This calculation is actually comparable to the previously presented simulation examples shown in Section 3.3.

Fig. 14 shows a comparison between the observed and the predicted settlements on the 1017th day. The observation data from the 103rd to the 696th day are used in the calculation. The predicted settlements are presented as surfaces, while the observed settlements are plotted as points with dotted lines showing the difference between them. Obviously, for the case in which the spatial correlation is not included in the calculation (Fig. 14a), represented by choosing a small value for η, i.e., $\eta = 0.025$ m ($\eta/s \approx 0.001$), the predicted settlements become uniform and cannot represent the variation in ground settlements. This is because the estimated values for the model parameters at each removed point, which need to be interpolated from the values at the other observation points by the kriging technique (see Section 2.3), are actually the arithmetic means of all the values in this case. On the other hand, for the case in which the estimated value of the auto-correlation distance, $\eta = 32$ m, is used (Fig. 14b), the estimation produces a relatively more realistic pattern for the settlement in the area.

Fig. 15 shows the plots of the mean absolute errors (Eq. (28)) of the settlement estimations at the removed observation points vs. the observation time. Two cases of calculations are presented, namely, the settlement estimation at the current observation day and the settlement prediction on the 1017th day. The former is an attempt to avoid temporal errors resulting from the prediction of future settlements; therefore, the estimated settlement and the observed settlement

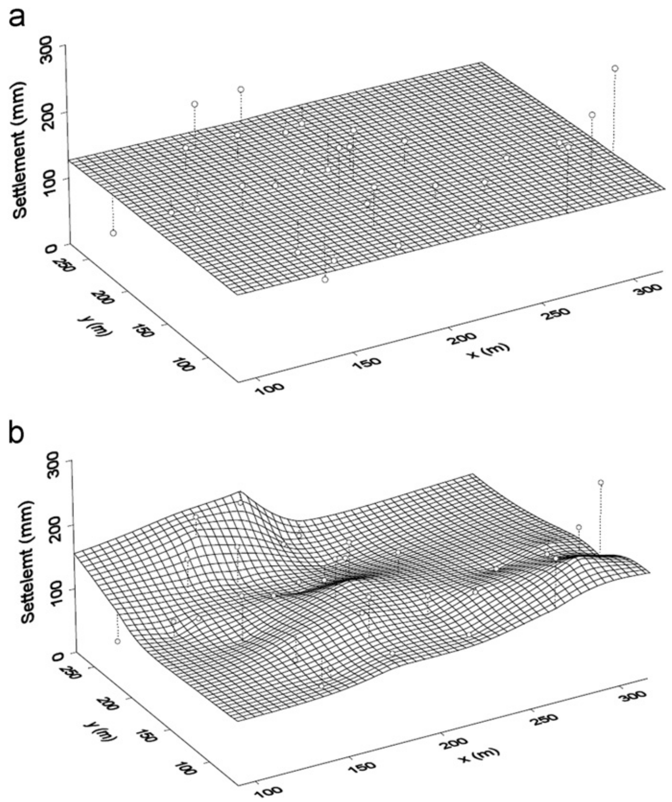

Fig. 14. Comparison between the estimated settlement, shown as surfaces, and the observed settlement, shown as points, using data from the 103rd to the 696th day for predicting the settlement on the 1017th day: for (a) without consideration of spatial correlation and (b) with consideration of spatial correlation.

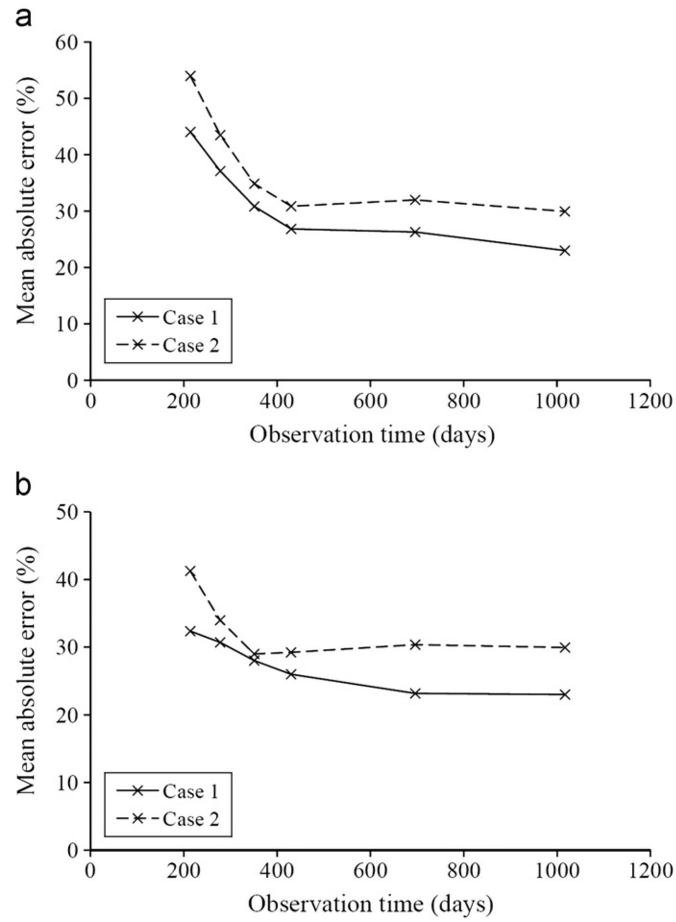

Fig. 15. Estimation errors versus observation times for estimations at removed observation points: for (a) settlement estimation on the observation day and (b) settlement prediction on the 1017th day. Case 1 refers to the case in which spatial correlation is considered. Case 2 refers to the case in which spatial correlation is ignored.

are compared at that observation time (Fig. 15a). The latter is the comparison between the predicted settlement at the last observation time step (the 1017th day) and the observed settlement at that time (Fig. 15b). It should be emphasized that, to consider the case of spatial correlation (Case 1), the estimated values for the auto-correlation distance and the observation error, which vary with the observation time as shown in Fig. 12, are used in the calculations, while it is assumed that $\eta = 0.025$ m ($\eta/s \approx 0.001$) for the case in which the spatial correlation is ignored (Case 2).

It can be seen from Fig. 15(a) and (b) that giving consideration to spatial correlation can improve the estimations in terms of reducing estimation errors. Depending on the characteristics of the observation data, this improvement may not be significant at the early stages of the observation. The estimation errors also decrease with the observation time. In other words, estimations can be improved if more observation data are given, which corresponds to the results of the simulation examples shown in Fig. 4.

5. Conclusion

A systematic approach for the spatial-temporal prediction of secondary compression settlements is proposed. Both prior information of the settlement model parameters and the observed settlements are used for the settlement estimations based on a Bayesian approach. The spatial correlation structure of the soil properties is introduced to the settlement prediction model through the spatial correlation of the model parameters. The estimation of the auto-correlation distance of the parameters and the observation error by Akaike's Bayesian Information Criterion (ABIC) are also proposed. For determining the statistics of the estimated model parameters at any arbitrary location, the kriging method is employed.

By performing a series of simulation examples for the prediction of secondary compression, the proposed spatial-temporal formulation is considered to have the following two main advantages:

(1) Settlement predictions can be improved by considering the spatial correlation structure by which a stronger spatial correlation structure yields a better estimation. According to the simulation, the effect of the spatial correlation on the prediction becomes more apparent when the observation spacing is less than half of the auto-correlation distance.

(2) A rigorous settlement prediction at any arbitrary point becomes possible through the interpolation of the soil parameters based on knowledge of the spatial correlation structure.

A case study on the field observation data for the secondary compression of alluvial deposits by preloading was carried out using the proposed approach. It was found that the estimation of the auto-correlation distance is relatively unstable at the early stage of observation, but that it becomes more stable as the observation data accumulates. However, even though it seems that the auto-correlation distance of the soil parameters is relatively short, in comparison to the observation point's spacing, the proposed method provides a rational estimation of the settlement at any time and at any location with quantified errors.

Appendix

This section shows the derivation of the ordinary kriging (Krige, 1966; Matheron, 1973; Wackernagel, 1998) using the Bayesian approach. Hoshiya and Yoshida (1996) have proved that the simple kriging can be derived based on the Bayesian formulation by maximizing the conditional probability. It was assumed that the observation vector (Y) and the unknown parameter (θ) are the same physical parameters, and that the observation error (ε) is zero. From these assumptions, we have zero covariance of the observation error

$$V_\varepsilon = 0_{n,n} \tag{A1}$$

and a linear function between Y and θ

$$Y = M\theta = \begin{bmatrix} I_{n,n} & 0_n \end{bmatrix} \begin{bmatrix} \theta_n \\ z^*(x_0) \end{bmatrix} \tag{A2}$$

where at n observation points $x_1, x_2, \ldots, x_n$, observation vector $Y = [y(x_1), y(x_2), \ldots, y(x_n)]^T$, and unknown parameter $\theta_n = [z^*(x_1), z^*(x_2), \ldots, z^*(x_n)]^T$. $z^*(x_0)$ is the unknown parameter at an arbitrary point x_0 to be estimated. $I_{n,n}$ denotes the $n \times n$ identity matrix. $0_{n,n}$ and 0_n denote the $n \times n$ and the $n \times 1$ zero matrices, respectively.

The best estimator of the θ conditional on Y, i.e., θ^*, can be determined by maximizing the conditional probability, $p(\theta \mid Y)$, which is expressed based on Bayes' theorems, as follows:

$$p(\theta \mid Y) \propto p(\theta) p(Y \mid \theta) \tag{A3}$$

where $p(\theta)$ is a Gaussian density function with mean θ_0 and covariance V_θ; $p(Y \mid \theta)$ is a Gaussian density function with mean $M\theta$ and covariance V_ε Thus, we have

$$p(\theta) = (2\pi)^{-(n+1)/2} \left| V_\theta \right|^{-1/2} \exp\left[-\frac{1}{2} (\theta - \theta_0)^T V_\theta^{-1} (\theta - \theta_0) \right] \tag{A4}$$

$$p(Y \mid \theta) = (2\pi)^{-n/2} \left| V_\varepsilon \right|^{-1/2} \exp\left[-\frac{1}{2} (Y - M\theta)^T V_\varepsilon^{-1} (Y - M\theta) \right] \tag{A5}$$

From Eqs. (A3)–(A5), it is found that the maximization of $p(\theta \mid Y)$ is equivalent to the minimization of the following objective function:

$$J = (\theta - \theta_0)^T V_\theta^{-1} (\theta - \theta_0) + (Y - M\theta)^T V_\varepsilon^{-1} (Y - M\theta) \tag{A6}$$

It can be proved that θ^*, which gives the minimum values of J, is expressed as

$$\theta^* = \theta_0 + V_\theta M^T (V_\varepsilon + M V_\theta M^T)^{-1} (Y - M\theta) \tag{A7}$$

Based on the simple kriging assumption of the second-order stationary, the mean of random variable z_m is assumed to be the same at any location, the value of which is known. From this assumption, we have

$$\theta_0 = z_m u_n \tag{A8}$$

where z_m is the constant mean of random variable θ; $u_n = [1, 1, \ldots, 1]^T_{n \times 1}$. Based on the assumptions previously presented in Eqs. (A1), (A2), and (A8), it was proved that the above equation can be rewritten in the form of 'simple kriging' for the estimation of $z^*(x_0)$ (Hoshiya and Yoshida, 1996), as follows:

$$z^*(x_0) - z_m = V_{\theta,x0}^T V_{\theta,xn}^{-1} (\theta_n - z_m u_n) = w_{sm} (\theta_n - z_m u_n) \tag{A9}$$

where w_{sm} is the weight factor for the simple kriging and

$$V_{\theta,xn} = \begin{bmatrix} \rho(|\underline{x}_1 - \underline{x}_1|) & \cdots & \rho(|\underline{x}_1 - \underline{x}_n|) \\ \vdots & \ddots & \vdots \\ \rho(|\underline{x}_n - \underline{x}_1|) & \cdots & \rho(|\underline{x}_n - \underline{x}_n|) \end{bmatrix} \tag{A10}$$

$$V_{\theta,x0} = \begin{bmatrix} \rho(|\underline{x}_1 - \underline{x}_0|) \\ \vdots \\ \rho(|\underline{x}_n - \underline{x}_0|) \end{bmatrix} \tag{A11}$$

$\rho(|\underline{x}_i - \underline{x}_j|)$ denotes the auto-correlation function where $\underline{x}_i$ and $\underline{x}_j$ are the spatial vector coordinates.

In this paper, the extension for the derivation of the 'ordinary kriging' will be presented. It is clear from the above equations that the simple kriging actually gives the estimation of the residual, i.e., $z^*(x_0) - z_m$, assuming constant mean z_m, is known. On the other hand, the ordinary kriging directly estimates the unknown value, i.e., $z^*(x_0)$, implying that the determination of z_m is already included in the ordinary kriging formulation. Therefore, prior to deriving the ordinary kriging, a formulation to estimate z_m has to be derived. The term 'kriging the mean' is used to represent this calculation in some literature (e.g., Wackernagel, 1998).

In this case, $z^*(x_0)$ will be excluded from the calculation, while z_m will be considered as an unknown parameter to be estimated. Thus, Eq. (A2) becomes

$$Y = M\theta = I_{n,n}\theta_n = \theta_n \tag{A12}$$

It is clear that, in this case, $V_\theta = V_{\theta,xn}$. By introducing this and Eqs. (A8) and (A12) into Eq. (A6), we have

$$J = (\theta_n - z_m u_n)^T V_{\theta,xn}^{-1} (\theta_n - z_m u_n) \quad \text{(A13)}$$

The best estimator of z_m, z_m^* can be determined by minimizing J as follows:

$$\frac{\partial J}{\partial z_m} = -2u_n^T V_{\theta,xn}^{-1}(\theta_n - z_m^* u_n) = 0 \quad \text{(A14)}$$

$$z_m^* = \{[u_n^T V_{\theta,xn}^{-1} u_n]^{-1} u_n^T V_{\theta,xn}^{-1}\}\theta_n = w_m \theta_n \quad \text{(A15)}$$

where w_m is the weight factor for 'kriging the mean'.

Substituting z_m^* for z_m in Eq. (A9), i.e., the simple kriging equation, we have

$$z^*(x_0) = (w_m + w_{sm} - w_{sm} u_n w_m)\theta_n = w_{or}\theta_n \quad \text{(A16)}$$

where w_{or} is actually equivalent to the weight factor for 'ordinary kriging'. As a result, the relationship between the weights for simple, mean, and ordinary kriging is obtained. In order to prove this, the common form of the ordinary kriging equation will be shown and rearranged in corresponding terms. Using the previously defined parameters, the common form of the weight factor of the ordinary kriging (Wackernagel, 1998) can be expressed as follows:

$$\begin{bmatrix} w_{or}^T \\ \mu \end{bmatrix} = \begin{bmatrix} V_{\theta,xn} & -u_n \\ u_n^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} V_{\theta,x0} \\ 1 \end{bmatrix} \quad \text{(A17)}$$

where μ denotes the Lagrange multiplier. Based on the technique of the matrix inversion by partitioning, it can be proved that

Considering the right-hand side of Eq. (A17), let

$$\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix}^{-1} = \begin{bmatrix} A_{11}^{-1} + A_{11}^{-1}A_{12}[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}A_{21}A_{11}^{-1} & \vdots & -A_{11}^{-1}A_{12}[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1} \\ \cdots & \vdots & \cdots \\ -[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}A_{21}A_{11}^{-1} & \vdots & [A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1} \end{bmatrix}^{-1} \quad \text{(A18)}$$

$$\begin{bmatrix} V_{\theta,xn} & \vdots & -u_n \\ \cdots & \vdots & \cdots \\ u_n^T & \vdots & 0 \end{bmatrix} = \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \vdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix} \quad \text{(A19)}$$

From Eqs. (A17)–(A19), we have

$$w_{or} = [u_n^T V_{\theta,xn}^{-1} u_n]^{-1} u_n^T V_{\theta,xn}^{-1} + V_{\theta,x0}^T V_{\theta,xn}^{-1} - V_{\theta,x0}^T V_{\theta,xn}^{-1} u_n \times [u_n^T V_{\theta,xn}^{-1} u_n]^{-1} u_n^T V_{\theta,xn}^{-1} \quad \text{(A20)}$$

which is equivalent to

$$w_{or} = w_m + w_{sm} - w_{sm} u_n w_m \quad \text{(A21)}$$

The above equation is identical to the one shown in Eq. (A16), which has been derived based on a Bayesian formulation. Therefore, the derivation of the ordinary kriging using a Bayesian approach has now been completely proved.

References

Akaike, H., 1980. Likelihood and Bayes procedure with discussion. In: Bernardo, J.M. (Ed.), Bayesian Statistics. Univ. Press, Valencia, Spain, pp. 143–166 185–203.

Asaoka, A., 1978. Observational procedure of settlement prediction. Soil and Foundations 18 (4), 87–101.

Baecher, G.B., Christian, J.T., 2003. Reliability and Statistics in Geotechnical Engineering. John Wiley & Sons, West Sussex.

Baecher, G.B., Christian, J.T., 2008. Spatial Variability and Geotechnical Reliability. Reliability-Based Design in Geotechnical Engineering. Taylor & Francis, Oxon 76–133.

Bjerrum, L., 1967. Engineering geology of Norwegian normally consolidated marine clays as related to settlement of buildings. Géotechnique 17 (2), 81–118.

DeGroot, D.J., Baecher, G.B., 1993. Estimating autocovariance of in-situ soil properties. Journal of Geotechnical Engineering 119 (1), 147–166.

Fenton, G.A., 1994. Error evaluation of three random-field generators. Journal of Engineering Mechanics ASCE 120 (12), 2478–2497.

Fenton, G.A., Griffiths, D.V., 2002. Probabilistic foundation settlement on spatially random soil. Journal of Geotechnical and Geoenvironmental Engineering ASCE 128 (5), 381–390.

Fenton, G.A., Vanmarcke, E.H., 1990. Simulation of random fields via local average subdivision. Journal of Engineering Mechanics ASCE 116 (8), 1733–1749.

Garlanger, J.E., 1972. The consolidation of soils exhibiting creep under constant effective stress. Géotechnique 22 (1), 71–78.

Griffiths, D.V., Fenton, G.A., 2004. Probabilistic slope stability by finite elements. Journal of Geotechnical and Geoenvironmental Engineering ASCE 130 (5), 507–518.

Honjo, Y., Kashiwagi, N., 1999. Matching objective and subjective information in groundwater inverse analysis by Akaike's Bayesian Information Criterion. Water Resources Research 35 (2), 435–447.

Hoshiya, M., Yoshida, I., 1996. Identification of conditional stochastic Gaussian field. Journal of Engineering Mechanics ASCE 122 (2), 101–108.

Krige, D.G., 1966. Two dimensional weighted moving averaging trend surfaces for ore evaluation. Proceedings of the Symposium on Mathematical Statistics and Computer Applications for Ore Evaluation.

Lumb, P., 1974. Application of statistics in soil mechanics. In: Lee, I.K. (Ed.), Soil Mechanics: New Horizons. Newnes-Butterworth, London, pp. 44–112 221–239.

Matheron, G., 1973. The intrinsic random functions and their applications. Advances in Applied Probability, 5.

Mesri, G., et al., 1997. Secondary compression of peat with or without surcharging. Journal of the Geotechnical Engineering Division ASCE 123 (5), 411–421.

Rungbanaphan, P., Honjo, Y., Yoshida, I., 2010. Settlement prediction by spatial-temporal random process using Asaoka's method. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards 4 (4), 174–185.

Shinozuka, M., 1971. Simulation of multivariate and multidimensional random processes. Journal of the Acoustical Society of America 49 (1, part 2), 357–367.

Shinozuka, M., Jan, C.-M., 1972. Digital simulation of random processes and its applications. Journal of Sound and Vibration 25 (1), 111–128.

Sridharan, A., Murthy, N.S., Prakash, K., 1987. Rectangular hyperbolar method of consolidation analysis. Géotechnique 37 (3), 355–368.

Tan, S.A., 1994. Hyperbolic method for settlements in clays with vertical drains. Canadian Geotechnical Journal 31, 125–131.

Ueda, T., Honjo, Y., Hatano, T., Sakaguchi, S., 1986. Estimation of the spatial distribution of residual settlement in a reclaimed area. Journal of Japanese Society of Geotechnical Society (Tsuchi to Kiso) 34 (5), 51–58 in Japanese.

Vanmarcke, E.H., 1977. Probabilistic modeling of soil profiles. Journal of the Geotechnical Engineering Division ASCE 103 (GT11), 1227–1246.

Wackernagel, H., 1998. Multivariate Geostatistics: An Introduction with Applications second ed. Springer-Verlag, Berlin, Heidelberg, Germany.