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$D^* D_s K$ and $D_s^* D K$ vertices in a QCD sum rule approach

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Abstract

We calculate the strong form factors and coupling constants of $D^* D_s K$ and $D_s^* D K$ vertices using the QCD sum rules technique. In each case we have considered two different cases for the off-shell particle in the vertex: the lightest meson and one of the heavy mesons. The method gives the same coupling constant for each vertex. When the results for different vertices are compared, they show that the SU(4) symmetry is broken by around 40%.

PACS: 14.40.Lb; 14.40.Nd; 12.38.Lg; 11.55.Hx

The knowledge of the form factors in hadronic vertices is of crucial importance to estimate hadronic amplitudes when hadronic degrees of freedon are used. When all the particles in a hadronic vertex are on-mass-shell, the effective fields of the hadrons describe pointlike physics. However, when at least one of the particles in the vertex is off-shell, the finite size effects of the hadrons become important.

In this work we study the $D^* D_s K$ and $D_s^* D K$ vertices, which are fundamental to the evaluation of the dissociation cross section of J/Ψ by kaons when using effective Lagrangians. The suppression of charmonium production is one of the most traditional signatures of the quark–gluon plasma (QGP) formation in relativistic heavy ion collisions [1]. The dissociation of charmoniums in the QGP due to color screening would lead to a reduction of its production in such collisions. However, using the charmonium suppression as a signature of QGP formation requires the understanding of J/Ψ production and absorption mechanisms in hadronic matter, because this suppression may be due to the interactions with the comovers during the collision [2,3].

One of the approaches used to study the interaction of charmonium with the hadronic medium, mainly in the low energy region ($\sqrt{s} < 10$ GeV), is based on effective SU(4) Lagrangians [4–9]. This technique, however, requires the detailed knowledge of the form factors in the hadronic vertices. The calculated cross section may change by a factor of two if a soft, instead of a hard, form factor is used in the vertices containing charmed mesons.

This situation gave us the motivation to start a program to calculate charmed form factors and coupling constants, using the QCD sum rules approach [10]. We have been continuously working on this problem and computing different vertices [11–17]. An interesting subproduct of such calculations [12–14,17], was the understanding of the behavior of the off-shell particle probing of the vertex: heavier particles resolves better the structure of the vertex, while lighter particles are more suitable for measuring its size. This conclusion is also supported in the present work.

As a part of this project we evaluate, in the present calculation, the form factors in the vertices $D^* D_s K$ and $D_s^* D K$, and compare the results with the predictions from the exact SU(4) symmetry [9].

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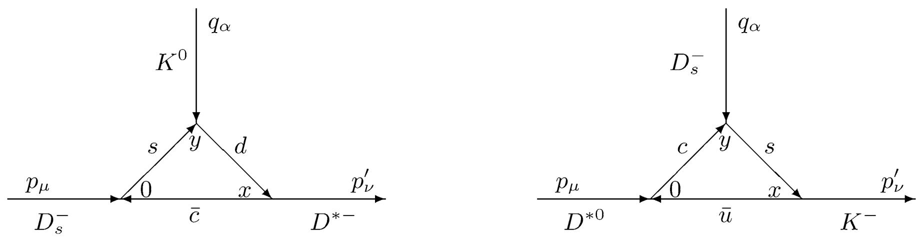

Fig. 1. Perturbative diagrams for K off-shell (left) and D_s off-shell (right) corresponding to the D^*D_sK vertex.

Following the QCDSR formalism described in our previous works [11–17], we write the three-point correlation function associated with the D^*D_sK vertex, which is given by

$$\Gamma_\mu^{(K)}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \langle 0|T\{j_\mu^{D^*}(x) j^{K\dagger}(y) j^{D_s^\dagger}(0)\}|0\rangle \tag{1}$$

for K meson off-shell, where the interpolating currents are $j_\mu^{D^*} = \bar{c}\gamma_\mu d$, $j^K = i\bar{s}\gamma_5 d$ and $j^{D_s} = i\bar{c}\gamma_5 s$, and

$$\Gamma_{\mu\nu}^{(D_s)}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \langle 0|T\{j_\mu^K(x) j^{D_s^\dagger}(y) j_\nu^{D^*\dagger}(0)\}|0\rangle \tag{2}$$

for D_s meson off-shell, with the interpolating currents $j_\mu^K = \bar{u}\gamma_\mu\gamma_5 s$, $j^{D_s} = i\bar{c}\gamma_5 s$, $j_\mu^{D^*} = \bar{u}\gamma_\mu c$, with u, d, s and c being the *up*, *down*, *strange* and *charm* quark fields, respectively. In both cases, each one of these currents has the same quantum numbers as the corresponding mesons.

Using the above currents to evaluate the correlation functions (1) and (2), the theoretical or QCD side is obtained. The framework to calculate the correlators in the QCD side is the Wilson operator product expansion (OPE). The Cutkosky's rule allows us to obtain the double discontinuity of the correlation function for each one of the Dirac structures appearing in the correlation function. Calling ρ_i the spectral density for the Dirac structure i, we can write the correlation function as a double dispersion relation over the virtualities p^2 and p'^2, holding $Q^2 = -q^2$ fixed. Therefore, the amplitudes Γ_i are given by:

$$\Gamma_i(p^2, p'^2, Q^2) = -\frac{1}{\pi^2} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du\, \frac{\rho_i(s,u,Q^2)}{(s-p^2)(u-p'^2)}, \tag{3}$$

where the spectral density $\rho_i(s,u,Q^2)$ equals the double discontinuity of the amplitude $\Gamma_i(p^2, p'^2, Q^2)$. The amplitudes receive contributions from all terms in the OPE. The leading contribution comes from the perturbative term, shown in Fig. 1. The phenomenological side of the sum rule, which is written in terms of the mesonic degrees of freedom, is parametrized in terms of the form factors, meson decay constants and meson masses. The QCD sum rule is obtained by matching both representations, using the universality principle. The matching is improved by performing a double Borel transform on both sides.

The perturbative contribution for both Eqs. (1) and (2), written in terms of Eq. (3), is given by

$$\begin{aligned}\rho_\mu^{(K)}(s,u,Q^2) = \frac{3}{2\pi\sqrt{\lambda}}\Big\{ & p_\mu\big[A\big(m_c^2 - m_c m_s - 2k\cdot p + p\cdot p'\big) + 2\pi\big(m_c^2 - k\cdot p'\big)\big] \\ & + p'_\mu\big[B\big(m_c^2 - m_c m_s - 2k\cdot p + p\cdot p'\big) + 2\pi\big(-m_c^2 + m_c m_s + k\cdot p\big)\big]\Big\}\end{aligned} \tag{4}$$

for K off-shell, and

$$\begin{aligned}\rho_{\mu\nu}^{(D_s)}(s,u,Q^2) = -\frac{3i}{2\pi\sqrt{\lambda}}\Big\{ & g_{\mu\nu}\big[\pi\big(m_s(s-m_c^2) - m_c(u-m_s^2)\big) + 2D(m_s - m_c)\big] \\ & + (p_\mu p'_\nu + p'_\mu p_\nu)\big[Am_c - Bm_s + 2C(m_s - m_c)\big] \\ & + p_\mu p_\nu 2\big[F(m_s - m_c) - Am_s\big] + p'_\mu p'_\nu 2\big[Bm_c + E(m_s - m_c)\big]\Big\}\end{aligned} \tag{5}$$

for D_s off-shell. Here $s = p^2$, $u = p'^2$, $t = -Q^2$, $\lambda \equiv \lambda(s,t,u) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$, $k\cdot p = \frac{s+m_c^2-m_s^2}{2}$, $p\cdot p' = \frac{s+u-t}{2}$, $k\cdot p' = \frac{u+m_c^2}{2}$, and A, B, C, D, E and F are functions of $\{s,t,u\}$, given by the following expressions:

$$A=\frac{2\pi}{\sqrt{s}}\left(\bar{k}_0-\frac{|\bar{\vec{k}}|p'_0}{|\vec{p}'|}\cos\bar{\theta}\right),\qquad B=2\pi\frac{|\bar{\vec{k}}|}{|\vec{p}'|}\cos\bar{\theta},$$
$$C=\frac{\pi\bar{k}_0^2}{\sqrt{s}|\vec{p}'|}\left[2\cos\bar{\theta}-\frac{p'_0}{|\vec{p}'|}\left(3\cos^2\bar{\theta}-1\right)\right],\qquad D=\pi\bar{k}_0^2\left(1-\cos^2\bar{\theta}\right),$$
$$E=\frac{\pi\bar{k}_0^2}{|\vec{p}'|^2}\left(3\cos^2\bar{\theta}-1\right),\qquad F=\frac{\pi\bar{k}_0^2}{s}\left[3-\frac{4p'_0}{|\vec{p}'|}\cos\bar{\theta}+\frac{p'^2_0}{|\vec{p}'|^2}\left(3\cos^2\bar{\theta}-1\right)-\cos^2\bar{\theta}\right],$$
$$p'_0=\frac{s+u-t}{2\sqrt{s}},\qquad |\vec{p}'|^2=\frac{\lambda}{4s},$$

where

$$\bar{k}_0=\frac{s+m_c^2-m_s^2}{2\sqrt{s}},\qquad |\bar{\vec{k}}|=\sqrt{\bar{k}_0^2-m_c^2},\qquad \cos\bar{\theta}=\frac{2p'_0\bar{k}_0-m_c^2-u}{2|\vec{p}'||\bar{\vec{k}}|},$$

for K off-shell, and

$$\bar{k}_0=\frac{s-m_c^2}{2\sqrt{s}},\qquad |\bar{\vec{k}}|=\bar{k}_0,\qquad \cos\bar{\theta}=\frac{2p'_0\bar{k}_0+m_s^2-u}{2|\vec{p}'||\bar{\vec{k}}|},$$

for D_s off-shell.

The phenomenological side of the vertex functions is obtained considering the contributions of the D_s and D^* mesons to the matrix element in Eq. (1) and the D^* and K mesons to the matrix element in Eq. (2). We introduce the meson decay constants f_K, f_{D_s} and f_{D^*}, which are defined by the following matrix elements:

$$\langle 0|j^K|K\rangle=\frac{m_K^2 f_K}{m_s+m_q},\tag{6}$$

$$\langle 0|j^{D_s}|D_s\rangle=\frac{m_{D_s}^2}{m_c+m_s}f_{D_s},\tag{7}$$

$$\langle 0|j_\nu^{D^*}|D^*\rangle=m_{D^*}f_{D^*}\epsilon_\nu^*,\tag{8}$$

where ϵ_ν is the polarization vector of the D^* meson.

In principle, we can work with any Dirac structure appearing in the amplitude in Eqs. (1) and (2). However, there are some points that one must follow: (i) the chosen structure must also appear in the phenomenological side and (ii) the chosen structure must have a stability that guarantees a good match between the two sides of the sum rule. The structures that obey these two points are p'_μ, in the case K off-shell, and $p'_\mu p'_\nu$ in the case D_s off-shell. The corresponding phenomenological amplitudes in these structures are

$$\Gamma^{(K)\mathrm{ph}}\left(p^2,p'^2,Q^2\right)=g_{D^*D_sK}^{(K)}\left(Q^2\right)\frac{f_{D^*}f_{D_s}f_Km_{D^*}m_{D_s}^2m_K^2}{(m_c+m_s)m_s(p^2-m_{D_s}^2)(p'^2-m_{D^*}^2)(Q^2+m_K^2)}\left(1+\frac{m_{D_s}^2+Q^2}{m_{D^*}^2}\right)\tag{9}$$

for the K off-shell, and

$$\Gamma^{(D_s)\mathrm{ph}}\left(p^2,p'^2,Q^2\right)=g_{D^*D_sK}^{(D_s)}\left(Q^2\right)\frac{(-2)if_{D^*}f_{D_s}f_Km_{D^*}m_{D_s}^2}{(m_c+m_s)(p^2-m_{D^*}^2)(p'^2-m_K^2)(Q^2+m_{D_s}^2)}\tag{10}$$

for D_s off-shell.

In the case of K off-shell the contribution of the quark condensate vanishes after the double Borel transform. In the case of the D_s off-shell, the quark condensate does not contribute to the chosen structure.

To write the sum rules we equate each phenomenological amplitude in Eqs. (9)–(10), with the expression obtained by substituting the corresponding spectral density in Eqs. (4)–(5) into Eq. (3). The matching between both sides is improved by performing a double Borel transformation [18] in the variables $P^2=-p^2\to M^2$ and $P'^2=-p'^2\to M'^2$. We get then the final form of the sum rule, which allow us to obtain the form factors $g_{D^*D_sK}^{(M)}(Q^2)$ appearing in Eqs. (9)–(10), where M stands for the off-shell meson.

We use Borel masses satisfying the constraint $M^2/M'^2=m_{\mathrm{in}}^2/m_{\mathrm{out}}^2$, where m_{in} and m_{out} are the masses of the incoming and outcoming meson respectively. In the case of the K meson off-shell, this constraint gives $M^2/M'^2=m_{D_s}^2/m_{D^*}^2$. For the D_s meson off-shell, the relation should be $M^2/M'^2=m_{D^*}^2/m_K^2$. However, the small value of the K mass spoils the stability of the Borel transformation. Thus, as is common in the literature, we change the K mass for the ρ mass. The resulting relation is then $M^2/M'^2=m_{D^*}^2/m_\rho^2$.

The values of the parameters used in the calculation of the D^*D_sK vertex are depicted in Table 1. The continuum thresholds s_0 and u_0, appearing in Eq. (3), are given by $s_0=(m_{\mathrm{in}}+\Delta_s)^2$ and $u_0=(m_{\mathrm{out}}+\Delta_u)^2$, where m_{in} and m_{out} are the masses of the

Table 1
Parameters used in the calculation of the QCD sum rule for the $D^* D_s K$ vertex. All quantities are in GeV

m_q	m_s	m_c	m_K	m_{D_s}	m_{D^*}	f_K [20]	f_{D_s} [21]	f_{D^*} [22]
0.0	0.13	1.2	0.498	1.97	2.01	0.160	0.280	0.240

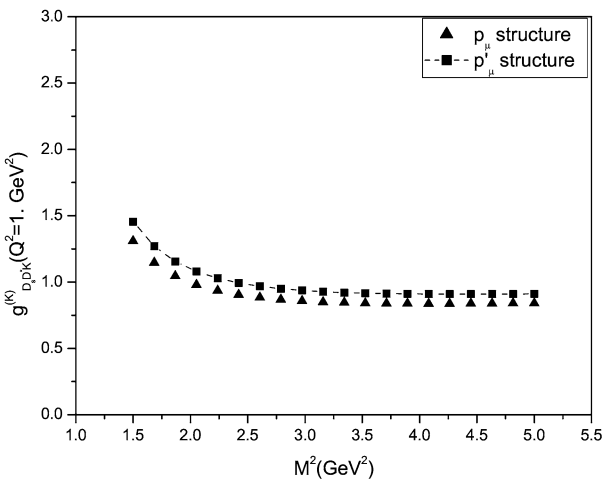

Fig. 2. Stability of $g^{(K)}_{D^* D_s K}(Q^2 = 1\ \text{GeV}^2)$, as a function of the Borel mass M^2.

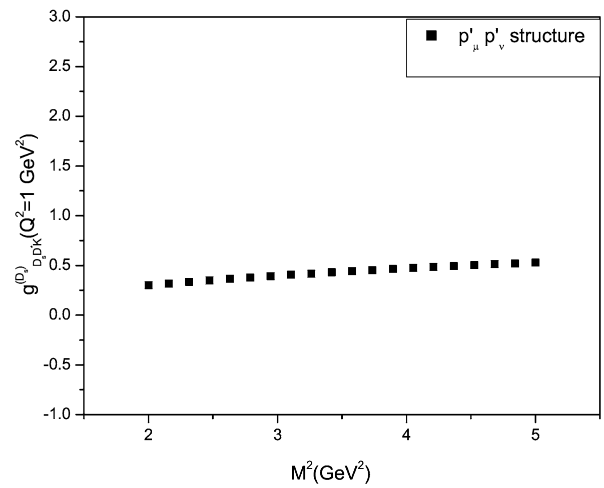

Fig. 3. Stability of $g^{(D_s)}_{D^* D_s K}(Q^2 = 1\ \text{GeV}^2)$, as a function of the Borel mass M^2.

incoming and outcoming mesons respectively. For the K off-shell we have $m_{\text{in}} = m_{D_s}$ and $m_{\text{out}} = m_{D^*}$, and for the D_s off-shell we have $m_{\text{in}} = m_{D^*}$ and $m_{\text{out}} = m_K$ (see Fig. 1).

Using $\Delta_s = \Delta_u = 0.5$ GeV for the continuum thresholds and fixing $Q^2 = 1\ \text{GeV}^2$, we found a good stability of the form factor $g^{(K)}_{D^* D_s K}$, as a function of the Borel mass M^2, in the interval $3 < M^2 < 5\ \text{GeV}^2$, as can be seen in Fig. 2. In the case of the form factor $g^{(D_s)}_{D^* D_s K}$ the interval for stability of the sum rule is $2 < M^2 < 5\ \text{GeV}^2$, as can be seen in Fig. 3. Fixing $\Delta_s = \Delta_u = 0.5$ GeV and $M^2 = 3\ \text{GeV}^2$, we evaluate the momentum dependence of both form factors. The results are shown in Fig. 4, where the squares corresponds to the $g^{(K)}_{D^* D_s K}(Q^2)$ form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g^{(D_s)}_{D^* D_s K}(Q^2)$ form factor. In the case of the K meson off-shell, our numerical results can be parametrized by an exponential function (dotted line in Fig. 4):

$$g^{(K)}_{D^* D_s K}(Q^2) = 2.83\, e^{-\frac{Q^2}{4.19}}. \tag{11}$$

As in Ref. [13], we define the coupling constant as the value of the form factor at $Q^2 = -m_M^2$, where m_M is the mass of the off-shell meson. For the K off-shell case the resulting coupling constant is:

$$g^{(K)}_{D^* D_s K} = 3.01. \tag{12}$$

In the case when the D_s meson is off-shell, our sum rule results can be parametrized by a monopole formula (solid line in Fig. 4):

$$g^{(D_s)}_{D^* D_s K}(Q^2) = \frac{9.01}{Q^2 + 6.86}, \tag{13}$$

giving the following coupling constant, obtained at the D_s pole:

$$g^{(D_s)}_{D^* D_s K} = 3.02. \tag{14}$$

The parametrization of the form factors used in Eqs. (11) and (13) are not unique, the more common ones are the monopolar, Gaussian and exponential parametrizations [19]. The better fit of our sum rules results are obtained with the exponential in the case when the lighter meson is off-shell, and with the monopole parametrization when the heavier meson is off-shell. Comparing the results in Eqs. (12) and (14) we see that the method used to extrapolate the QCDSR results in both cases, K and D_s off-shell, allows us to extract values for the coupling constant which are in very good agreement with each other.

In order to study the dependence of this results with the continuum threshold, we vary $\Delta_s = \Delta_u$ in the interval $0.4 \leqslant \Delta_s = \Delta_u \leqslant 0.6$ GeV, as can be seen in Fig. 5. This procedure give us uncertainties in such a way that the final results for the couplings in each case are: $g^{(K)}_{D^* D_s K} = 3.02 \pm 0.15$ and $g^{(D_s)}_{D^* D_s K} = 3.03 \pm 0.14$.

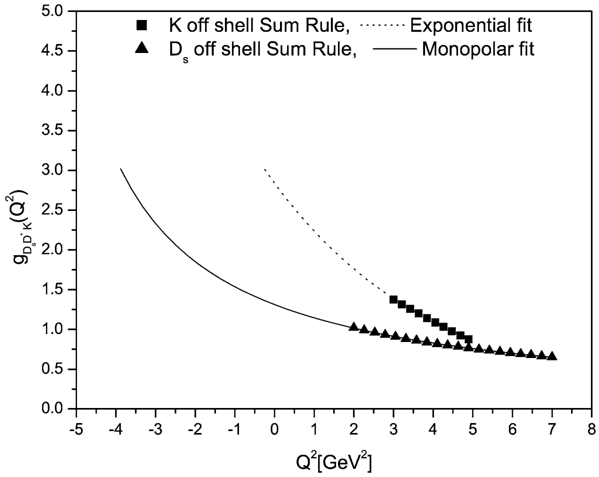

Fig. 4. $g^{(K)}_{D^*D_sK}$ (squares) and $g^{(D_s)}_{D^*D_sK}$ (triangles) form factors as a function of Q^2 from the QCDSR calculation of this work. The solid (dotted) line corresponds to the monopole (exponential) parametrization of the QCDSR results for each case.

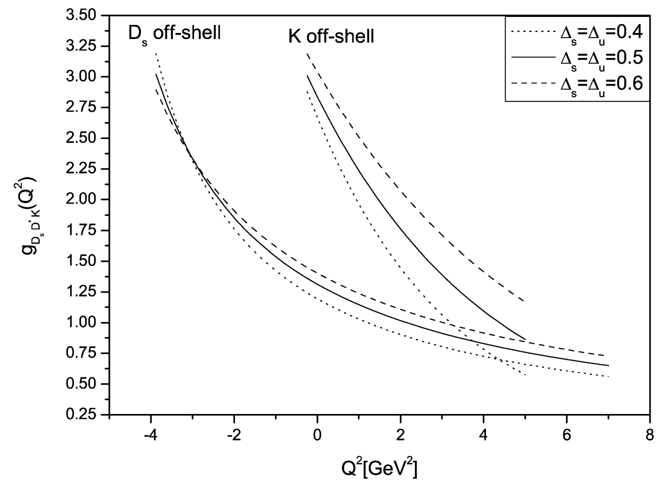

Fig. 5. Dependence of the form factor with the continuum threshold, for K and D_s off-shell cases. The dotted line corresponds to $\Delta_s = \Delta_u = 0.4$ GeV, the solid line corresponds to $\Delta_s = \Delta_u = 0.5$ GeV and the dashed one corresponds to $\Delta_s = \Delta_u = 0.6$ GeV.

Now we study the $D_s^* DK$ vertex. The treatment is similar to the previous case. The correlation functions are

$$\Gamma^{(K)}_{\mu}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \langle 0|T\{ j_\mu^{D_s^*}(x) j^{K^\dagger}(y) j^{\bar{D}^\dagger}(0)\}|0\rangle \tag{15}$$

for K meson off-shell, where the interpolating currents are $j_\mu^{D_s^*} = \bar{c}\gamma_\mu s$, $j^K = i\bar{u}\gamma_5 s$ and $j^D = i\bar{c}\gamma_5 u$, and

$$\Gamma^{(D)}_{\mu\nu}(p,p') = \int d^4x\, d^4y\, e^{ip'\cdot x} e^{-i(p'-p)\cdot y} \langle 0|T\{ j_\mu^{K}(x) j^{D^\dagger}(y) j_\nu{}^{D_s^{*\dagger}}(0)\}|0\rangle \tag{16}$$

for D meson off-shell, with the interpolating currents $j_\mu^K = \bar{u}\gamma_\mu\gamma_5 s$, $j_\nu^{D_s^*} = \bar{c}\gamma_\nu s$, and $j^D = i\bar{u}\gamma_5 c$. See Fig. 6 for understanding the perturbative contribution with these currents.

For each correlation function, Eqs. (15) and (16), the corresponding perturbative spectral density which enters Eq. (3) is:

$$\begin{aligned}\rho^{(K)}_{\mu}(s,u,Q^2) = \frac{3}{2\pi\sqrt{\lambda}}\big\{ & p_\mu\big[A\big(m_c^2 + m_c m_s - 2k\cdot p + p\cdot p'\big) + 2\pi\big(m_c^2 - m_c m_s - k\cdot p'\big)\big] \\ & + p'_\mu\big[B\big(m_c^2 + m_c m_s - 2k\cdot p + p\cdot p'\big) + 2\pi\big(k\cdot p - m_c^2\big)\big]\big\}\end{aligned} \tag{17}$$

for K off-shell, where $k\cdot p = \frac{s+m_c^2}{2}$ and $k\cdot p' = \frac{u+m_c^2-m_s^2}{2}$, and

$$\begin{aligned}\rho^{(D)}_{\mu\nu}(s,u,Q^2) = -\frac{3i}{2\pi\sqrt{\lambda}}\big\{ & g_{\mu\nu}\big[2\pi\big(m_s^2(m_c - m_s) + m_s(k\cdot p + k\cdot p' - p\cdot p') - m_c k\cdot p\big) - 2m_c D\big] \\ & + p_\mu p'_\nu\big[A(m_c + m_s) + Bm_s - 2Cm_s - 2\pi m_s\big] + p'_\mu p_\nu\big[A(m_c - m_s) - Bm_s - 2Cm_s + 2\pi m_s\big] \\ & - p_\mu p_\nu 2m_c F + p'_\mu p'_\nu 2m_c(B - E)\big\}\end{aligned} \tag{18}$$

for D off-shell, where $k\cdot p = \frac{s+m_s^2-m_c^2}{2}$ and $k\cdot p' = \frac{u+m_s^2}{2}$. The definitions of the other quantities are the same as for the $D^* D_s K$ vertex, with

$$\bar{k}_0 = \frac{s+m_c^2}{2\sqrt{s}}, \qquad |\vec{\bar{k}}| = \sqrt{\bar{k}_0^2 - m_c^2}, \qquad \cos\bar{\theta} = \frac{2p'_0\bar{k}_0 + m_s^2 - m_c^2 - u}{2|\vec{p}'||\vec{\bar{k}}|},$$

for K off-shell, and

$$\bar{k}_0 = \frac{s+m_s^2-m_c^2}{2\sqrt{s}}, \qquad |\vec{\bar{k}}| = \sqrt{\bar{k}_0^2 - m_s^2}, \qquad \cos\bar{\theta} = \frac{2p'_0\bar{k}_0 - m_s^2 - u}{2|\vec{p}'||\vec{\bar{k}}|},$$

for D off-shell.

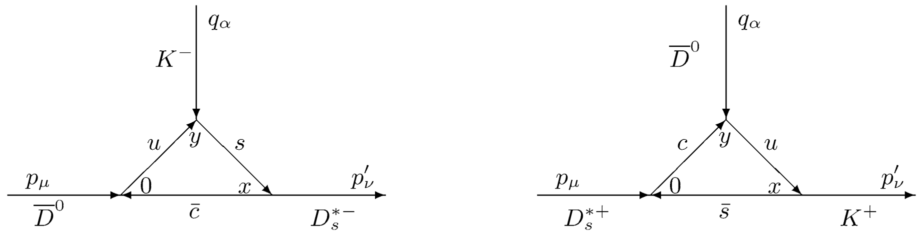

Fig. 6. Perturbative diagrams for K off-shell (left) and D off-shell (right) corresponding to the $D_s^* DK$ vertex.

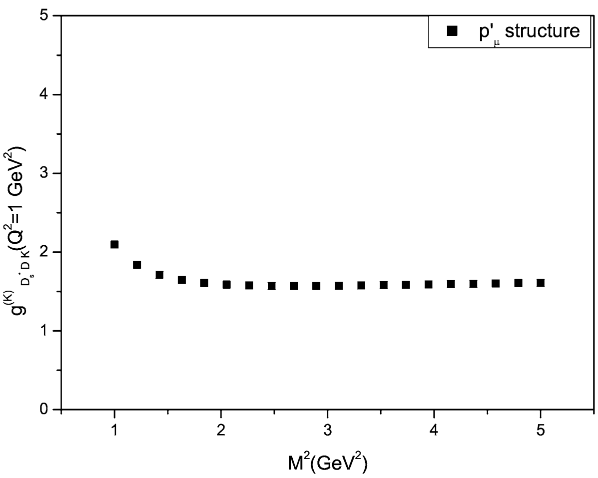

Fig. 7. Stability of $g^{(K)}_{D_s^* DK}(Q^2 = 1\ \mathrm{GeV}^2)$, as a function of the Borel mass M^2.

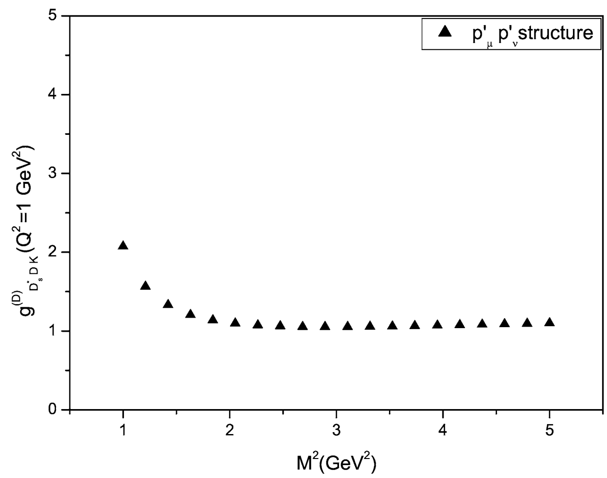

Fig. 8. Stability of $g^{(D)}_{D_s^* DK}(Q^2 = 1\ \mathrm{GeV}^2)$, as a function of the Borel mass M^2.

The phenomenological side of the vertex functions are obtained by considering the contributions of the D and D_s^* mesons to the matrix element in Eq. (15) and the D_s^* and K mesons to the matrix element in Eq. (16). We introduce the decay constants f_D and $f_{D_s^*}$, which are defined by the following matrix elements:

$$\langle 0|j^D|D\rangle = \frac{m_D^2}{m_c + m_q} f_D, \tag{19}$$

$$\langle 0|j_\nu^{D_s^*}|D_s^*\rangle = m_{D_s^*} f_{D_s^*} \epsilon_\nu^*, \tag{20}$$

where ϵ_ν is the polarization vector of the D_s^* meson. The f_K decay constant was already defined in Eq. (6). Again we have to choose a Dirac structure for each case in Eqs. (17)–(18). Following the points discussed before, the chosen Dirac structures are p'_μ for the off-shell K, and $p'_\mu p'_\nu$ for the off-shell D. The corresponding phenomenological amplitudes in these structures are

$$\Gamma^{(K)\mathrm{ph}}\left(p^2, p'^2, Q^2\right) = g^{(K)}_{D_s^* DK}\left(Q^2\right) \frac{f_{D_s^*} f_D f_K m_{D_s^*} m_D^2 m_K^2}{m_c m_s (p^2 - m_D^2)(p'^2 - m_D^2)(Q^2 + m_K^2)} \left(1 + \frac{m_D^2 + Q^2}{m_{D_s^*}^2}\right) \tag{21}$$

for K off-shell, and

$$\Gamma^{(D)\mathrm{ph}}\left(p^2, p'^2, Q^2\right) = g^{(D)}_{D_s^* DK}\left(Q^2\right) \frac{(-2) i f_{D_s^*} f_D f_K m_{D_s^*} m_D^2}{m_c (p^2 - m_{D_s^*}^2)(p'^2 - m_K^2)(Q^2 + m_D^2)} \tag{22}$$

for D off-shell. As in the case of the $D^* D_s K$ vertex, the quark condensate does not contribute to the sum rule for these structures.

The procedure to obtain the QCD sum rule is the same used in the case of the $D^* D_s K$ vertex studied before. In this case we use the following relations between the Borel masses: $M^2/M'^2 = m_D^2/m_{D_s^*}^2$ for K off-shell and $M^2/M'^2 = m_{D_s^*}^2/m_\rho^2$ for D off-shell. The values of the parameters used in the calculation of the $D_s^* DK$ vertex are given in Table 2, where we have used the relation $f_{D_s^*} = f_{D^*} f_{D_s}/f_D$ and the value of f_{D_s}/f_D from Ref. [23] in order to obtain the D_s^* decay constant.

Using $\Delta_s = \Delta_u = 0.5$ GeV for the continuum thresholds and fixing $Q^2 = 1\ \mathrm{GeV}^2$, we found a good stability of the sum rule for $g^{(K)}_{D_s^* DK}$, as a function of the Borel mass M^2, in the interval $2 < M^2 < 5\ \mathrm{GeV}^2$, as can be seen in Fig. 7. In the case of $g^{(D)}_{D_s^* DK}$, the interval for stability is also $2 < M^2 < 5\ \mathrm{GeV}^2$, as can be seen in Fig. 8.

Table 2
Parameters used in the calculation of the QCD sum rule for the $D_s^* DK$ vertex. All quantities are in GeV

m_q	m_s	m_c	m_K	m_D	$m_{D_s^*}$	f_K [20]	f_D [24]	$f_{D_s^*}$
0.0	0.13	1.2	0.498	1.87	2.11	0.160	0.200	0.330

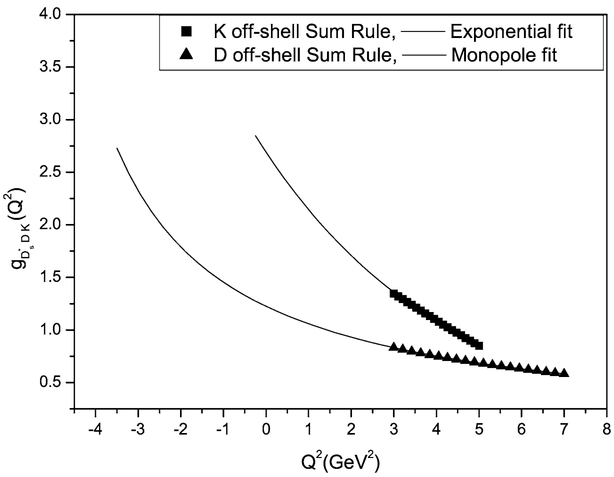

Fig. 9. $g^{(K)}_{D_s^* DK}$ (squares) and $g^{(D)}_{D_s^* DK}$ (triangles) form factors as a function of Q^2 from the QCDSR calculation of this work. The dashed (solid) line corresponds to the exponential (monopole) parametrization of the QCDSR results for each case.

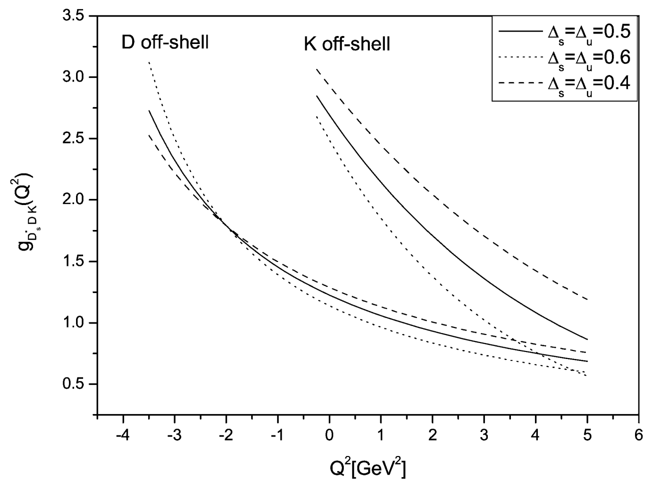

Fig. 10. Dependence of the form factor with the continuum threshold, for K and D off-shell cases. The dotted line corresponds to $\Delta_s = \Delta_u = 0.4$ GeV, the solid corresponds to $\Delta_s = \Delta_u = 0.5$ GeV and the dashed corresponds to $\Delta_s = \Delta_u = 0.6$ GeV.

Fixing $\Delta_s = \Delta_u = 0.5$ GeV and $M^2 = 3\ \text{GeV}^2$ in both cases, we calculate the momentum dependence of the form factors which are shown in Fig. 9. The squares corresponds to the $g^{(K)}_{D_s^* DK}(Q^2)$ form factor in the interval where the sum rule is valid. The triangles are the result of the sum rule for the $g^{(D)}_{D_s^* DK}(Q^2)$ form factor. In the case when the K meson is off-shell, our numerical results can be parametrized by an exponential function (dashed curve in Fig. 9):

$$g^{(K)}_{D_s^* DK}\left(Q^2\right) = 2.69 e^{-\frac{Q^2}{4.39}}. \tag{23}$$

The coupling constant was obtained as the value of the form factor at $Q^2 = -m_K^2$. In this case the resulting coupling constant is

$$g^{(K)}_{D_s^* DK} = 2.87. \tag{24}$$

In the case when the D meson is off-shell, the sum rule results are represented by the triangles in Fig. 9, and they can be parametrized by a monopole formula (solid line in the figure):

$$g^{(D)}_{D_s^* DK}\left(Q^2\right) = \frac{7.78}{Q^2 + 6.34}, \tag{25}$$

giving the following coupling constant, obtained at the D pole:

$$g^{(D)}_{D_s^* DK} = 2.72. \tag{26}$$

Studing the dependence of our results with the continuum threshold, for $\Delta_{s,u}$ varying in the interval $0.4 \leqslant \Delta_{s,u} \leqslant 0.6$ GeV, as can be seen in Fig. 10, we obtain the following values, with errors, for the couplings in each case: $g^{(K)}_{D_s^* DK} = 2.87 \pm 0.19$ and $g^{(D)}_{D_s^* DK} = 2.72 \pm 0.31$.

Concluding, we have studied the form factors and coupling constants of $D^* D_s K$ and $D_s^* DK$ vertices in a QCD sum rule calculation. For each case we have considered two particles off-shell, the lightest and one of the heavy ones: the K and D_s mesons for the $D^* D_s K$ vertex, and the K and D mesons for the $D_s^* DK$ vertex. In the two situations, the off-shell particles give compatible results for the coupling constant in each vertex. The results are:

$$g_{D^* D_s K} = 3.02 \pm 0.14, \tag{27}$$

$$g_{D_s^* DK} = 2.84 \pm 0.31. \tag{28}$$

In a similar calculation, this time on the light cone [25], the same coupling constants were obtained, with the results $g_{D^*D_sK} = 2.02^{+0.84}_{-0.56}$ and $g_{D_s^*DK} = 1.84^{+0.91}_{-0.63}$. Although somewhat larger, our values are compatible with the ones from that reference.

We can compare our result with the prediction of the exact SU(4) symmetry [5,7,9], which would give the following relation among these numbers [9]: $g_{D^*D_sK} = g_{D_s^*DK} = 5$. Eqs. (27) and (28) show that the coupling constants in the vertices D^*D_sK and D_s^*DK are consistent one with the other, but that they are relatively far from the value given by the SU(4) symmetry in the cited reference. Therefore, we conclude that the SU(4) symmetry is broken by approximately 40% in the calculation performed here. We can also extract the cutoff parameter, Λ, from the parametrizations in Eqs. (11) and (23) for K off-shell, Eq. (13) for D_s off-shell and Eq. (25) for D off-shell. We get $\Lambda \approx 2.07$ GeV for the K meson off-shell, $\Lambda \approx 2.61$ GeV for the D_s meson off-shell, and $\Lambda \approx 2.51$ GeV for the D meson off-shell. Comparing the values of the cutoffs, we see that the form factor is harder if the off-shell meson is heavier, implying that the size of the vertex depends on the mass of the exchanged meson: the heavier is the meson, the more as a point like particle is its behavior when probing the target, as observed in Refs. [12–14,17].

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