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The effects of magnetic field on the fluid flow through a rotating straight duct with large aspect ratio

Md. Kamruzzaman[a], Mohammad Wahiduzzaman[b], Md. Mahmud Alam[b] and Lyazid Djenidi[a,] *

[a]Department of Mechanical Engineering, The University of Newcastle, Newcastle-2308, Australia
[b]Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh

Abstract

This paper presents a numerical study of an investigation of a fluid flow through a rotating rectangular straight duct in the presence of magnetic field. The straight duct of rectangular cross-section rotates at a constant angular velocity about the centre of the duct cross-section is same as the axis of the magnetic field along the positive direction in the stream wise direction of the flows. Numerical calculation is based on the Magneto hydrodynamics incompressible viscous steady fluid model whereas Spectral method is applied as a main tool. Flow depends on the Magnetic parameter, Dean number and Taylor number. One of the interesting phenomena of the fluid flow is the solution curve and the flow structures in case of rotation of the duct axis. The calculation are carried out for $5 \le M_g \le 50000$, $50 \le T_r \le 100000$, $D_n = 500$, 1000, 1500 and 2000 where the aspect ratio $\gamma = 3.0$. The maximum axial flow will be shifted to the centre from the wall and turn into the ring shape under the effects of high magnetic parameter and large Taylor number whereas the fluid particles strength is weak.

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Keywords: Magnetic parameter; Taylor number; Dean number; aspect ratio

Nomenclature

γ	Aspect ratio
M_g	Magnetic parameter
D_n	Dean number
T_r	Taylor number
Q'	Dimensional total flow
Q	Non-Dimensional Total flow

* Corresponding author. Tel.:+62-04132681792; fax: +0-000-000-0000 .
*E-mail address:*zaman9208@gmail.com; alam_mahmud200@yahoo.com

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1. Introduction

Fluid flows in a straight duct are of great importance. It has large applications in chemical and mechanical engineering. The purpose of this paper is to make some numerical calculations on the fluid flow through a rotating rectangular straight duct in the presence of magnetic field which has been interested to the engineering communication. The results of this investigation may not have direct practical applications but are relevant to the problems mentioned above. The fluid flows through a rectangular straight duct to rotate at a constant angular velocity about an axis normal to a plane. Such rotation passages are used in cooling systems for conductors of electric generators. The earliest work on the flow in rotating straight pipe was carried out for the asymptotic limits of weak and strong rotations by Barua [2]. Benton and Baltimore [3] used a perturbation expansion to the Hagen-Poiseuille flow. Ito and Nanbu [4] and Alam, et al. [1] have used spectral method to describe the flow through a rotating straight pipe with large aspect ratio. MHD flow in an insulating rectangular duct under a non-uniform magnetic field is studied by [5]. Numerical simulations of MHD flows past obstacles in a duct under externally applied magnetic field is studied by [6]. [7] Investigates the study of surface and bulk instabilities in MHD duct flow with imitation of insulator coating imperfection. [8] Investigates the natural convective flow phenomena under the influence of magnetic field. [9] studied the rotational MHD flow field of unity magnetic Prandtl number in the effects of regional magnetic field. [10] has been observed the stability of viscous flow between rotating cylinders in the presence of a magnetic field.

Hence our aim is to study through the direct numerical simulation, the response of the magnetic effects on the fluid flow through a rotating rectangular straight duct with large aspect ratio**.**

2. Governing equations

The fully developed laminar flow of an incompressible viscous fluid in a straight duct that is subjected to a steady rotation $\mathbf{\Omega}$ with rectangular cross-section in the presence of magnetic field has been considered. Let $2a$ be the width of the duct cross-section and $2b$ its height. Cartesian co-ordinate system (x', y', z') has been considered to describe the motion of the fluid particles in the duct with the center O at the centralism of the rectangular cross-section duct as illustrated in Figure 1. The system rotates at a constant angular velocity $\mathbf{\Omega} = (0, -\Omega, 0)$ around the y' – axis. The flow is derived by the pressure gradient $\dfrac{\partial p}{\partial z} = G$ along the centerline of the duct in the presence of magnetic field. u', v', w' are the dimensional velocity components along x', y', z' direction respectively and u, v, w are the dimensionless velocity along x', y', z' direction respectively. p' is the modified pressure which includes gravitational and centrifugal force.

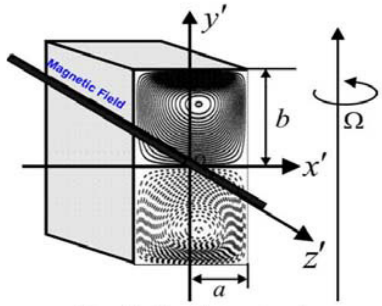

Figure 1: Co-ordinate system in a Rotating Straight Duct

The assumption of fully developed flow means that except for the pressure derivatives z' are all set to zero. The dependent and independent variables are non dimensionlized as follows:

$$u' = \frac{\upsilon}{a} u\,; \quad x' = xa\,; \quad p' = \frac{\upsilon^2}{a^2} \rho\, p\,; \quad v' = \frac{\upsilon}{a} v\,; \quad y' = y\,a\,; \quad w' = \frac{\upsilon}{a} w\,; \quad z' = 0,$$

where the variables with prime are dimensional quantities and "a" be the half width of the cross section of the duct. Under the above assumption, the governing equations are;

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - T_r w - M_g u \tag{1}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - M_g v \tag{2}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=D_n+\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+T_r u \tag{3}$$

where, Taylor number $T_r=2\left(\frac{a^2\Omega}{\upsilon}\right)$, Magnetic parameter , $M_g=\frac{\sigma'}{\mu_e}a^2B_0^2=\sigma'\mu_e a^2H_0^2$

Dean number $D_n=\frac{Ga^3}{\rho\upsilon^2}$ and equation of continuity $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

The boundary condition is that the velocities are zero at $x=\pm 1$ and $y=\pm\left(\frac{b}{a}\right)=\pm\gamma$ (aspect ratio).

The new variable $\bar{y}=\left(\frac{y}{\gamma}\right)$ is introduced where γ is the aspect ratio, $\gamma=\left(\frac{b}{a}\right)$. $u=-\left(\frac{\partial\psi}{\partial y}\right)$ and $v=\left(\frac{\partial\psi}{\partial x}\right)$ which satisfies the continuity equation The basic equations (1)-(3) become for ψ and w as:

$$\frac{\partial^4\psi}{\partial x^4}+\frac{2}{\gamma^2}\frac{\partial^4\psi}{\partial\bar{y}^2\partial x^2}+\frac{1}{\gamma^4}\frac{\partial^4\psi}{\partial\bar{y}^4}=-\frac{1}{\gamma^3}\frac{\partial\psi}{\partial\bar{y}}\frac{\partial^3\psi}{\partial\bar{y}^2\partial x}-\frac{1}{\gamma}\frac{\partial\psi}{\partial\bar{y}}\frac{\partial^3\psi}{\partial x^3}+\frac{1}{\gamma^3}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial\bar{y}^3}+\frac{1}{\gamma}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial x^2\partial\bar{y}}$$
$$-\frac{1}{\gamma}\frac{\partial w}{\partial\bar{y}}T_r+\left(\frac{1}{\gamma^2}\frac{\partial^2\psi}{\partial\bar{y}^2}+\frac{\partial^2\psi}{\partial x^2}\right)M_g \tag{4}$$

$$\frac{\partial^2 w}{\partial x^2}+\frac{1}{\gamma^2}\frac{\partial^2 w}{\partial\bar{y}^2}=-\frac{1}{\gamma}\frac{\partial\psi}{\partial\bar{y}}\frac{\partial w}{\partial x}+\frac{1}{\gamma}\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial\bar{y}}-D_n+\frac{1}{\gamma}\frac{\partial\psi}{\partial\bar{y}}T_r \tag{5}$$

The boundary conditions for ψ and w are given by

$$w(\pm 1,\bar{y})=w(x,\pm 1)=\psi(\pm 1,\bar{y})=0$$
$$\left(\frac{\partial\psi}{\partial x}\right)(\pm 1,\bar{y})=\psi(x,\pm 1)=\left(\frac{\partial\psi}{\partial\bar{y}}\right)(x,\pm 1)=0$$

Flux through the Straight Duct

The dimensional total flux Q' through the duct is $Q'=\int_{-b}^{b}\int_{-a}^{a}w\,dx'\,dy'=\upsilon aQ$

where $Q=\int_{-\gamma}^{\gamma}\int_{-1}^{1}w\,dx\,d\bar{y}$ is the non- dimension flux.

3. Calculation technique

The simulations are based on the Spectral method is used as a numerical technique to obtain the solution. It is necessary to discuss the method briefly. The basic ideas of the Spectral and collocation methods are given below. The expansion by polynomial functions is utilized to obtain steady or unsteady solution. The series of the Chebyshev polynomial is used in the x and $\bar{y}$ directions where, x and $\bar{y}$ are coordinate variables. Assuming the flow is symmetric along the axial direction. The expansion function $\phi_n(x)$ and $\psi_n(x)$ are expressed as ;

$$\phi_n(x)=(1-x^2)T_n(x) \tag{6}$$

$$\psi_n(x)=(1-x^2)^2T_n(x) \tag{7}$$

where, $T_n(x)=\cos(n\cos^{-1}(x))$ is the Chebyshev polynomial.

The functions $w(x,\bar{y})$ and $\psi(x,\bar{y})$ are expanded as;

$$w(x,\bar{y}) = \sum_{m=0}^{M}\sum_{n=0}^{N} w_{mn}\phi_m(x)\phi_n(\bar{y}) \quad (8); \quad \psi(x,\bar{y}) = \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{mn}\psi_m(x)\psi_n(\bar{y}) \quad (9)$$

where, M and N are the truncation numbers in the x and $\bar{y}$ directions respectively. The collocation method (Gottlieb and Orszag [5]) applied in x and $\bar{y}$ directions yields a set of nonlinear differential equations for w_{mn} and ψ_{mn}. The collocation points are taken as $(x_i, \bar{y}_j)$

$$x_i = \cos\left[\pi\left(1 - \frac{i}{M+2}\right)\right] \qquad (i = 1, 2, ..., M+1) \quad (10)$$

$$\bar{y}_j = \cos\left[\pi\left(1 - \frac{j}{N+2}\right)\right] \qquad (j = 1, 2, ..., N+1) \quad (11)$$

The details calculation technique and arc-length method for critical calculations are not shown for brevity.

4. Results and discussion

The steady solution has been obtained by the graphical representation of the total flux (Q) versus Taylor number (T_r) at Magnetic parameter $(M_g) = 5000$, corresponding Dean number $(D_n) = 500$, 1000, 1500 and 2000 respectively where the aspect ratio $\gamma = 3.0$. The steady solution curves have been drawn by the path continuation technique in the range $50 \le T_r \le 100000$. The graphical representation has shown in Figure 2 for the total flux (Q) versus Taylor number (T_r) in the range $50 \le T_r \le 32000$. For sufficient accuracy, $M = 10$ and $N = 30$ in the numerical calculations have been considered. The steady solution curves have been obtained for aspect ratio $(\gamma) = 3.0$ and M_g = 5000 in the range $50 \le T_r \le 32000$. These solution curves denoted by t_1, t_2, t_3 and t_4 at the Dean number $(D_n) = 500$, 1000, 1500 and 2000 respectively for graph of the total flux (Q) versus Taylor number (T_r). For brevity, plots of the flow pattern are not shown in actual format. The flow pattern of the secondary flow and contours plot of the axial flow at several Taylor numbers (T_r) on the solution curve for constant ψ and w are shown in Figures (3)-(6). We look the figures from the upstream. Therefore in these figures, the structures of the secondary flow and the axial flow can be understood. T_r = (500,2500, 5000, 6000, 9000 , 16000) on t_1 curve (see Figure 3); T_r = (500, 2500, 3000, 6000, 7500 14300, 16000) on t_2 curve (see Figure 4); T_r = (500, 1500, 3000, 4300, 9700, 16000) on t_3 curve (see Figure 5); T_r = (500, 1500, 3000, 4300, 9700, 16000) on g_8 curve (see Figure 6) have been taken where the stream lines of the secondary flow (top) and the contour plots of the axial flow (bottom) in each row from left to right with the increment $\Delta\psi$ = 0.045, 0.075, 0.070, 0.10 and $\Delta w = 6.0$, 10.0, 10.0, 20.0 at Dean number $(D_n) = 500$, 1000, 1500 and 2000 respectively.

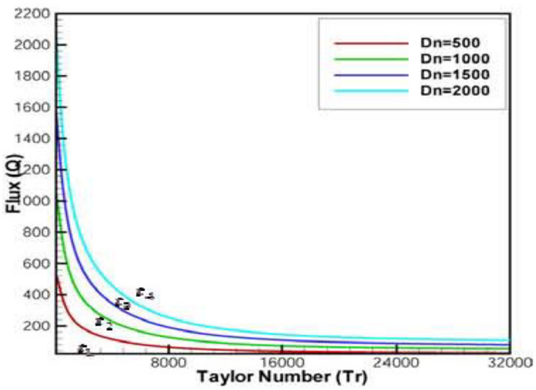

Figure 2: Steady solution for $M_g = 5000$, $D_n = 500$, 1000, 1500 and 2000, at $50 \le T_r \le 32000$

In Figures (3)-(6), the secondary flow, $(\psi \geq 0)$ in the upper region of the duct is the clock wise direction and counter clock wise in the lower part when $(\psi < 0)$. We have observed that the symmetric solution obtained in the range $50 \leq T_r \leq 32000$. The stream lines of the secondary flow are shown at various Taylor number (T_r) in the development of the vortex. 3-vortex, 4-vortex, 5-vortex and 6-vortex solution have been found in the secondary flow which is depend on the Taylor numbers (T_r) and Magnetic parameter (M_g). The contour plots of the axial flow has been formed the ring shape which are either single or double ring shape that appeared depends on the variation of Taylor number (T_r) as well as Magnetic parameter (M_g).

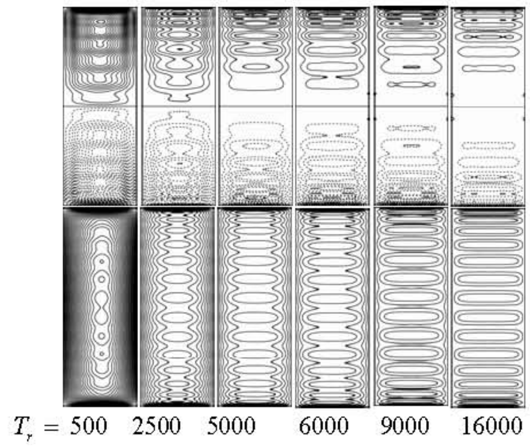

Fig. 3. Stream lines of the Secondary Flow (top) and contours plot of Axial flow (bottom) in each row at Dean number $(D_n) = 500$ and $M_g = 5000$ for Flux (Q) versus Taylor number (T_r) at $T_r = 500$, 2500, 5000, 6500, 9000 and 16000.

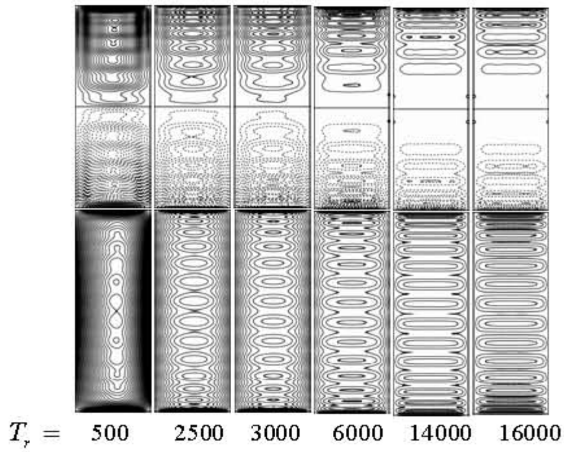

Fig. 4. Stream lines of the Secondary Flow (top) and contours plot of Axial flow (bottom) in each row at Dean number $(D_n) = 1000$ and $M_g = 5000$ for Flux (Q) versus Taylor number (T_r) at $T_r = 500$, 2500, 3000, 6000, 14000 and 16000.

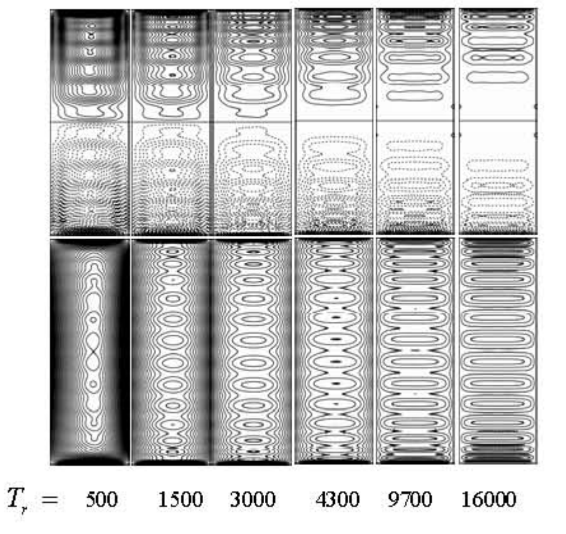

Fig. 5. Stream lines of the Secondary Flow (top) and contours plot of Axial flow (bottom) in each row at Dean number $(D_n) = 1500$ and $M_g = 5000$ for Flux (Q) versus Taylor number (T_r) at $T_r = 500$, 1500, 3000, 4300, 9700 and 16000.

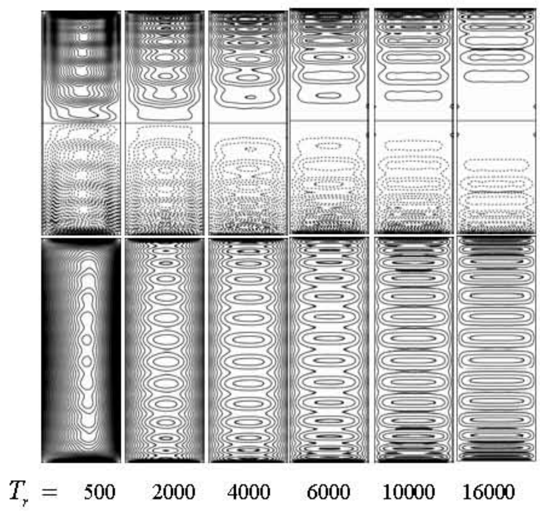

Fig. 6. Stream lines of the Secondary Flow (top) and contours plot of Axial flow (bottom) in each row at Dean number $(D_n) = 2000$ and $M_g = 5000$ for Flux (Q) versus Taylor number (T_r) at $T_r = 500$, 2000, 4000, 6000, 10000 and 16000.

5. Conclusion

According to the results, we have obtained the following important view:

1. For Magnetic parameter (M_g) and Taylor number (T_r) in both cases at high Dean number (D_n), steady solution has been obtained.
2. Anomalous vortices solution has been found for the maximum secondary flow pattern which depends on the Magnetic parameter (M_g) and Taylor number (T_r).
3. The symmetric flow structures at the maximum total flow region show almost the same flow behaviour in the range of $5 \le M_g \le 50000$. The strength of the secondary flows are decreases with the gradually increases of magnetic parameter (M_g).
4. Tendency of the axial flow structures to turn into the single, double and triple ring shape that appeared of course depends on the various Taylor number (T_r) at Dean number $(D_n) = 500$, 1000, 1500 and 2000 and Magnetic parameter (M_g).

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