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High energy neutrino spin light

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Abstract

The quantum theory of spin light (electromagnetic radiation emitted by a Dirac massive neutrino propagating in dense matter due to the weak interaction of a neutrino with background fermions) is developed. In contrast to the Cherenkov radiation, this effect does not disappear even if the medium refractive index is assumed to be equal to unity. The formulas for the transition rate and the total radiation power are obtained. It is found out that radiation of photons is possible only when the sign of the particle helicity is opposite to that of the effective potential describing the interaction of a neutrino (antineutrino) with the background medium. Due to the radiative self-polarization the radiating particle can change its helicity. As a result, the active left-handed polarized neutrino (right-handed polarized antineutrino) converting to the state with inverse helicity can become practically "sterile". Since the sign of the effective potential depends on the neutrino flavor and the matter structure, the spin light can change a ratio of active neutrinos of different flavors. In the ultra relativistic approach, the radiated photons averaged energy is equal to one third of the initial neutrino energy, and two thirds of the energy are carried out by the final "sterile" neutrinos.

A Dirac massive neutrino has nontrivial electromagnetic properties. In particular, it possesses nonzero magnetic moment [1]. Therefore a Dirac massive neutrino propagating in dense matter can emit electromagnetic radiation due to the weak interaction of a neutrino with background fermions [2,3]. As a result of the radiation, neutrino can change its helicity due to the radiative self-polarization. In contrast to the Cherenkov radiation, this effect does not disappear even if the refractive index of the medium is assumed to be equal to unity. This conclusion is valid for any model of neutrino interactions breaking spatial parity. The phenomenon was called the neutrino spin light in analogy with the effect, related with the synchrotron radiation power depending on the electron spin orientation (see [4]).

The properties of spin light were investigated basing upon the quasi-classical theory of radiation and self-polarization of neutral particles [5,6] with the use of the Bargmann–Michel–Telegdi (BMT) equation [7] and its

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generalizations [8,9]. This theory is valid when the radiated photon energy is small as compared with the neutrino energy, and this narrows the range of astrophysical applications of the obtained formulas.

In the present Letter, the properties of spin light are investigated basing upon the consistent quantum theory, and this allows the neutrino recoil in the act of radiation to be considered for. The above mentioned restriction is eliminated in this way.

On the other hand, the detailed analysis of the results of our investigations shows that the features of the effect depend on the neutrino flavor, helicity and the matter structure [10]. This fact leads to the conclusion that the spin light can initiate transformation of a neutrino from the active state to a practically "sterile" state, and the inverse process is also possible.

When the interaction of a neutrino with the background fermions is considered to be coherent, the propagation of a massive neutrino in the matter is described by the Dirac equation with the effective potential [11,12]. In what follows, we restrict our consideration to the case of a homogeneous and isotropic medium. Then in the frameworks of the minimally extended standard model, the form of this equation is uniquely determined by the assumptions similar to those adopted in [13]

$$\left(i\hat{\partial} - \frac{1}{2}\hat{f}\left(1+\gamma^5\right) - m_\nu\right)\Psi_\nu = 0. \tag{1}$$

The function f^μ is a linear combination of fermion currents and polarizations. The quantities with hats denote scalar products of Dirac matrices with 4-vectors, i.e., $\hat{a} \equiv \gamma^\mu a_\mu$.

If the medium is at rest and unpolarized then $\mathbf{f} = 0$. The component f^0 calculated in the first order of the perturbation theory is as follows [14–16]

$$f^0 = \sqrt{2}G_{\mathrm{F}}\left\{\sum_f \left(I_{e\nu} + T_3^{(f)} - 2Q^{(f)}\sin^2\theta_{\mathrm{W}}\right)(n_f - n_{\bar{f}})\right\}. \tag{2}$$

Here, $n_f, n_{\bar{f}}$ are the number densities of background fermions and antifermions, $Q^{(f)}$ is the electric charge of the fermion and $T_3^{(f)}$ is the third component of the weak isospin for the left-chiral projection of it. The parameter $I_{e\nu}$ is equal to unity for the interaction of electron neutrino with electrons. In other cases $I_{e\nu} = 0$. Summation is performed over all fermions f of the background.

Let us obtain a solution of Eq. (1). Since function $f^\mu = \mathrm{const}$, Eq. (1) commutes with operators of canonical momentum $i\partial_\mu$. However, the commonly adopted choice of eigenvalues of this operator as quantum numbers in this problem is not satisfactory. Kinetic momentum components of a particle, related to its group 4-velocity u^μ by the relation $q^\mu = m_\nu u^\mu$, $q^2 = m_\nu^2$, are more suitable to play the role of its quantum numbers. This choice can be justified, since it is the particle kinetic momentum that can be really observed.

The explicit form of the kinetic momentum operator for the particle with spin is not known beforehand, and hence, in order to find the appropriate solutions, we have to use the correspondence principle.

It was shown in [8] that when the effects of the neutrino weak interaction are taken into account, the Lorentz invariant generalization of the BMT equation for spin vector S^μ is as follows:

$$\dot{S}^\mu = 2\mu_0\left\{\left(F^{\mu\nu} + G^{\mu\nu}\right)S_\nu - u^\mu u_\nu\left(F^{\nu\lambda} + G^{\nu\lambda}\right)S_\lambda\right\}, \tag{3}$$

where

$$G^{\mu\nu} = \frac{1}{2\mu_0}e^{\mu\nu\rho\lambda}f_\rho u_\lambda, \tag{4}$$

and a dot denotes the differentiation with respect to the proper time τ.

Let us introduce the quasi-classical spin wave functions. Such wave functions can be constructed as follows [6,9]. Suppose the Lorentz equation is solved, i.e., the dependence of particle coordinates on proper time is found. Then the BMT equation transforms to ordinary differential equation, whose resolvent determines a one-parametric subgroup of the Lorentz group. The quasi-classical spin wave function is represented by a spin-tensor, whose evolution is determined by the same one-parametric subgroup.

In the case when the effect of an external electromagnetic field can be neglected as compared with the effect of the neutrino interaction with the background matter, the equation for the neutrino quasi-classical wave function $\Psi(\tau)$ is

$$\dot{\Psi}(\tau) = i\mu_0\gamma^5{}^{\star}G^{\mu\nu}u_\nu\gamma_\mu\hat{u}\Psi(\tau), \tag{5}$$

where ${}^{\star}G^{\mu\nu} = -\frac{1}{2}e^{\mu\nu\rho\lambda}G_{\rho\lambda}$ is a tensor dual to $G^{\mu\nu}$. Obviously, the quasi-classical density matrix of a polarized neutrino takes the form

$$\rho(\tau,\tau') = \frac{1}{2}U(\tau,\tau_0)\bigl(\hat{q}(\tau_0)+m\bigr)\bigl(1-\gamma^5\hat{S}(\tau_0)\bigr)U^{-1}(\tau',\tau_0), \tag{6}$$

where $U(\tau,\tau_0)$ is the resolvent of Eq. (5), and the equation for $U(\tau,\tau_0)$ is

$$\dot{U}(\tau,\tau_0) = \frac{i}{4}\gamma^5(\hat{f}\hat{u}-\hat{u}\hat{f})U(\tau,\tau_0). \tag{7}$$

We note that the operator $U(\tau,\tau_0)$ is defined up to a phase factor $e^{-iF(x)}$, with the derivative of the exponent with respect to the proper time is equal to zero:

$$\dot{F}(x) = 0. \tag{8}$$

Let us choose the solution of Eq. (1) in the form [9]

$$\Psi(x) = U\bigl(\tau(x)\bigr)\Psi_0(x), \tag{9}$$

where Ψ_0 is a solution of the Dirac equation for a free particle

$$\Psi_0(x) = e^{-i(qx)}(\hat{q}+m_\nu)\bigl(1-\gamma^5\hat{S}_0\bigr)\psi^0. \tag{10}$$

Here ψ^0 is constant bispinor and $\Psi_0(x)$ normalized by the condition

$$\bar{\Psi}_0(x)\Psi_0(x) = 2m_\nu.$$

Substitution of the expression (9) in Eq. (1) results in the relation

$$\left\{\hat{q}+(\hat{\partial}F)-\frac{1}{2}\hat{f}+\frac{1}{2}\gamma^5\hat{f}+\frac{1}{4}\gamma^5\hat{N}(\hat{f}\hat{u}-\hat{u}\hat{f})-m_\nu\right\}e^{-iF(x)}U\bigl(\tau(x)\bigr)\Psi_0 = 0, \tag{11}$$

where $N^\mu = \partial^\mu\tau$. Since the commutator $[\hat{q},U]=0$, and the matrix U is nondegenerate, then for this relation to hold the following condition is required

$$(\hat{\partial}F)-\frac{1}{2}\hat{f}+\frac{1}{2}\gamma^5\hat{f}+\frac{1}{4}\gamma^5\hat{N}(\hat{f}\hat{u}-\hat{u}\hat{f}) = 0. \tag{12}$$

It is easy to find out that Eq. (12) is valid only if

$$\partial^\lambda F = \frac{1}{2}f^\lambda, \qquad e^{\mu\nu\rho\lambda}N_\mu f_\nu u_\rho = 0, \qquad \bigl(1-(Nu)\bigr)f^\lambda+(Nf)u^\lambda = 0. \tag{13}$$

From two latter equations it follows that

$$N^\mu = \frac{f^\mu(fu)-u^\mu f^2}{(fu)^2-f^2u^2}. \tag{14}$$

So $f^\mu = \text{const}$, then

$$\tau = (Nx), \qquad F = \frac{1}{2}(fx), \tag{15}$$

and we can write

$$U(x) = e^{-i(fx)/2} \sum_{\zeta=\pm 1} e^{i\zeta\varphi} \Lambda_\zeta. \tag{16}$$

Here

$$\Lambda_\zeta = \frac{1}{2}\big[1 - \zeta\gamma^5 \hat{S}_{tp}\hat{q}/m_\nu\big], \quad \zeta \pm 1, \tag{17}$$

are spin projection operators with eigenvalues $\zeta \pm 1$ respectively, and

$$\varphi = \frac{\tau}{2}\sqrt{(fq)^2 - f^2 m_\nu^2} = \frac{(fq)(fx) - f^2(qx)}{2\sqrt{(fq)^2 - f^2 m_\nu^2}}, \qquad S^\mu_{tp} = \frac{q^\mu (fq)/m_\nu - f^\mu m_\nu}{\sqrt{(fq)^2 - f^2 m_\nu^2}}. \tag{18}$$

From the obtained formulas it follows that the eigenvalues of the operator of canonical momentum $i\partial^\mu$ are

$$P^\mu = q^\mu\left(1 + \frac{\zeta f^2}{2\sqrt{(fq)^2 - f^2 m_\nu^2}}\right) + \frac{f^\mu}{2}\left(1 - \frac{\zeta (fq)}{\sqrt{(fq)^2 - f^2 m_\nu^2}}\right). \tag{19}$$

The dispersion law follows from Eq. (19) in the form

$$P^2 = m_\nu^2 + (Pf) - f^2/2 - \zeta\sqrt{\big((Pf) - f^2/2\big)^2 - f^2 m_\nu^2}. \tag{20}$$

If the medium is at rest and unpolarized then the neutrino total energy and canonical momentum are determined by the formulas

$$\varepsilon = q^0 + f^0/2, \qquad \mathbf{P} = \mathbf{q}\Delta_{q\zeta}, \tag{21}$$

where $\Delta_{q\zeta} = 1 + \zeta f^0/2|\mathbf{q}|$, and

$$S^\mu_{tp} = \frac{1}{m_\nu}\big\{|\mathbf{q}|, q^0\mathbf{q}/|\mathbf{q}|\big\}, \tag{22}$$

i.e., the eigenvalues $\zeta = \pm 1$ determine the helicity of the particle. Consequently, the dispersion law is

$$\varepsilon = \sqrt{\big(\Delta|\mathbf{P}| - \zeta f^0/2\big)^2 + m_\nu^2} + f^0/2, \tag{23}$$

where $\Delta = \mathrm{sign}(\Delta_{q\zeta})$. Obviously

$$\frac{\partial\varepsilon}{\partial\mathbf{P}} = \frac{\mathbf{q}}{q^0}$$

is the particle group velocity.

The relation (23) differs those used in previous papers (see, for example, [18]) by the multiplier Δ. This is due to the fact that, in these papers the projection of the particle spin on the canonical momentum **P** and not the helicity of the particle was used as the spin quantum number ζ. The helicity is the projection of the spin on the direction of its kinetic momentum [19–21], because the rest frame of the particle is determined by the condition that its group velocity is equal to zero. In our problem the directions of canonical and kinetic momenta, generally speaking, are different, and hence, the projection of particle spin on the canonical momentum does not coincide with its helicity.

From formulas (21), it is seen that if the energy is expressed in terms of the kinetic momentum, then it does not depend on the particle helicity, while the particle canonical momentum is a function of the helicity. Therefore, the statement of the authors of [17], i.e., that the radiation of photons in the process of the spin light emission takes place due to neutrino transitions from the "exited" helicity state to the low-lying helicity state in matter, is not correct.

Let us consider the process of emitting photons by a massive neutrino in unpolarized matter at rest. In this case, the orthonormalized system of solutions for Eq. (1) is

$$\Psi(x)=\frac{|\Delta_{q\zeta}|}{\sqrt{2q^0}}e^{-i(q^0+f^0/2)x^0}e^{i\mathbf{qx}\Delta_{q\zeta}}(\hat{q}+m_\nu)\big(1-\zeta\gamma^5\hat{S}_{tp}\big)\psi^0. \tag{24}$$

The formula for the spontaneous radiation transition probability of a neutral fermion with anomalous magnetic moment μ_0 is[1]

$$P=-\frac{1}{2p^0}\int d^4x\,d^4y\int\frac{d^4q\,d^4k}{(2\pi)^6}\delta\big(k^2\big)\delta\big(q^2-m_\nu^2\big)\,\mathrm{Sp}\big\{\Gamma_\mu(x)\varrho_i(x,y;p,\zeta_i)\Gamma_\nu(y)\varrho_f(y,x;q,\zeta_f)\big\}$$
$$\times\,\varrho_{ph}^{\mu\nu}(x,y;k). \tag{25}$$

Here, $\varrho_i(x,y;p)$, $\varrho_f(y,x;q)$ are density matrices of the initial (i) and final (f) states of the fermion, $\varrho_{ph}^{\mu\nu}(x,y;k)$ is the radiated photon density matrix, $\Gamma^\mu=-\sqrt{4\pi}\mu_0\sigma^{\mu\nu}k_\nu$ is the vertex function. The density matrix of longitudinally polarized neutrino in the unpolarized matter at rest constructed with the use of the solutions of Eq. (1) has the form

$$\varrho(x,y;p,\zeta)=\frac{1}{2}\Delta_{p\zeta}^2(\hat{p}+m_\nu)\big(1-\zeta\gamma^5\hat{S}_p\big)e^{-i(x^0-y^0)(p^0+f^0/2)+i(\mathbf{x}-\mathbf{y})\mathbf{p}\Delta_{p\zeta}}. \tag{26}$$

After summing over photon polarizations[2] and integrating with respect to coordinates we obtain the expression for the transition rate under investigation:

$$W=\frac{\mu_0^2}{p^0}\int\frac{d^4q\,d^4k}{(2\pi)}\delta\big(k^2\big)\delta\big(q^2-m_\nu^2\big)\delta\big(p^0-q^0-k^0\big)\delta^3(\mathbf{p}\Delta_{p\zeta_i}-\mathbf{q}\Delta_{q\zeta_f}-\mathbf{k})T(p,q), \tag{27}$$

where

$$T(p,q)=4\Delta_{p\zeta_i}^2\Delta_{q\zeta_f}^2\big\{(pk)(qk)-\zeta_i\zeta_f\big[k^0|\mathbf{p}|-p^0(\mathbf{pk})/|\mathbf{p}|\big]\big[k^0|\mathbf{q}|-q^0(\mathbf{qk})/|\mathbf{q}|\big]\big\}. \tag{28}$$

After integrating over $\mathbf{k}$, k^0, $|\mathbf{q}|$ we obtain the spectral-angular distribution of the final neutrino

$$W=-\zeta_i\zeta_f\frac{\mu_0^2}{\pi p^0|\mathbf{p}|}\int_{m_\nu}^{p^0}dq^0\Delta_{p\zeta_i}\Delta_{q\zeta_f}\int dO\,\delta\big(\big(p^0-q^0\big)^2+2|\mathbf{p}||\mathbf{q}|\Delta_{p\zeta_i}\Delta_{q\zeta_f}\cos\vartheta_\nu-|\mathbf{p}|^2\Delta_{p\zeta_i}^2-|\mathbf{q}|^2\Delta_{q\zeta_f}^2\big)$$
$$\times\big\{\big(f^0/2\big)^2\big[\zeta_f|\mathbf{p}||\mathbf{q}|+\zeta_i\big(m_\nu^2-p^0q^0\big)\big]^2+\big[\big(f^0/2\big)\big(\zeta_iq^0|\mathbf{p}|-\zeta_fp^0|\mathbf{q}|\big)+m_\nu^2\big(p^0-q^0\big)\big]^2\big\}, \tag{29}$$

where

$$|\mathbf{q}|=\sqrt{\big(q^0\big)^2-m_\nu^2}.$$

It is convenient to express the results of integrating over angular variables using dimensionless quantities. Introducing the notations

$$x=q^0/m_\nu,\qquad \gamma=p^0/m_\nu,\qquad d=\big|f^0\big|/2m_\nu,\qquad \bar{\zeta}_{i,f}=\zeta_{i,f}\,\mathrm{sign}\big(f^0\big) \tag{30}$$

[1] In the expression for the radiation energy $\mathcal{E}$, the additional factor k, i.e., the energy of radiated photon, appears in the integrand.

[2] We do not consider the polarization of spin light photons here. In the quasi-classical approximation, this problem was investigated in [17].

we have

$$W_{\bar{\zeta}_f} = \frac{\mu_0^2 m_\nu^3}{\gamma(\gamma^2-1)} \int \frac{dx}{\sqrt{x^2-1}} \big\{ d^2 \big[\bar{\zeta}_f \sqrt{\gamma^2-1}\sqrt{x^2-1} - \bar{\zeta}_i(\gamma x - 1) \big]^2 + \big[\gamma - x + d\big(\bar{\zeta}_i x \sqrt{\gamma^2-1} - \bar{\zeta}_f \gamma \sqrt{x^2-1}\big)\big]^2 \big\}. \tag{31}$$

The integration bounds in the formula (31) are

$$x \in \emptyset, \qquad \gamma \in [1, \infty), \tag{32}$$

if $\bar{\zeta}_i = 1$,

$$\begin{aligned} &x \in \emptyset, && \gamma \in [1, \gamma_0), \\ &x \in [\omega_1, \omega_2], && \gamma \in [\gamma_0, \gamma_1), \\ &x \in [1, \omega_2], && \gamma \in [\gamma_1, \gamma_2), \\ &x \in \emptyset, && \gamma \in [\gamma_2, \infty), \end{aligned} \tag{33}$$

if $\bar{\zeta}_i = -1$, $\bar{\zeta}_f = -1$, and

$$\begin{aligned} &x \in \emptyset, && \gamma \in [1, \gamma_1), \\ &x \in [1, \omega_1], && \gamma \in [\gamma_1, \gamma_2), \\ &x \in [\omega_2, \omega_1], && \gamma \in [\gamma_2, \infty), \end{aligned} \tag{34}$$

if $\bar{\zeta}_i = -1$, $\bar{\zeta}_f = 1$.

Here

$$\omega_1 = \frac{1}{2}\big(z_1 + z_1^{-1}\big), \qquad \omega_2 = \frac{1}{2}\big(z_2 + z_2^{-1}\big), \tag{35}$$

where

$$z_1 = \gamma + \sqrt{\gamma^2-1} - 2d, \qquad z_2 = \gamma - \sqrt{\gamma^2-1} + 2d, \tag{36}$$

and

$$\begin{aligned} &\gamma_0 = \sqrt{1+d^2}, \\ &\gamma_1 = \frac{1}{2}\big\{(1+2d) + (1+2d)^{-1}\big\}, \\ &\gamma_2 = \frac{1}{2}\big\{(1-2d) + (1-2d)^{-1}\big\}, \quad d < 1/2, \\ &\gamma_2 = \infty, \quad d \geqslant 1/2. \end{aligned} \tag{37}$$

The integration is carried out elementary. The transition rate under investigation is defined as

$$W_{\bar{\zeta}_f} = \frac{\mu_0^2 m_\nu^3}{4} \big\{ (1+\bar{\zeta}_f)\big[Z(z_1,1)\Theta(\gamma-\gamma_1) + Z(z_2,-1)\Theta(\gamma-\gamma_2)\big] + (1-\bar{\zeta}_f)\big[Z(z_1,1)\Theta(\gamma_1-\gamma) + Z(z_2,-1)\Theta(\gamma_2-\gamma)\big]\Theta(\gamma-\gamma_0)\big\}(1-\bar{\zeta}_i). \tag{38}$$

Here

$$Z(z,\bar\zeta_f)=\frac{1}{\gamma(\gamma^2-1)}\Big\{\ln z\big[\gamma^2+d\sqrt{\gamma^2-1}+d^2+1/2\big]+\frac{1}{4}\big(z^2-z^{-2}\big)\big[d^2\big(2\gamma^2-1\big)+d\sqrt{\gamma^2-1}+1/2\big]$$
$$+\frac{\bar\zeta_f}{4}\big(z-z^{-1}\big)^2\big[2d\sqrt{\gamma^2-1}+1\big]d\gamma-\big(z-z^{-1}\big)\big[d^2+d\sqrt{\gamma^2-1}+1\big]\gamma$$
$$-\bar\zeta_f\big(z+z^{-1}-2\big)\big[d\sqrt{\gamma^2-1}+\gamma^2\big]d\Big\}. \tag{39}$$

Therefore, the transition rate after summation over polarizations of the final neutrino becomes

$$W_{\bar\zeta_f=1}+W_{\bar\zeta_f=-1}=\frac{\mu_0^2 m_\nu^3}{2}(1-\bar\zeta_i)\big\{Z(z_1,1)+Z(z_2,-1)\big\}\Theta(\gamma-\gamma_0). \tag{40}$$

If $d\gamma\ll 1$, then expression (38) leads to the formula

$$W_{\bar\zeta_f}=\frac{16\mu_0^2 m_\nu^3 d^3}{3\gamma}\big(\gamma^2-1\big)^{3/2}(1-\bar\zeta_i)(1+\bar\zeta_f), \tag{41}$$

obtained in the quasi-classical approximation in [3].

In the ultra-relativistic limit ($\gamma\gg 1, d\gamma\gg 1$), the transition rate is given by the expression

$$W_{\bar\zeta_f}=\mu_0^2 m_\nu^3 d^2\gamma(1-\bar\zeta_i)(1+\bar\zeta_f). \tag{42}$$

Let us consider now the radiation power. If we introduce the function

$$\tilde Z(z,\bar\zeta_f)=\gamma Z(z,\bar\zeta_f)-Y(z,\bar\zeta_f), \tag{43}$$

where

$$Y(z,\bar\zeta_f)=\frac{1}{\gamma(\gamma^2-1)}\Big\{-\ln z\big[d^2+d\sqrt{\gamma^2-1}+1\big]\gamma-\frac{1}{4}\big(z^2-z^{-2}\big)\big[d^2+d\sqrt{\gamma^2-1}+1\big]\gamma$$
$$+\frac{1}{12}\big(z-z^{-1}\big)^3\big[d^2\big(2\gamma^2-1\big)+d\sqrt{\gamma^2-1}+1/2\big]$$
$$+\frac{1}{2}\big(z-z^{-1}\big)\big[2d^2\gamma^2+2d\sqrt{\gamma^2-1}+\gamma^2+1\big]$$
$$+\frac{\bar\zeta_f}{12}\big(\big(z+z^{-1}\big)^3-8\big)\big[2d\sqrt{\gamma^2-1}+1\big]d\gamma-\frac{\bar\zeta_f}{12}\big(z-z^{-1}\big)^2\big[d\sqrt{\gamma^2-1}+\gamma^2\big]d\Big\}, \tag{44}$$

then the formula for the total radiation power can be obtained from (38), (40) by the substitution $Z(z,\bar\zeta_f)\to\tilde Z(z,\bar\zeta_f)$. It can be verified that if $d\gamma\ll 1$ then the radiation power is

$$I_{\bar\zeta_f}=\frac{32\mu_0^2 m_\nu^4 d^4}{3}\big(\gamma^2-1\big)^2(1-\bar\zeta_i)(1+\bar\zeta_f). \tag{45}$$

This result was obtained in the quasi-classical approximation in [2]. In the ultra-relativistic limit, the radiation power is equal to

$$I_{\bar\zeta_f}=\frac{1}{3}\mu_0^2 m_\nu^4 d^2\gamma^2(1-\bar\zeta_i)(1+\bar\zeta_f). \tag{46}$$

It can be seen from Eqs. (42) and (46) that in the ultra-relativistic limit the averaged energy of emitted photons is $\langle\varepsilon_\gamma\rangle=\varepsilon_\nu/3$. It should be pointed out that the obtained formulas are valid both for a neutrino and for an antineutrino. The charge conjugation operation leads to the change of the sign of the effective potential and the replacement of the left-hand projector by the right-hand one in Eq. (1). Thus the sign in front of the γ^5 matrix remains invariant.

Using Eq. (27), it is possible to find the dependence of the radiated photon energy on the angle ϑ_γ between the direction of the neutrino propagation and the photon wave vector

$$\frac{k^0}{m_\nu} = 2d\frac{\beta X - d/\gamma}{(X + d/\gamma)(X - d/\gamma)}. \tag{47}$$

Here $\beta = \sqrt{\gamma^2 - 1}/\gamma$ is the neutrino velocity and $X = 1 - (\beta - d/\gamma)\cos\vartheta_\gamma$. In the quasi-classical approximation, this formula reduces to the relation

$$\frac{k^0}{m_\nu} = \frac{2d\beta}{1 - \beta\cos\vartheta_\gamma}, \tag{48}$$

which follows from the results of [3] after Lorentz transformation to the laboratory frame.

The following conclusions can be made from the obtained results. A neutrino (antineutrino) can emit photons due to coherent interaction with matter only when its helicity has the sign opposite to the sign of the effective potential f^0. Otherwise, radiation transitions are impossible. In the case of low energies of the initial neutrino, only radiation without spin-flip is possible and the probability of the process is very low. At high energies, the main contribution to radiation is given by the transitions with the spin-flip, the transitions without spin-flip are either absent or their probability is negligible. This leads to total self-polarization, i.e., the initially left-handed polarized neutrino (right-handed polarized antineutrino) is transformed to practically "sterile" right-handed polarized neutrino (left-handed polarized antineutrino). For "sterile" particles, the situation is opposite. They can be converted to the active form in the medium "transparent" for the active neutrino.

With the use of the effective potential calculated in the first order of the perturbation theory (2), the following conclusions can be made. If the matter consists only of electrons then, in the framework of the minimally extended standard model in the ultra-relativistic limit (here we use Gaussian units), we have for the transition rate

$$W_{\bar\zeta_f} = \frac{\alpha\varepsilon_\nu}{32\hbar}\left(\frac{\mu_0}{\mu_{\rm B}}\right)^2\left(\frac{\tilde G_{\rm F} n_e}{m_e c^2}\right)^2(1 - \bar\zeta_i)(1 + \bar\zeta_f), \tag{49}$$

and for the total radiation power

$$I_{\bar\zeta_f} = \frac{\alpha\varepsilon_\nu^2}{96\hbar}\left(\frac{\mu_0}{\mu_{\rm B}}\right)^2\left(\frac{\tilde G_{\rm F} n_e}{m_e c^2}\right)^2(1 - \bar\zeta_i)(1 + \bar\zeta_f). \tag{50}$$

Here ε_ν is the neutrino energy, $\mu_{\rm B} = e/2m_e$ is the Bohr magneton, α is the fine structure constant, m_e is the electron mass and $\tilde G_{\rm F} = G_{\rm F}(1 + 4\sin^2\theta_{\rm W})$, where $G_{\rm F}$, $\theta_{\rm W}$ are the Fermi constant and the Weinberg angle respectively. Thus, after the radiative transition, two thirds of the initial active neutrino energy are carried away by the final "sterile" one.

At the same time, as it can be seen from Eq. (2), a muon neutrino in the electron medium does not emit any radiation. Moreover, a muon neutrino does not emit radiation in an electrically neutral medium, when the number density of protons is equal to the electron number density. And an electron neutrino can emit radiation if the electron number density is greater than the neutron number density. An example of such medium is provided by the Sun. Therefore the spin light can change the ratio of active neutrino of different flavors.

It is obviously that the above conclusions change to opposite if the matter consists of antiparticles. Therefore the neutrino spin light can serve as a tool for determination of the type of astrophysical objects, since neutrino radiative transitions in dense matter can result in radiation of photons of super-high energies, even exceeding the GZK cutoff. Indeed, the neutron medium is "transparent" for all active neutrinos, but an active antineutrino emits radiation in such a medium, the transition rate and the total radiation power can be obtained from Eqs. (49) and (50) after substitution $\tilde G_{\rm F} \to G_{\rm F}$, $n_e \to n_n$. If the density of the neutron star is assume to be $n \approx 10^{38}$, the transition rate is estimated as

$$W = 10^{22}\frac{\varepsilon_\nu}{\varepsilon_{\rm GZK}}\left(\frac{\mu_0}{\mu_{\rm B}}\right)^2, \tag{51}$$

where $\varepsilon_{\mathrm{GZK}} = 5 \times 10^{19}$ eV is GZK cutoff energy. Although the transition rate determined by Eq. (51) is extremely low, this effect can still serve as one of a possible explanations of the cosmic ray paradox.

The spin light can also be important for the understanding of the "dark matter" formation mechanism in the early stages of evolution of the Universe.

When the present Letter was already submitted for publication, we came across an article [22], where the spin light theory was also considered. The formulas of [22] in the ultra-relativistic limit of physical interest reproduce the results for the transition rate and the total power of spin light already obtained in our earlier publication [10].

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