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Research Note

On Sandewall's paper: Nonmonotonic inference rules for multiple inheritance with exceptions

Geneviève Simonet *

L.I.R.M.M., 161, rue Ada, 34392 Montpellier cedex 5, France

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Abstract

Sandewall (1986) presents a theory of multiple inheritance with exceptions (also called non-monotonic inheritance) based on a set of nonmonotonic inference rules, taking advantage at the same time of theories based on nonmonotonic logic as proposed by Etherington and Reiter and of path-based theories as proposed by Touretzky, Horty and Thomason. Flaws in Sandewall's set of rules are shown and a revised set of rules is proposed. This revised set is shown to provide the same conclusion sets on a hierarchy as path-based theories for three classical variants of preclusion. Moreover, although most approaches to inheritance leave it in the metalanguage, it is shown that putting preclusion in the object language provides extensions with desirable general properties which are not always true in the restricted language of conclusion sets.

Keywords: Knowledge representation; Semantic networks; Multiple inheritance hierarchies; Exceptions; Nonmonotonic inference rules; Path-based theories

1. Introduction

In artificial intelligence, knowledge is often organized in hierarchies in which the nodes represent concepts (which may be individual concepts), and the positive (*is-a*) links represent a relation of generalization/specialization. In these hierarchies, more specific concepts inherit properties from more general ones. Inheritance is said to be *multiple* when a concept may directly inherit from several other concepts. It is said to admit *exceptions* (or to be nonmonotonic) when inheritance can be cancelled by adding information to the hierarchy; usually, this is done by allowing exclusion links (*is-not-a* links) as well as *is-a* links. The fundamental problem in a multiple inheritance

* E-mail: simonet@lirmm.fr.

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hierarchy with exceptions Γ is to define the conclusion sets of Γ, and to find such sets. The problem was raised in Fahlman [3], and put in a formal setting in Touretzky [15], who provided a path-based inheritance definition. There are many approaches to this problem: Etherington and Reiter [2] propose a theory based on default logic, and Sandewall [10] proposes a definition that combines elements of both the path-based and logical accounts.

Sandewall's theory is interesting in two ways. First, he allows hierarchies to contain cycles, which are generally disallowed in nonmonotonic inheritance hierarchies. Second, he puts preclusion and contradiction in the object language, i.e., in the language from which extensions are built, whereas most approaches to inheritance leave these in the metalanguage. [1] Here, I will show that putting preclusion in the object language provides extensions with desirable general properties.

In Section 2, I will recall Sandewall's set of rules for constructing an extension of a hierarchy Γ. In Section 3, I show that this rule set gives unintuitive results on some typical examples from the literature, analyze the flaws that cause these examples, and propose a revision of Sandewall's rules. In Section 4, I show that three variants of my revised rules correspond to alternatives developed in the path-based framework of [6,7]. I then discuss some desirable general properties of extensions, and show that these properties are not always true for the conclusion sets generated by these extensions. Thus, we obtain these properties with the more expressive language containing preclusion, but not with the less expressive language without preclusion. This shows the advantage of putting preclusion in the object language.

2. Sandewall's set of inference rules

We use the following notation to discuss Sandewall's multiple inheritance hierarchies with exceptions. We use Γ for the hierarchies, v, w, x, y, z for nodes, $(x, y, +)$ for a positive link from x to y, and $(x, y, -)$ for a negative link from x to y. s denotes one of the signs $+$ and $-$; $-s$ denotes the opposite sign to s.

Sandewall uses the notion of an extension to define a conclusion set of Γ—but there is some ambiguity in his paper about the word "extension". Actually, he defines it in three different ways: (1), as a hierarchy whose set of nodes is identical to that of Γ and whose set of links contains those of Γ; (2), as a set of propositions; and (3) as a set of positive and negative paths. To each sense of "extension" corresponds a sense of *conclusion set* of an extension, where a conclusion set is a set of propositions of the form $isa(x, y, s)$. In case (1), the conclusion set of an extension is the set of propositions $isa(x, y, s)$ such that (x, y, s) is a link of the extension; in case (2) it is the set of propositions $isa(x, y, s)$ belonging to the extension; and in case (3) it is the set of propositions $isa(x, y, s)$ such that there is a path from x to y of polarity s in the

[1] Some approaches, such as [4,5], put preclusion and contradiction in the object language indirectly, through abnormality predicates. But these have not been much developed and we do not have theoretical results about them.

extension. To be clear, I myself will refer to extensions as sets of propositions and to expansions as sets of paths. [2]

Inference rules deal with sets of propositions, each proposition being an atom or the negation of an atom, where an atom has one of the following forms:

$$isax(x, y, s),$$
$$isa(x, y, s),$$
$$precl(x, y, z, s),$$
$$cntr(x, y, z, s).$$

$isax(x, y, s)$ means that (x, y, s) is a link in the hierarchy. $isa(x, y, s)$ means that x inherits from y with polarity s. *precl* and *cntr* are related to Touretzky's preclusion and contradiction relations, respectively. The notion of preclusion has been discussed in much detail in the literature, in particular by Horty [7]. The idea of preclusion is that a conclusion $isa(x, z, s)$ should not be inferred from a path from x through y to z of polarity s if there is a positive path from x to y passing through a node inheriting from z with polarity $-s$ (with some conditions of inheritance varying according to the definition of preclusion). The idea of contradiction is that a conclusion $isa(x, v, s)$ should not be inferred from a path from x through z to v of polarity s if x negatively inherits from z or x inherits from v with polarity $-s$.

Sandewall writes inference rules in the general form "If D_1 is known and D_2 is not known then infer D_3", where D_1, D_2 and D_3 are sets of propositions. One may determine extensions for sets of such rules as follows. Construct a sequence of increasing sets of propositions,

$$E_0, E_1, \ldots, E_i, \ldots,$$

where E_0 is the set of propositions $isax(x, y, s)$ such that (x, y, s) is a link in the hierarchy and each E_{i+1} is constructed from E_i by selecting an instantiation of a rule where D_1 is a subset of E_i and D_2 is disjoint from E_i, and then choosing E_{i+1} as $E_i \cup D_3$. The process is continued to its (possibly infinite) limit. We obtain an extension by constructing a sequence $E_0, E_1, \ldots, E_i, \ldots$ on condition that the limit E of this sequence is consistent, i.e. does not contain the propositions q and $\neg q$ for some q, and E is a fixpoint for the set of rules, i.e. that the set D_3 of any rule applicable to E is a subset of E. [3]

The number of nodes in a hierarchy being finite, the total number of propositions available for a given hierarchy is also finite. For a given hierarchy, the sequence (E_i) is stationary, as it is a sequence of increasing subsets of a finite set. Therefore, a set E of propositions is an extension of the hierarchy iff there is a finite sequence $(E_0, E_1, \ldots, E_n)$ constructed as explained above, such that E is equal to E_n and E is consistent and a fixpoint for the given set of rules. Depending on the order in which the rules are applied, we may obtain several different extensions.

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[2] I will not need to use the definition of an extension as a hierarchy.

[3] To be precise, this last condition appears in the definition of a correct extension in [9], but has been forgotten in [10]. The absence of this condition would permit the production of incomplete extensions, for instance by endlessly using the same rule instantiation while others are available.

The following is Sandewall's set of inference rules.

Sandewall's set of inference rules

1. If $isax(x,y,s)$ is in E
   then add $isa(x,y,s)$ to E
2. If $isa(x,y,s)$ is in E
   then add $\neg isa(x,y,-s)$ to E
3. If $isa(x,y,+)$ and $isa(y,z,s)$ are in E
   and $isa(x,z,-s)$ and $cntr(x,y,z,s)$ are not in E
   then add $isa(x,z,s)$, $precl(x,y,z,s)$ and $\neg cntr(x,y,z,s)$ to E
4. If $precl(x,y,z,s)$, $isa(x,v,+)$ and $isa(v,y,+)$ are in E
   then add $precl(x,v,z,s)$ to E
5. If $precl(x,y,z,s)$ and $isa(y,z,-s)$ are in E
   then add $\neg isa(y,z,-s)$ to E
6. If $isa(x,y,+)$, $isa(y,z,+)$, $isa(x,z,-)$ and $isa(z,v,s)$ are in E
   and $isax(y,v,s)$ is not in E
   then add $cntr(x,y,v,s)$ to E

From this set of rules, we can reconstruct the appropriate preclusion and contradiction conditions in Sandewall's theory. The predicate $precl(x,y,z,s)$ is inferred if there is a path from x through y to z of polarity s passing through some nodes $y_0, y_1, \ldots, y_k$ such that y_0 is equal to y, x positively inherits from y_k, y_k inherits from z with polarity s and for any i from 0 to $k-1$, x positively inherits from y_i and y_i positively inherits from y_{i+1} (rules 3 and 4). The conclusion $isa(x,z,s)$ is precluded if the predicates $precl(x,y,z,s)$ and $isa(y,z,-s)$ are inferred. The conclusion $isa(x,z,s)$ is inferred in rule 3 without testing the nonpreclusion condition, and inconsistency follows from rule 5 in case preclusion is detected afterwards.

The predicate $cntr(x,y,v,s)$ is inferred if there is a path from x through y and some node z to v of polarity s such that x positively inherits from y, y positively inherits from z, x negatively inherits from z, z inherits from v with polarity s and (y,v,s) is not a link of Γ (rule 6). The conclusion $isa(x,z,s)$ is contradicted if the predicate $cntr(x,y,z,s)$ or $isa(x,z,-s)$ is inferred. The noncontradiction condition is tested before inferring the conclusion $isa(x,z,s)$ in rule 3, and the proposition $\neg cntr(x,y,z,s)$ is inferred in rule 3 in order to produce inconsistency, in case contradiction is detected afterwards.

However, as the predicates *precl* and *cntr* are explained in terms of paths, what we have for each of these predicates is a proof theory, not a semantics. This is a general problem with path-based inheritance theories. Sandewall may have been hoping to present a version of inheritance theory that would be as semantically transparent as default logic. But his importation of notions like preclusion from Touretzky's proof-theoretic formalism into the object language makes the semantics of his system problematic. Preclusion may have some connection with preferences between extensions in an extension of Reiter's logic that allows for such preferences, but so far, attempts to provide this sort of semantics for inheritance networks have not been entirely successful. Horty discusses these matters in [7]. Nevertheless, there are technical reasons for including preclusion in the object language, which I will develop in Section 4.

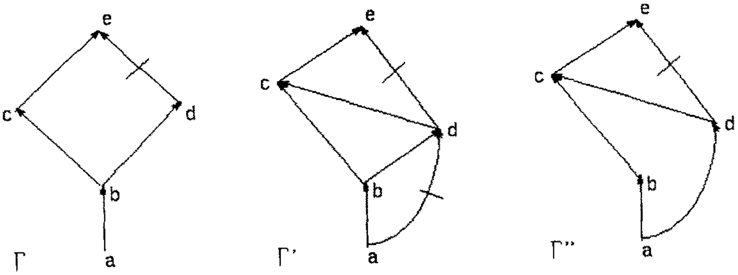

Fig. 1. Coupling or decoupling.

3. Revision of Sandewall's set of rules

3.1. Flaws in Sandewall's set of rules

Sandewall's inference rules give the intuitively correct answer on the elementary structures of Type 1, 2 and 3 presented in [10]. Let us see how they behave on some typical other examples given in the literature.

In [16], the debate between downward and upward construction is illustrated by the hierarchies Γ and Γ' of Fig. 1 (except that Γ' does not contain the link $(d,c,+)$ in [16]). The downward construction gives the intuitively correct answer on Γ. It yields two credulous conclusion sets, one containing the propositions $isa(b,e,+)$ and $isa(a,e,+)$ and the other containing the coupled conclusions $isa(b,e,-)$ and $isa(a,e,-)$. But with the upward construction, Γ has four credulous conclusion sets: the two preceding ones, plus one containing the propositions $isa(b,e,+)$ and $isa(a,e,-)$ and one containing the propositions $isa(b,e,-)$ and $isa(a,e,+)$. (These are decoupled conclusions.)

The upward construction gives the intuitively correct answer on Γ'. It produces a unique conclusion set containing the propositions $isa(b,e,-)$ and $isa(a,e,+)$. In contrast, the unique conclusion set obtained with the downward construction contains the proposition $isa(b,e,-)$ and no conclusion from a to e.

Sandewall's inference rules give the intuitively correct answer on Γ, the Type 2 structure that is represented in Fig. 6 in [10]. This is due to the fact that if the decoupled conclusions $isa(b,e,-s)$ and $isa(a,e,s)$ are inferred then $precl(a,b,e,s)$ is inferred too, which leads to inconsistency through rule 5. As for Γ', Sandewall's rules give no extension on this hierarchy! In any sequence of consistent E_i, there is no way to avoid inferring $isa(b,e,-)$, $isa(a,c,+)$, $precl(a,c,e,+)$. This is because there is no way to infer $isa(a,e,-)$ or $cntr(a,c,e,+)$; we therefore obtain $precl(a,b,e,+)$, which leads to inconsistency through rule 5.

A similar situation occurs in Fig. 2. This example, which concerns path-based theories, is presented in [17] to illustrate the debate between preemption with and without reinstatement. The question here is: should the positive path $xcbf$ belong to the unique expansion of Γ? The answer to this question does not affect the conclusion set of the expansion which intuitively contains the proposition $isa(x,b,+)$ in addition to the propositions $isa(x,y,s)$ such that (x,y,s) is a link of Γ. However, once again,

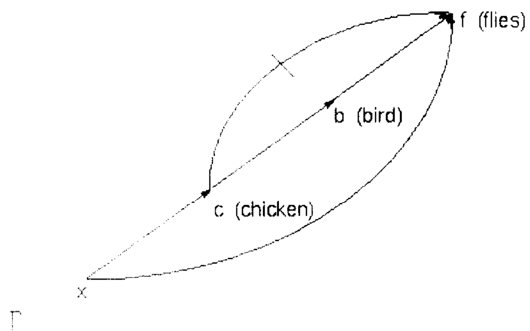

Fig. 2. Reinstatement.

Sandewall's rules give no extension on this hierarchy. In any sequence of consistent E_i, there is no way to avoid inferring $isa(c, f, -)$, $isa(x, b, +)$, $precl(x, b, f, +)$, as there is no way to infer $isa(x, f, -)$ or $cntr(x, b, f, +)$. We then have $precl(x, c, f, +)$, which leads to inconsistency through rule 5.

The common flaw in the examples of Figs. 1 (hierarchy Γ') and 2 is that in any situation where the propositions $isa(x, v, +)$, $isa(v, y, +)$, $isa(y, z, s)$, $isa(x, y, +)$ and $isa(v, z, -s)$ have to be inferred, but the propositions $isa(x, z, -s)$ and $cntr(x, y, z, s)$ may not be inferred, then $precl(x, y, z, s)$ through rule 3 and $precl(x, v, z, s)$ through rule 4 have to be inferred, which leads to inconsistency through rule 5. It seems that Sandewall did not foresee this situation and thought that inconsistency through rule 5 could be avoided by inferring $isa(x, z, -s)$ instead of $isa(x, z, s)$. He did not consider the possibility that the argument supporting the link $(x, z, -s)$—or a prefix of this argument—would itself be victim of preclusion or contradiction. In order to remedy this, the proposition $precl(x, y, z, s)$ should be used preventively. This idea leads to three changes in Sandewall's formalization: first, $precl(x, y, z, s)$ should be introduced in a rule devoted to that purpose (rather than being combined with $isa(x, z, s)$ in rule 3); second, $precl(x, y, z, s)$ should be added to the set D_2 of rule 3 (to block $isa(x, z, s)$ in case of preclusion); and third, $\neg precl(x, y, z, s)$ should be added to the set D_3 of rule 3 (to lead to inconsistency in case preclusion is detected after concluding $isa(x, z, s)$).

To stay as close as possible to Sandewall's set of rules, the proposition $precl(x, y, z, s)$ should be inferred when $isa(x, y, +)$ and $isa(y, z, s)$ are in E and there is a sequence $(v_0, v_1, \ldots, v_k)$ such that $isa(v_0, z, -s)$ is in E, v_k is equal to y and for any i from 0 to $k-1$, $isa(x, v_i, +)$ and $isa(v_i, v_{i+1}, +)$ are in E.

This preventive strategy can lead to counterintuitive results of another sort. Consider for instance the hierarchy Γ'' in Fig. 1 (called Type 1C structure in [10]). Intuitively (according to Sandewall's offpath form of preemption), Γ'' should have a unique extension, containing the proposition $isa(a, e, -)$. However, if $precl(x, y, z, s)$ is used preventively, $precl(a, c, e, +)$ is inferred, but not $precl(a, b, e, +)$, so that $isa(a, e, +)$ may be inferred from $isa(a, b, +)$ and $isa(b, e, +)$ through rule 3, leading to an intuitively incorrect extension containing the proposition $isa(a, e, +)$. This is remedied by replacing the proposition $isa(y,z,s)$ by $isax(y,z,s)$ in the set D_1 of rule 3. (In a path-based theory, this would correspond to the choice of an upward construction.) This modification renders the presence of the proposition $cntr(x,y,z,s)$ in the set D_2 of rule 3 useless (because $cntr(x,y,z,s)$ cannot be inferred when (y,z,s) is a link of Γ). So *cntr* may be eliminated from the formalism, thereby simplifying the set of rules.

A less important flaw of Sandewall's rules is their inability to produce any extension on hierarchies containing contradictory links, i.e. links in the form $(x,y,+)$ and $(x,y,-)$. Rules 1 and 2 render inconsistency unavoidable in such hierarchies. If we do not wish to restrict ourselves to consistent hierarchies, rule 2 should be suppressed and the proposition $\neg isa(x,z,-s)$ should be added to the set D_3 of rule 3.

3.2. Revised set of inference rules

The following is the revised set of inference rules as it emerges from the preceding remarks.

Revised set of inference rules

1. If $isax(x,y,s)$ is in E
   then add $isa(x,y,s)$ to E
2. Inference of $precl(x,y,z,s)$
   1. If $isa(x,y,+)$ and $isax(y,z,s)$ are in E
      then add $p(x,y,y,z,s)$ to E
   2. If $p(x,w,y,z,s)$, $isa(x,v,+)$ and $isa(v,w,+)$ are in E
      then add $p(x,v,y,z,s)$ to E
   3. If $p(x,v,y,z,s)$ and $isa(v,z,-s)$ are in E
      then add $precl(x,y,z,s)$ to E
3. If $isa(x,y,+)$ and $isax(y,z,s)$ are in E
   and $precl(x,y,z,s)$ and $isa(x,z,-s)$ are not in E
   then add $isa(x,z,s)$, $\neg precl(x,y,z,s)$ and $\neg isa(x,z,-s)$ to E

The revised set of inference rules produces the same conclusion sets as path-based credulous theories using downward construction and offpath preemption on the examples of Figs. 1 and 2.

4. Inference rules for path-based theories

4.1. Path-based defeasible credulous theories

In path-based theories proposed by Touretzky, Horty and Thomason [6–8,14,15], a conclusion set of a hierarchy Γ is defined from a set Φ of paths in Γ as the set of conclusions supported by the paths in Φ. A path is either positive or negative. A positive path is only composed of positive links, and a negative path is composed of positive links except for the last one, which is negative. The conclusion supported

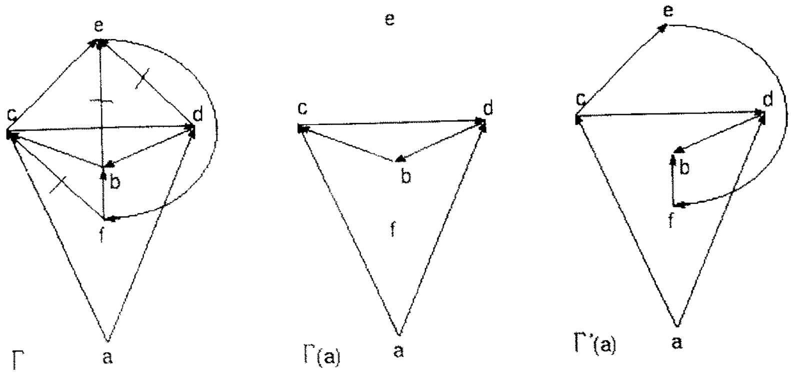

Fig. 3. Offpath preemption.

by a positive (respectively negative) path from x to y is the proposition $isa(x, y, +)$ (respectively $isa(x, y, -)$). The set Φ of paths is called a *grounded expansion* in [15], an *extension* in subsequent papers. It is called an *expansion* here, as an extension denotes a set of propositions in Sandewall's theory. An expansion Φ is defined as a fixpoint for the notion of inheritability in (Γ, Φ), which is itself defined from the notions of construction, preemption (also called preclusion) and contradiction in (Γ, Φ).[4] There are upward and downward variants of the construction. For any path α in Γ formed by concatenation of the path α_1 and the link u_1 and by concatenation of the link u_2 and the path α_2,

- α is *upward constructible* in (Γ, Φ) iff α_1 is in Φ,
- α is *downward constructible* in (Γ, Φ) iff α_1 and α_2 are in Φ.

There are multiple variants of preemption, in particular several variants of onpath and several variants of offpath preemption. Offpath preemption, the most classical variant, was originally proposed by Sandewall [10] as intuitively preferable to Touretzky's onpath proposal in [15]. We will consider here three variants of offpath preemption: namely, offpath preemption, offpath preemption with reinstatement and preemption by general subsumption. In the following definitions, Φ is a set of paths in Γ and α is a compound path in Γ (i.e., a path composed of more than one link), with start node x and last link (y, z, s).

Offpath preemption is defined in [6–8,11] as follows (see Fig. 3):

- α is *offpath preempted* in (Γ, Φ) iff $(x, z, -s)$ is a link of Γ or there is a node v of Γ such that $(v, z, -s)$ is a link of Γ and there is a positive path μ_1 from x to v and a positive path μ_2 from v to y such that the concatenation of μ_1 and μ_2 is in Φ.

The notion of reinstatement has been discussed in [1,7,12,17]. Offpath preemption with reinstatement is defined as follows:

[4] An expansion Φ is explicitly defined as a fixpoint in [6,7] and may be defined in an equivalent way as a fixpoint in [8,14,15].

- α is *offpath preempted with reinstatement* in (Γ,Φ) iff $(x,z,-s)$ is a link of Γ or there is no path in Γ of same start node, end node and polarity as α that is a link or a compound path constructible and not offpath preempted in (Γ,Φ).

The third variant has been called preemption by general subsumption in [7] and HC2-preemption in [12]. It is defined as follows:

- α is *preempted by general subsumption* in (Γ,Φ) iff $(x,z,-s)$ is a link of Γ or there is a node v of Γ such that $(v,z,-s)$ is a link of Γ and there is a positive path μ_1 in Φ from x to v and a positive path μ_2 in Φ from v to y.

Contradiction in (Γ,Φ) is naturally defined as follows:

- α is *contradicted* in (Γ,Φ) iff there is a path in Φ of same start node and end node as α and opposite polarity to α.

In a credulous theory, inheritability in (Γ,Φ) of a path α in Γ is defined as follows:

- if α is a link then α is *inheritable* in (Γ,Φ),
- if α is a compound path then α is *inheritable* in (Γ,Φ) iff it is constructible and neither preempted nor contradicted in (Γ,Φ).

Finally, for any set Φ of paths in Γ,

- Φ is an *expansion* of Γ iff it is exactly the set of paths in Γ that are inheritable in (Γ,Φ).

4.2. Set S of inference rules

The following is the set—called S—of inference rules that immediately emerges from the definition of path-based credulous theory using upward construction.

Set S of inference rules

1. If $isax(x,y,s)$ is in E
   then add $isa(x,y,s)$ to E
2. Inference of $precl(x,y,z,s)$
   (depends on the definition of preemption)
3. If $isa(x,y,+)$ and $isax(y,z,s)$ are in E
   and $precl(x,y,z,s)$ and $isa(x,z,-s)$ are not in E
   then add $isa(x,z,s)$, $\neg precl(x,y,z,s)$ and $\neg isa(x,z,-s)$ to E

The set S appears as a generalization of the revised one. Rule 2′ (respectively 2″, 2‴) is the instantiation of rule 2 for offpath preemption (respectively offpath preemption with reinstatement, preemption by general subsumption).

Inference rule 2′ for offpath preemption

2′. Inference of $precl(x,y,z,s)$
   1. If $isa(x,v,+)$, $isax(v,y,+)$, $isa(x,y,+)$ and $isax(v,z,-s)$ are in E
      and $precl(x,v,y,+)$ and $isax(x,y,-)$ are not in E
      then add $precl(x,y,z,s)$ and $\neg precl(x,v,y,+)$ to E
   2. If $precl(x,w,z,s)$, $isax(w,y,+)$ and $isa(x,y,+)$ are in E
      and $precl(x,w,y,+)$ and $isax(x,y,-)$ are not in E
      then add $precl(x,y,z,s)$ and $\neg precl(x,w,y,+)$ to E

Inference rule $2''$ for offpath preemption with reinstatement

$2''$. Inference of $precl(x, y, z, s)$

1. If $isa(x, v, +)$, $isax(v, y, +)$, $isa(x, y, +)$ and $isax(v, z, -s)$ are in E
   and $isax(x, y, -)$ is not in E
   then add $precl(x, y, z, s)$ to E
2. If $precl(x, w, z, s)$, $isax(w, y, +)$ and $isa(x, y, +)$ are in E
   and $isax(x, y, -)$ is not in E
   then add $precl(x, y, z, s)$ to E

Inference rule $2'''$ for preemption by general subsumption

$2'''$. Inference of $precl(x, y, z, s)$

1. If $isa(x, v, +)$, $isa(v, y, +)$ and $isax(v, z, -s)$ are in E
   then add $precl(x, y, z, s)$ to E

4.3. Equivalence of theories

Theorem 1 states that the theory defined by the set S of nonmonotonic inference rules with the adequate instantiation of rule 2 gives the same conclusion sets as the path-based credulous theory using upward construction and one of the variants of offpath preemption presented above.

Theorem 1. *For any hierarchy Γ, the conclusion sets of the extensions of Γ obtained by the set S of inference rules where rule 2 is instantiated as rule $2'$ (respectively $2''$, $2'''$) are exactly the conclusion sets of the expansions of Γ in path-based credulous theory using upward construction and offpath preemption (respectively offpath preemption with reinstatement, preemption by general subsumption).*

The proof of Theorem 1 is given in the Appendix.

For any hierarchy Γ containing no contradictory links, the set obtained from the revised set of inference rules by replacing $isa(v, w, +)$ by $isax(v, w, +)$ in subrule 2.2 and $isa(v, z, -s)$ by $isax(v, z, -s)$ in subrule 2.3 exactly produces the conclusion sets of the expansions of Γ in path-based credulous theory using upward construction and offpath preemption with reinstatement. The presence of proposition $isa(v, z, -s)$ instead of $isax(v, z, -s)$ in subrule 2.3 makes rule 2 related not only to offpath preemption, but to downward construction too. That explains why the conclusion sets produced by the revised set on the examples of Figs. 1 and 2 are those produced by path-based theories using downward construction.

Sandewall presents two interesting properties of the extensions, recalled here:

Proposition 2. *Let E and E' be two extensions of a hierarchy Γ, for which $E \subseteq E'$. Then $E = E'$.*

Proposition 3. *Every union of distinct extensions of a hierarchy is inconsistent.*

We may consider the properties expressed in Propositions 2 and 3 for expansions or conclusion sets instead of extensions with the following definitions of inconsistency. An expansion is inconsistent iff it contains two contradictory paths that are not both links of Γ. A conclusion set is inconsistent iff it contains two propositions in the form $isa(x,y,+)$ and $isa(x,y,-)$ such that $(x,y,+)$ and $(x,y,-)$ are not both links of Γ. Sandewall points out that Touretzky proved Propositions 2 and 3 for expansions, but only in the case of a hierarchy containing no positive cycle for Proposition 3 (Touretzky proved these results for downward construction and onpath preemption, but his proof still holds for upward construction and the variants of offpath preemption considered here). The hierarchy Γ in Fig. 4 is a conterexample of Proposition 3 for expansions. If preemption is offpath preemption or preemption by general subsumption then Γ has two credulous expansions Φ and Φ'. The paths in Φ (respectively Φ') starting from a are the paths in $\Gamma(a)$ (respectively $\Gamma'(a)$) starting from a. Φ and Φ' are distinct expansions of Γ, but their union is consistent. Note that the union of the corresponding extensions E and E' of Γ is inconsistent, as E contains the proposition $\neg precl(a,b,c,+)$ and E' contains $precl(a,b,c,+)$. What about the conclusion sets C and C' of the extensions E and E' of Γ (or, in an equivalent way from Theorem 1, of the expansions Φ and Φ')? C is a strict subset of C' and the union of C and C' is consistent, so that neither Proposition 2 nor Proposition 3 holds for the conclusion sets of the hierarchy Γ in Fig. 4. It follows from Touretzky's results that Propositions 2 and 3 are true for the conclusion sets of any hierarchy Γ containing no positive cycle, as a set Φ of paths in Γ is inconsistent iff the set C of conclusions of Φ is inconsistent and Proposition 3 implies Proposition 2 (the conclusion set of an expansion being consistent). It also follows from his results that Proposition 2 is true for the conclusion sets of any hierarchy Γ if the preemption is an offpath one with reinstatement. Indeed, in that case, for any conclusion sets C and C' supported by Φ and Φ' respectively, the inclusion of C in C' implies that of Φ in Φ'. In summary, Propositions 2 and 3 hold for extensions, expansions and conclusion sets as long as the hierarchy Γ contains no positive cycle. But if Γ contains positive cycles then Propositions 2 and 3 still hold for extensions, but not for expansions and conclusion sets. This shows that to get the desirable properties expressed in Propositions 2 and 3 for any hierarchy Γ, we have to use the more expressive object language of extensions containing the predicate *precl*, and not the less expressive one of conclusion sets not containing the predicate *precl*. In other words, it appears that to get these desirable properties, we have to put preclusion in the object language.

5. Conclusion

Sandewall [10] presents a theory of multiple inheritance with exceptions based on a set of nonmonotonic inference rules, taking advantage at the same time of theories based on nonmonotonic logic as proposed by Etherington and Reiter [2] and of path-based theories as proposed by Touretzky, Horty and Thomason [6–8,14,15]. Besides, he allows hierarchies to contain cycles, which are generally disallowed in nonmonotonic inheritance hierarchies. He puts preclusion in the object language, i.e., in the language from which extensions are built, whereas most approaches to inheritance leave preclusion

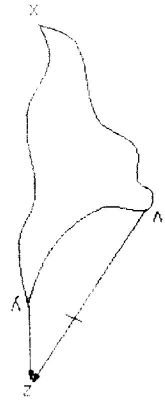

Fig. 4. Counterexample of Propositions 2 and 3 for expansions and conclusion sets.

in the metalanguage. I show in this paper that putting preclusion in the object language provides extensons with desirable general properties which are not always true in the restricted language of conclusion sets not containing preclusion. Moreover, I show that some classical variants of path-based theories provide the same conclusion sets on a hierarchy as some appropriate sets of nonmonotonic inference rules. The general conclusion seems to be that path-based theories may profit by desirable properties established in nonmonotonic logic based on sets of nonmonotonic inference rules.

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Appendix A

Theorem 1. *For any hierarchy Γ, the conclusion sets of the extensions of Γ obtained by the set S of inference rules where rule 2 is instantiated as rule $2'$ (respectively $2''$, $2'''$) are exactly the conclusion sets of the expansions of Γ in path-based credulous theory using upward construction and offpath preemption (respectively offpath preemption with reinstatement, preemption by general subsumption).*

Proof. Let Γ be a hierarchy. In the following, the set S is the set of inference rules where rule 2 is instantiated as rule $2'$ (respectively $2''$, $2'''$), an extension of Γ is an extension obtained by the set S and in absence of precision, preemption is offpath preemption (respectively offpath preemption with reinstatement, preemption by general subsumption). A path formed with the k links $(x_0, x_1, s_1), \ldots, (x_{i-1}, x_i, s_i), \ldots, (x_{k-1}, x_k, s_k)$ is noted $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ (s_i is equal to $+$ for any $i < k$). A path whose start node is x and last link is (y, z, s) is said to be in the form $(x, \ldots, y, z, s)$. For any set Φ of paths in Γ, $C(\Phi)$ denotes the set of conclusions of Φ and for any set E of propositions, $C(E)$ denotes the set of conclusions of E.

For any set E of propositions, let $f(E)$, $g(E)$ and $h(E)$ be the sets of paths in Γ defined as follows:

- $f(E)$ is the set of paths $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ in Γ such that for any i, $2 \leqslant i \leqslant k$, $isa(x_0, x_i, s_i)$ is in E and neither $precl(x_0, x_{i-1}, x_i, s_i)$ nor $isa(x_0, x_i, -s_i)$ are in E,
- $g(E)$ is the set of paths $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ in Γ such that for any i, $2 \leqslant i \leqslant k$, $isa(x_0, x_i, s_i)$ is in E and neither $precl(x_0, x_{i-1}, x_i, s_i)$ nor $isax(x_0, x_i, -s_i)$ are in E,
- $h(E)$ is the set of paths $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ in Γ such that for any i, $2 \leqslant i \leqslant k$, $isa(x_0, x_i, s_i)$ is in E and $isax(x_0, x_i, -s_i)$ is not in E.

Lemma A.1. *Let E be an extension of Γ.*

(a) $C(f(E)) = C(g(E)) = C(h(E)) = C(E)$,

(b) *for any compound path α in Γ in the form $(x, \ldots, y, z, s)$, α is offpath preempted in $(\Gamma, g(E))$ (respectively offpath preempted in $(\Gamma, h(E))$, preempted by general subsumption in $(\Gamma, g(E))$) iff $isax(x, z, -s)$ or $precl(x, y, z, s)$ is in E.*

Proof. (a) $C(f(E)) \subseteq C(g(E)) \subseteq C(h(E)) \subseteq C(E)$ is evident.

It remains to prove $C(E) \subseteq C(f(E))$. Let $(E_0, \ldots, E_i, \ldots, E_n)$ be the sequence such that $E = E_n$. It is easy to show by induction on i that for any i, $0 \leqslant i \leqslant n$, $C(E_i) \subseteq C(f(E))$. Then $E \subseteq C(f(E))$.

(b) follows from the definitions of preemption and rule 2, from (a) and from the fact that E is a fixpoint for the set S. □

(A) Let us show that the conclusion set of any extension of Γ is that of a credulous expansion of Γ.

Let E be an extension of Γ and Φ be the path set $g(E)$ (respectively $h(E)$, $g(E)$). We know from Lemma A.1(a) that $C(E)$ is equal to $C(\Phi)$. It remains to show that Φ is a credulous expansion of Γ.

Let α be a path in Γ. Let us show that α is in Φ iff it is inheritable in (Γ, Φ). If α is a link then α is in Φ and is inheritable in (Γ, Φ). We suppose α is a compound path in the form $(x, \ldots, y, z, s)$.

(1) We suppose α is in Φ. Let us show that it is inheritable in (Γ, Φ). α is constructible in (Γ, Φ) (from the definition of Φ) and not contradicted in (Γ, Φ) (because $C(\Phi) = C(f(E))$ from Lemma A.1(a) and $isa(x, z, -s)$ is not in $C(f(E))$). Let us show that α is not preempted in (Γ, Φ). If preemption is offpath preemption or preemption by general subsumption then it follows from Lemma A.1(b) that α is not preempted in (Γ, Φ). If preemption is offpath preemption with reinstatement then let α' be a path in $g(E)$ supporting the conclusion (x, z, s) (α' exists from Lemma A.1(a)). If α' is a link then α is not preempted in (Γ, Φ). If α' is a compound path in the form $(x, \ldots, v, z, s)$ then α' is constructible in (Γ, Φ) and it follows from Lemma A.1(b) that α' is

not offpath preempted in (Γ, Φ). Then α is not preempted in (Γ, Φ). Then α is inheritable in (Γ, Φ).

(2) We suppose α is inheritable in (Γ, Φ). Let us show that it is in Φ. α is not contradicted in (Γ, Φ) then $isax(x, z, -s)$ is not in E. α is not preempted in (Γ, Φ) then if preemption is offpath preemption or preemption by general subsumption, it follows from Lemma A.1(b) that $precl(x, y, z, s)$ is not in E. It remains to show that $isa(x, z, s)$ is in E. Let α' be equal to α if preemption is offpath preemption or preemption by general subsumption, and a path supporting the conclusion $isa(x, z, s)$ and being a link or a compound path constructible and not offpath preempted in (Γ, Φ) otherwise. If α' is a link then $isa(x, z, s)$ is in E. If α' is a compound path in the form $(x, \ldots, v, z, s)$ then $isa(x, v, +)$ and $isax(v, z, s)$ are in E and neither $precl(x, v, z, s)$ nor $isa(x, z, -s)$ are. As E is a fixpoint for the set S, the set D_3 of rule 3 is a subset of E. Then $isa(x, z, s)$ is in E. Then α is in Φ.

(B) Let us show that the conclusion set of any credulous expansion of Γ is that of an extension of Γ.

Lemma A.2. *For any hierarchy Γ, any expansion Φ of Γ in path-based credulous theory using upward construction and offpath preemption with reinstatement and any link (x, y, s) in $C(\Phi)$, there is a path $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ in Φ supporting (x, y, s) such that for any i, $2 \leqslant i \leqslant k$, the path $(x_0, \ldots, x_j, \ldots, x_{i-1}, x_i, s_i)$ is not offpath preempted in (Γ, Φ).*

The proof of Lemma A.2 is easy if Γ contains no positive cycle and rather technical and not of much interest here otherwise. It is written in detail in [13].

Lemma A.3. *For any hierarchy Γ, any expansion Φ of Γ in path-based credulous theory using upward construction and offpath preemption (respectively offpath preemption with reinstatement) and any path $(x_0, \ldots, x_i, \ldots, x_{k-1}, x_k, s_k)$ in Γ, α is in Φ iff for any i, $2 \leqslant i \leqslant k$,*

(1) *$isa(x_0, x_i, s_i)$ is in $C(\Phi)$,*

(2) *any (or, in a equivalent way, some) path in Γ in the form $(x_0, \ldots, x_{i-1}, x_i, s_i)$ is not offpath preempted in (Γ, Φ) (respectively $(x_0, x_i, -s_i)$ is not a link of Γ).*

The proof of Lemma A.3 is easy. It is written in detail in [13].

Let Φ be a credulous expansion of Γ and $(E_0, E_1, \ldots, E_n)$ a sequence obtained from the set S with the following decreasing order of priority on the choice of the instantiation of rule:

(1) An instantiation of rule 1.

(2) An instantiation of rule 3 such that Φ contains a compound path in the form $(x, \ldots, y, z, s)$ which is not offpath preempted (respectively offpath preempted, preempted by general subsumption) in (Γ, Φ).

(3) An instantiation of a subrule of rule 2 such that, in case this subrule is 2'.1 (respectively 2'.2), no compound path in Γ in the form $(x, \ldots, v, y, +)$ (respectively $(x, \ldots, w, y, +)$)is offpath preempted in (Γ, Φ).

(4) Any other instantiation of rule.

It follows from Lemmas A.2 and A.3 that the sequence $(E_0, E_1, \ldots, E_n)$ is in the form $(E_0, \ldots, E_{i_1}, \ldots, E_{i_2}, \ldots, E_{i_3})$, with

$$E_0 = \{isax(x,y,s): (x,y,s) \text{ is a link of } \Gamma\},$$

$$E_{i_1} = \{isa(x,y,s): (x,y,s) \text{ is a link of } \Gamma\},$$

$$E_{i_2} = E_{i_1} \cup \left(\bigcup \{isa(x,z,s), \neg precl(x,y,z,s), \neg isa(x,z,-s)\}\right),$$

(x,y,z,s) verifying the condition for choice (2),

$$E_{i_3} = E_{i_2} \cup \left(\bigcup \{precl(x,y,z,s), [\neg precl(x,v,y,+)], [\neg precl(x,w,y,+)]\}\right),$$

$(x,[v],[w],y,z,s)$ verifying the condition for choice (3), where $C(E_{i_2})$ is exactly $C(\Phi)$ (from Lemma A.2), a proposition $precl(x,y,z,s)$ is in E_{i_3} iff any compound path in Γ' in the form $(x,\ldots,y,z,s)$ is offpath preempted (respectively offpath preempted, preempted by general subsumption) in (Γ', Φ), where Γ' is the hierarchy obtained from Γ by adding the link (y,z,s) and E_{i_3} is consistent and a fixpoint for the set S (from Lemma A.3). Then E_{i_3} is an extension Γ and $C(E_{i_3})$ is equal to $C(\Phi)$. □

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