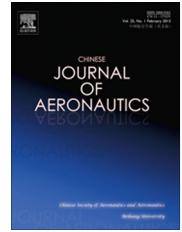




Chinese Society of Aeronautics and Astronautics
& Beihang University
Chinese Journal of Aeronautics

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Trajectory reshaping based guidance with impact time and angle constraints



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Received 12 September 2015; revised 29 February 2016; accepted 10 April 2016

Available online 22 June 2016

KEYWORDS

Guidance;
Homing missiles;
Impact angle;
Impact time;
Trajectory reshaping

Abstract This study presents a novel impact time and angle constrained guidance law for homing missiles. The guidance law is first developed with the prior-assumption of a stationary target, which is followed by the practical extension to a maneuvering target scenario. To derive the closed-form guidance law, the trajectory reshaping technique is utilized and it results in defining a specific polynomial function with two unknown coefficients. These coefficients are determined to satisfy the impact time and angle constraints as well as the zero miss distance. Furthermore, the proposed guidance law has three additional guidance gains as design parameters which make it possible to adjust the guided trajectory according to the operational conditions and missile's capability. Numerical simulations are presented to validate the effectiveness of the proposed guidance law.

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1. Introduction

A major objective of terminal guidance is to achieve a minimum miss distance.^{1,2} Current guidance applications, however, also require to impose additional terminal constraints like impact angle and impact time to improve the guidance performance. In order to enhance the effectiveness of the warhead, a particular terminal impact angle should be specified. By com-

parison, the impact time constraint is imposed to achieve a salvo attack or a cooperative attack for homing missiles, which can greatly enhance the survivability of the missile against advanced defense systems. Due to these important reasons, the guidance laws with multiple terminal constraints have been extensively studied in the past few decades.

Since the concept of impact angle guidance was initially reported in 1973,³ a large amount of work has been performed towards solving this problem. As a typical work in this area, modified proportional navigation guidance (PNG) laws were investigated to fulfill the impact angle constraints.^{4–8} Except for the modified PNG, some other control methods, such as optimal control,^{9–11} suboptimal control^{12–14} and sliding mode control (SMC),^{15–18} have also been utilized to derive the impact angle constrained guidance laws. Compared with the impact angle control laws, the studies on the impact time control guidance law are relatively rare.^{19–22}

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Peer review under responsibility of Editorial Committee of CJA.



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The guidance scheme proposed in Ref.²³ initiated the research area where both impact angle and impact time are constrained. From then on, there have been some valuable contributions made in this field.^{24–27} Through a combination of line-of-sight (LOS) rate shaping process and a second-order SMC approach, an impact time and angle (ITA) guidance law was developed in Ref.²⁴ for engaging a modern warfare ship. A closed-form ITA guidance law was presented in Ref.²⁵, where a feedback term was added to a specially constructed biased PNG to satisfy the ITA constraints. In Ref.²⁶, an optimal ITA guidance scheme for the nonlinear missile model was derived, which not only ensured the ITA requirements but achieved the minimum integral square control efforts as well. Based on the polynomial guidance law analyzed in Refs.^{28,29}, an augmented polynomial guidance law was devised in Ref.²⁷ to solve the ITA guidance problem. With the proper selection of the guidance gains, the generated homing trajectory turned out to be quite similar to the optimal solution.

In this paper, a new closed-form ITA guidance law is developed for homing missiles. The focus is first placed on engaging a stationary target. To derive the proposed guidance law, a trajectory reshaping process is introduced. This process results in defining a specific polynomial function with two unknown coefficients. One is tuned to adjust the length of the homing trajectory so as to achieve the impact time requirement. The other is determined to make the polynomial function equal to zero at each time step, so that the terminal impact angle constraint as well as the zero miss distance can be satisfied. Using the obtained solutions of the two coefficients, the guidance command can be expressed as a combination of an impact angle control law and a bias term, which is incorporated to annul the impact time error. After well developed, the guidance law is further extended to deal with maneuvering targets using the notion of predicted interception point (PIP). The associated modification in the guidance command with respect to a maneuvering target is also illustrated.

With respect to the previously published ITA guidance methods, the proposed guidance scheme could provide several advantages in the following aspects. Firstly, the approach developed in Ref.²⁴ requires an optimization routine to generate feasible LOS angle and rate profiles that meet the impact time and angle constraints, whereas such process is not needed in the implementation of the proposed ITA guidance law. Secondly, as long as the engagement conditions are determined, the guidance laws in Refs.^{23,25} would result in certain homing trajectories. The proposed law, however, has three guidance gains as design parameters which can be utilized to shape the homing trajectory and command profile in accordance with the missile's capability. In particular, the guided trajectories could exhibit similar behavior to the energy optimal solutions by choosing proper guidance gains for a given engagement. Although the numerical guidance strategy investigated in Ref.²⁶ could minimize the integral square control efforts, it requires to solve the two point boundary value problem on line and additional computational burdens would be imposed on the missile-borne computer. So its practical application is limited. Thirdly, while the works in Refs.^{23,25–27} just focused on stationary targets, this work also lays emphasis on engaging targets that are maneuvering.

2. System model and problem formulation

Consider a planar homing engagement between the missile M and the target T as depicted in Fig. 1. The missile is assumed to be traveling at a constant velocity V and the target is assumed to be stationary. The positions of the missile and the target in the inertial X - Y coordinate are denoted as (x, y) and (x_f, y_f) , respectively. The missile's heading angle is represented by θ . The acceleration command u is applied normal to the missile's velocity vector.

The equations of motion for the homing engagement are given as⁹

$$\begin{cases} \dot{x} = V \cos \theta \\ x(t_0) = x_0, \quad x(t_f) = x_f \end{cases} \quad (1)$$

$$\begin{cases} \dot{y} = V \sin \theta \\ y(t_0) = y_0, \quad y(t_f) = y_f \end{cases} \quad (2)$$

$$\begin{cases} \dot{\theta} = \frac{u}{V} \\ \theta(t_0) = \theta_0, \quad \theta(t_f) = \theta_f \end{cases} \quad (3)$$

where t_0 and t_f are the initial launch time and the designated impact time, respectively; θ_f is the desired impact angle. Note that the values of t_f and θ_f are determined before the missile is launched.

The design goal of the ITA guidance law can be summarized as follows. It is equivalent to designing a controller u such that

$$\lim_{t \rightarrow t_f} x \rightarrow x_f, \quad \lim_{t \rightarrow t_f} y \rightarrow y_f, \quad \lim_{t \rightarrow t_f} \theta \rightarrow \theta_f \quad (4)$$

It can be observed from Eqs. (1)–(3) that the equations of motion treat t as the independent variable. In terms of the homing guidance problem, however, the downrange x is a better choice for the independent variable due to the fact that x_f always corresponds to the target's location while t_f varies with different selections of impact time.²⁴ Hence, with respect to the independent variable x , a new set of equations of motion can be derived as

$$\begin{cases} y' = \tan \theta \\ y(x_0) = y_0, \quad y(x_f) = y_f \end{cases} \quad (5)$$

$$\begin{cases} \theta' = \frac{u}{V^2 \cos \theta} \\ \theta(x_0) = \theta_0, \quad \theta(x_f) = \theta_f \end{cases} \quad (6)$$

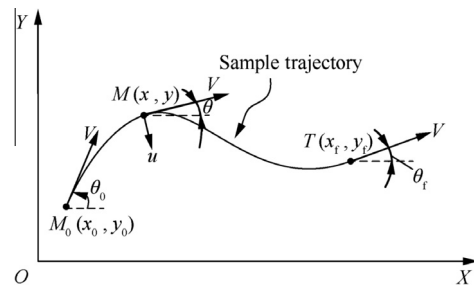


Fig. 1 Engagement geometry.

$$\begin{cases} t' = \frac{1}{V \cos \theta} \\ t(x_0) = t_0, \quad t(x_f) = t_f \end{cases} \quad (7)$$

If the X -axis is defined in a way that makes the desired impact angle θ_f equal to zero, as in the general impact angle control problems, the equations of motion, i.e., Eqs. (5) and (6), can be further transformed to the following linearized form:^{19,23,27}

$$y' = \theta \quad (8)$$

$$\theta' = \frac{u}{V^2} \quad (9)$$

Correspondingly, the design goal can be equivalently described as follows:

$$\lim_{x \rightarrow x_f} y \rightarrow y_f, \quad \lim_{x \rightarrow x_f} \theta \rightarrow 0^\circ, \quad \lim_{x \rightarrow x_f} t \rightarrow t_f \quad (10)$$

3. Guidance law with only impact angle constraint

In this section, the state feedback guidance law for engaging a stationary target with only impact angle constraint is derived. Before moving on, two variables are designed as follows:

$$\sigma = y - y_f = -y_{go}, \quad \sigma' = y' = \theta \quad (11)$$

From Eq. (11), it can be drawn that the miss distance will be minimized and the desired impact angle can be achieved as long as σ and σ' are both driven to zero as the downrange x goes to x_f . Therefore, the guidance law should be designed to drive both σ and σ' to zero as the missile approaches the target.

To devise the guidance law, a specific polynomial function is first defined as follows:

$$\zeta = \sigma' + \frac{n}{x_{go}} \sigma - Ax_{go}^p - Bx_{go}^q \quad (12)$$

where $x_{go} = x_f - x$; n, p, q, A and B are constants that need to be determined.

Assuming that $\zeta = 0$, one can get the following first-order linear differential equation:

$$\sigma' + \frac{n}{x_{go}} \sigma - Ax_{go}^p - Bx_{go}^q = 0 \quad (13)$$

By solving this equation, the analytic solution for σ can be obtained as

$$\sigma = Cx_{go}^n - \frac{A}{p-n+1} x_{go}^{p+1} - \frac{B}{q-n+1} x_{go}^{q+1} \quad (14)$$

where C is a constant determined by the initial condition as follows:

$$C = \frac{\sigma_0}{x_{go}^n} + \frac{A}{p-n+1} x_{go}^{p-n+1} + \frac{B}{q-n+1} x_{go}^{q-n+1} \quad (15)$$

where σ_0 and x_{go0} represent the initial values of σ and x_{go} .

Then, differentiating Eq. (14) with respect to x , we can obtain the analytic solution of σ' as

$$\sigma' = -Cnx_{go}^{n-1} + \frac{A(p+1)}{p-n+1} x_{go}^p + \frac{B(q+1)}{q-n+1} x_{go}^q \quad (16)$$

By inspecting Eqs. (14) and (16), it can be drawn that both σ and σ' will reach zero as x goes to x_f (i.e. x_{go} goes to zero) as long as the conditions $\zeta = 0$, $n > 1$, $p > 0$, $q > 0$, $p \neq n-1$ and $q \neq n-1$ hold.

For the sake of involving the impact time constraint into the guidance law design in the next section, the coefficients A and B need to be determined separately and thus p and q should not be equal to each other. Without loss of generality, the coefficients n, p and q are selected such that the condition $q > p > n-1 > 0$ holds.

Remark 1. The above analysis reveals that Eq. (13) essentially defines a reasonable homing trajectory for the missile; that is, as long as the system states satisfy Eq. (13), both σ and σ' would be driven to zero at $x = x_f$, which guarantees a minimum miss distance as well as a zero impact angle error. Hence, the design goal of the impact angle control guidance law can be accordingly transformed to maintaining ζ at zero during the entire homing phase.

To achieve this requirement at the initial time (i.e., $\zeta = 0$ at $x = x_0$), the condition $\sigma'_0 + \frac{n}{x_{go0}} \sigma_0 - Ax_{go0}^p - Bx_{go0}^q = 0$ must hold, from which the coefficient A can be determined as

$$\begin{aligned} A &= \frac{\sigma'_0}{x_{go0}^p} + \frac{n\sigma_0}{x_{go0}^{p+1}} - Bx_{go0}^{q-p} \\ &= \frac{\theta_0}{x_{go0}^p} - \frac{ny_{go0}}{x_{go0}^{p+1}} - Bx_{go0}^{q-p} \end{aligned} \quad (17)$$

where y_{go0} represents the initial value of y_{go} .

Then, the guidance command should be derived to make ζ identically equal to zero during the remaining flight. Since the coefficient A has been tuned to meet $\zeta = 0$, the guidance command just needs to make ζ' equal to zero. Taking the derivative of Eq. (12) and letting it equal to zero yield

$$\begin{aligned} \zeta' &= \sigma'' + \frac{n\sigma'}{x_{go}} + \frac{n\sigma}{x_{go}^2} + Ap x_{go}^{p-1} + Bq x_{go}^{q-1} \\ &= \theta' - \frac{ny_{go}}{x_{go}^2} + \frac{n\theta}{x_{go}} + Ap x_{go}^{p-1} + Bq x_{go}^{q-1} = 0 \end{aligned} \quad (18)$$

Associating this result with Eq. (9), we can obtain the guidance command as

$$u = V^2 \left(\frac{ny_{go}}{x_{go}^2} - \frac{n\theta}{x_{go}} - Ap x_{go}^{p-1} - Bq x_{go}^{q-1} \right) \quad (19)$$

Remark 2. From Eqs. (17) and (19), it can be observed that although the guidance command is expressed in a state feedback form, the coefficient A is calculated using the initial system states. Because the linearized equations of motion, i.e., Eqs. (8) and (9), are utilized in the guidance law design, large approximation errors may be produced in the initial homing phase, which would lead to terminal state errors. Further note that in practical implementation, the guidance command cannot be achieved immediately after its generation owing to the autopilot lag. In addition, the guidance command that the missile can generate is bounded according to the missile's capability. These limitations may also cause large miss distance or violation of terminal constraints. To enhance the robustness of the guidance law, the coefficient A is suggested to be initialized and recalculated at each step of x as follows:

$$A = \frac{\theta}{x_{go}^p} - \frac{ny_{go}}{x_{go}^{p+1}} - Bx_{go}^{q-p} \quad (20)$$

The aforementioned equation ensures that the condition $\xi = 0$ holds at each step of x , which implies that the homing trajectory is reshaped continuously.

By substituting Eq. (20) into Eq. (19), the associated guidance command can be transformed to the following form:

$$u = V^2 \left[\frac{n(p+1)y_{go}}{x_{go}^2} - \frac{(n+p)\theta}{x_{go}} + B(p-q)x_{go}^{q-1} \right] \quad (21)$$

It can be clearly seen from Eq. (21) that the guidance command consists of three components. The first component controls the missile to the collision course, thus enabling it to hit the target. The second component contributes to the achievement of the desired impact angle. The third component corresponds to a bias term that accounts for some other requirements, if needed. To further analyze the characteristics of the guidance command Eq. (21), the associated closed-form solutions for the system states will be derived in the following discussion.

Using Eqs. (9), (11) and (21), the following linear second-order ordinary differential equation can be obtained:

$$\sigma'' + \frac{n+p}{x_{go}} \sigma' + \frac{n(p+1)}{x_{go}^2} \sigma - B(p-q)x_{go}^{q-1} = 0 \quad (22)$$

Solving Eq. (22) yields

$$\sigma = m_1 x_{go}^n + m_2 x_{go}^{p+1} - \frac{B}{q-n+1} x_{go}^{q+1} \quad (23)$$

Then, taking the derivative of Eq. (23), we can obtain the solution for σ' as

$$\sigma' = -m_1 n x_{go}^{n-1} - m_2 (p+1) x_{go}^p + \frac{(q+1)B}{q-n+1} x_{go}^q \quad (24)$$

The coefficients m_1 and m_2 in Eqs. (23) and (24) are both constants that are determined by the initial conditions as

$$\begin{cases} m_1 = \frac{1}{(p-n+1)x_{go}^n} [-(p+1)y_{go0} + x_{go0}\theta_0 \\ + \frac{p-q}{q-n+1} B x_{go0}^{q+1}] \\ m_2 = \frac{1}{(p-n+1)x_{go}^{p+1}} (n y_{go0} - x_{go0}\theta_0 + B x_{go0}^{q+1}) \end{cases} \quad (25)$$

Hence, the closed-form solutions for the guidance system states can be obtained from Eqs. (11), (23) and (24) as

$$\begin{cases} y = m_1 x_{go}^n + m_2 x_{go}^{p+1} - \frac{B}{q-n+1} x_{go}^{q+1} + y_f \\ \theta = -m_1 n x_{go}^{n-1} - m_2 (p+1) x_{go}^p + \frac{(q+1)B}{q-n+1} x_{go}^q \end{cases} \quad (26)$$

It can be drawn from these solutions that the terminal requirements are always satisfied even through the coefficient B is an arbitrary constant. This result reveals that in addition to the constraints on miss distance and impact angle, the guidance law Eq. (21) is able to provide an additional degree of freedom by B , which can be used to shape the homing trajectory to fulfill the impact time constraint. Note that in terms of the impact angle control guidance problem, the coefficient B can be simply set to zero, thus reducing the guidance law Eq. (21) to the following simple form:

$$u_{1A} = V^2 \left[\frac{n(p+1)y_{go}}{x_{go}^2} - \frac{(n+p)\theta}{x_{go}} \right] \quad (27)$$

4. Guidance law with impact time and angle constraints

Based on the analysis in Section 3, the ITA guidance law will be derived in this section. Note that regardless of the selection of B , the missile can be steered to intercept the target along the desired impact angle by employing the guidance law Eq. (21). In the sequel, a proper value of B will be found to meet the impact time requirement.

Because the missile maintains a constant velocity, the impact time constraint can be equivalently achieved by adjusting the length of the homing trajectory. That is, the length of the predicted flight trajectory at the current time should be equal to the desired distance to go, i.e., $S_{go} = S_{go}^*$, where

$$S_{go}^* = V(t_f - t) \quad (28)$$

As suggested by Kim et al.²⁹, the calculation of the length of the homing trajectory can be approximated as follows:

$$\begin{aligned} S_{go} &= \int_x^{x_f} \sqrt{1 + (y')^2} d\eta \approx \int_x^{x_f} \left(1 + \frac{1}{2} \theta^2(\eta) \right) d\eta \\ &= x_{go} + \frac{1}{2} \int_x^{x_f} \theta^2(\eta) d\eta \end{aligned} \quad (29)$$

Note that the trajectory length S_{go} is calculated at each step of x , and the closed-form solution θ in Eq. (26) should be transformed to the following form:

$$\begin{aligned} \theta(\eta) &= -\hat{m}_1 n (x_f - \eta)^{n-1} - \hat{m}_2 (p+1) (x_f - \eta)^p \\ &+ \frac{(q+1)B}{q-n+1} (x_f - \eta)^q \end{aligned} \quad (30)$$

where

$$\begin{cases} \hat{m}_1 = \frac{1}{(p-n+1)x_{go}^n} [-(p+1)y_{go} + x_{go}\theta \\ + \frac{p-q}{q-n+1} B x_{go}^{q+1}] \\ \hat{m}_2 = \frac{1}{(p-n+1)x_{go}^{p+1}} (n y_{go} - x_{go}\theta + B x_{go}^{q+1}) \end{cases} \quad (31)$$

Substitute Eq. (30) into Eq. (29), and the estimated trajectory length can be obtained as

$$S_{go} = M_1 x_{go}^{2q+1} B^2 - (M_2 y_{go} x_{go}^q + M_3 \theta x_{go}^{q+1}) B + \bar{S}_{go} \quad (32)$$

where

$$\begin{cases} M_1 = \frac{(p-q)^2 [np+ng+(2n-1)pq+n]}{(2n-1)(2p+1)(2q+1)(n+p)(n+q)(p+q+1)} \\ M_2 = \frac{n(p-q)(p+1)(2p+q-2np+1)}{(2n-1)(2p+1)(n+p)(n+q)(p+q+1)} \\ M_3 = \frac{pq(p-q) - n(p-q)(p+q+2pq+1)}{(2n-1)(2p+1)(n+p)(n+q)(p+q+1)} \\ \bar{S}_{go} = \frac{n^2(p+1)^2}{(2n-1)(2p+1)(n+p)} \cdot \frac{y_{go}^2}{x_{go}} \\ + \frac{n+p(2n-1)}{2(2n-1)(2p+1)(n+p)} x_{go} \theta^2 \\ - \frac{n(p+1)}{(2n-1)(2p+1)(n+p)} y_{go} \theta + x_{go} \end{cases} \quad (33)$$

The term \bar{S}_{go} in Eq. (33) denotes the trajectory length that is estimated by Eq. (29) with $B = 0$. It is worth noticing that \bar{S}_{go} also corresponds to the distance to go under the guidance law u_{1A} .

As mentioned before, the estimated trajectory length should be equal to the desired distance to go. Hence, from Eqs. (28) and (32), the following quadratic equation can be derived in terms of B :

$$M_1 x_{go}^{2q+1} B^2 - (M_2 y_{go} x_{go}^q + M_3 \theta x_{go}^{q+1}) B - \varepsilon_d = 0 \quad (34)$$

where $\varepsilon_d = S_{go}^* - \bar{S}_{go}$ represents the error of distance to go.

Apparently, to achieve the desired impact time requirement, the value of B should be chosen to satisfy Eq. (34). Therefore, B can be obtained as

$$B = \frac{M_2 y_{go} + M_3 \theta x_{go}}{2M_1 x_{go}^{q+1}} \pm \frac{\sqrt{(M_2 y_{go} + M_3 \theta x_{go})^2 + 4M_1 x_{go} \varepsilon_d}}{2M_1 x_{go}^{q+1}} \quad (35)$$

Substitute Eq. (35) into Eq. (21), and the ITA guidance law can be finally obtained. Note that to ensure the existence of a real value of B , the discriminant of Eq. (34) must be equal to or larger than zero, i.e., $(M_2 y_{go} + M_3 \theta x_{go})^2 + 4M_1 x_{go} \varepsilon_d \geq 0$. From Eq. (33), it can be concluded that M_1 is larger than zero since $q > p > n - 1 > 0$. Hence, a real value of B will always exist as long as $\varepsilon_d \geq 0$. In particular, considering this condition at the initial time, one can get the following result:

$$t_f \geq \frac{\bar{S}_{go0}}{V} \quad (36)$$

where \bar{S}_{go0} is the initial estimation of distance to go under the guidance law u_{IA} . Essentially, Eq. (36) defines the principle for the selection of the desired impact time. In this way, the coefficient ε_d has always a positive value at the initial time, and the proposed ITA guidance law can be utilized to enforce ε_d to zero.

For numerical stability in practical implementation, the imaginary roots of the quadratic equation should be avoided. In this regard, ε_d can be modified to be the following form:²³

$$\varepsilon_d = \max(\varepsilon_d, 0) \quad (37)$$

In this case, the quadratic Eq. (34) will always have two real roots. Note that when ε_d is null, the impact time error is eliminated. This result implies that the distance to go under the application of u_{IA} is equal to the desired distance to go and thus the corresponding value of B is equal to zero. From the above analysis, it can be concluded that the sign of the second term on the right hand side of Eq. (35) should be determined to fulfill such a condition that $B = 0$ when $\varepsilon_d = 0$. This conclusion also results in choosing the smaller one in absolute value out of the two real solutions of B .

Remark 3. From the above analysis, it can be drawn that the proposed ITA guidance law is able to achieve the desired impact time and angle requirements as long as the parameters n , p and q are selected in accordance with $q > p > n - 1 > 0$. This conclusion indicates that the guidance law has three guidance gains as design parameters to shape the homing trajectory without violation of the desired terminal constraints. Accordingly, the proposed ITA guidance law possesses the capabilities to generate feasible homing trajectories in relation to the operational conditions as well as the missile's capability.

Remark 4. In some cases, it is difficult to make the heading angle be small all the time during the flight, which implies that the linearized equations (i.e., Eqs. (8) and (9)) would be inaccurate. Fortunately, the error caused by linearization can be gradually reduced by using the proposed guidance law. This is because the homing trajectory is continuously reshaped according to the instantaneous states and the desired terminal constraints during the flight. This statement is verified by numerical simulations in Section 6.2.

5. Application of ITA guidance law to maneuvering targets

This section details the steps in the application of the ITA guidance law to the maneuvering target scenario. The target is assumed to be traveling at a constant velocity V_t which is much smaller than that of the missile.²⁴ The target's heading angle is denoted as θ_t and it is changed by the lateral acceleration a_t .

The associated equations of motion of the target are given as

$$\begin{cases} \dot{x}_t = V_t \cos \theta_t \\ \dot{y}_t = V_t \sin \theta_t \\ \dot{\theta}_t = \frac{a_t}{V_t} \end{cases} \quad (38)$$

where a_t , the target's lateral acceleration, is assumed to be a constant during the entire homing phase.

In the sequel, the ITA guidance law will be extended to deal with a maneuvering target using the idea of PIP. The PIP is defined as the point at which the missile is expected to intercept the target. For the sake of clarity, a sample trajectory of the target is shown in Fig. 2.

Because the target executes a constant maneuver, the associated trajectory turns out to be a circle arc with $O'(o_x, o_y)$ as the center. The radius, central angle and length of the circle arc are represented by r_c , C_a and l_c respectively. From the geometrical relationship depicted in Fig. 2, the interception point can be predicted as³⁰

$$\begin{cases} x_f = x_t + \frac{V^2}{a_t} \left[\cos\left(\theta_t - \frac{\pi}{2}\right) - \cos\left(\frac{a_t t_{go}}{V_t} + \frac{\pi}{2} - \theta_t\right) \right] \\ y_f = y_t + \frac{V^2}{a_t} \left[\sin\left(\theta_t - \frac{\pi}{2}\right) + \sin\left(\frac{a_t t_{go}}{V_t} + \frac{\pi}{2} - \theta_t\right) \right] \end{cases} \quad (39)$$

where (x_t, y_t) represents the instantaneous target position and t_{go} the estimated time to go. As is well known, the most widely used t_{go} calculation method is $t_{go} = r/V_c$, where r and V_c represent the range and the closing velocity between the missile and the target, respectively. However, this method can provide good estimates of t_{go} only when the missile is slightly deviated from the collision course. For the ITA guidance problem, the homing trajectory is highly curved in general and thus the term r/V_c may give poor t_{go} estimation. To overcome this difficulty, an improved t_{go} estimation method is utilized in this work,³¹ which is expressed as

$$t_{go} = \frac{r}{V} \left[1 + \frac{(\theta - \lambda)^2}{4N - 2} \right] \quad (40)$$

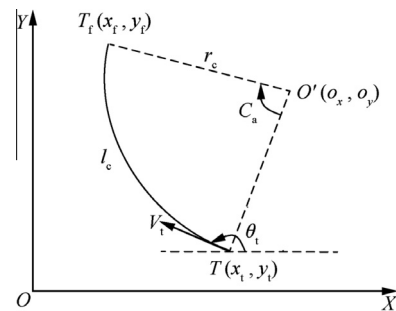


Fig. 2 Sample trajectory of target.

where λ is the LOS angle and N the proportional navigation constant.

Note that if the target's lateral acceleration is zero, i.e., $a_t = 0 \text{ m/s}^2$, the heading angle of the target would maintain a constant value and a straight flight would be resulted in. In this case, the predicted interception point can be simply determined by the following equations:²²

$$\begin{cases} x_f = x_t + V_1 \cos \theta_t t_{go} \\ y_f = y_t + V_1 \sin \theta_t t_{go} \end{cases} \quad (41)$$

In terms of intercepting a maneuvering target with a designated impact angle at a desired impact time, the variables x_f and y_f in Eq. (21) for a stationary target, should be updated using Eq. (39) (or Eq. (41)) at each time step.

Note that Eq. (40) guarantees that the estimated time to go would converge to a neighborhood of the actual time to go. So the predicted interception point would also converge to a neighborhood of the actual interception point. Fig. 3 depicts a sample trajectory of the predicted x_f . As can be seen, the predicted x_f trajectory would first approach the actual x_f and then oscillate around it. Such phenomena, however, may lead to singularities in the solution of B at the later stage of the homing phase. The reasons are explained as follows. Substituting Eq. (26) into Eq. (34) yields

$$\begin{aligned} & \left[M_1 - \frac{M_2}{q-n+1} - \frac{M_3(q+1)}{q-n+1} \right] x_{go}^{2q+1} B^2 \\ & + [m_1(M_2 + nM_3)x_{go}^{n+q} + m_2(M_2 + pM_3 \\ & + M_3)x_{go}^{p+q+1}] B - \varepsilon_d = 0 \end{aligned} \quad (42)$$

By solving this equation, the solution for B can be obtained as

$$\begin{aligned} B = & \left[-(C_2 + C_3 x_{go}^{p-n+1}) \pm \sqrt{(C_2 + C_3 x_{go}^{p-n+1})^2 + 4C_1 x_{go}^{1-2n} \varepsilon_d} \right] \\ & \cdot (2C_1 x_{go}^{q-n+1})^{-1} \end{aligned} \quad (43)$$

where $C_1 = M_1 - \frac{M_2}{q-n+1} - \frac{M_3(q+1)}{q-n+1}$, $C_2 = m_1(M_2 + nM_3)$ and $C_3 = m_2(M_2 + pM_3 + M_3)$ are constants. Since the predicted x_f would oscillate around the actual x_f , it may be equal to the instantaneous downrange x at the later stage of the homing phase (e.g., the point $T_p(t_p, x_{ip})$), which implies that $x_{go} = 0 \text{ m}$ occurs before the time t_f . Associating this result with Eq. (43), we can find that a singularity would appear in the solution of B .

For numerical stability in the guidance law implementation, the calculation of B is modified to be the following form:

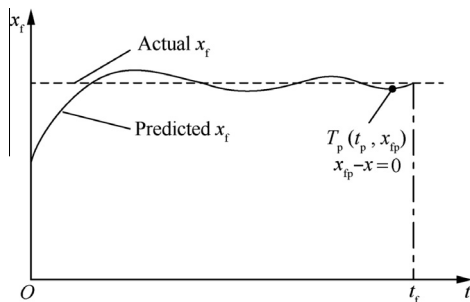


Fig. 3 Sample trajectory of predicted x_f .

$$B = \begin{cases} \frac{M_2 y_{go} + M_3 \theta x_{go}}{2M_1 x_{go}^{q+1}} \pm \frac{\sqrt{(M_2 y_{go} + M_3 \theta x_{go})^2 + 4M_1 x_{go} \varepsilon_d}}{2M_1 x_{go}^{q+1}} & 0 \leq t \leq t_f - \Delta \\ B(t_f - \Delta) & t_f - \Delta < t \leq t_f \end{cases} \quad (44)$$

where Δ is a constant that needs to be determined, and $B(t_f - \Delta)$ the value of B at the time $t_f - \Delta$.

Remark 5. (Choice of Δ) In order to ensure a satisfactory guidance performance, the value of Δ should be chosen neither too large nor too small. On the one hand, a relatively large Δ should be selected to avoid the singularities in the solution of B . On the other hand, a small Δ is required to guarantee the impact time accuracy. Therefore, there is a trade-off in the selection of Δ .

6. Numerical simulations

To verify the effectiveness of the proposed ITA guidance law, numerical simulations are performed for a variety of scenarios in this section. The parameters used in the following cases, unless specified, are listed in Table 1. Note that all simulations are terminated when the distance between the missile and target is less than 0.5 m.

6.1. Case study for different impact time

The simulations in this case are performed with different impact time, and the results are shown in Fig. 4. Note that results regarding the engagement of the target with only impact angle constraint (i.e., without impact time constraint) are also involved in the simulations, which are represented by the solid lines.

From the observation of the solid lines, it can be found that the missile is able to intercept the target with the desired impact angle at roughly 41 s and the magnitude of the acceleration is about -13 m/s^2 . According to the discussion in Section 4 (see Eq. (36)), the desired impact time should be chosen larger than 41 s in the ITA guidance law application. As a consequence, the impact time 50, 60 and 70 s are selected in this case, and the associated results are represented by dashed lines.

As can be drawn from Fig. 4(a) and (b), the proposed ITA guidance law enables the missile to change its course to meet different impact time demands. The corresponding impact time errors and impact angle errors are listed in Table 2. From these results, it can be found that the impact time and angle constraints are both satisfied. From Fig. 4(c), it can be clearly seen that as the desired impact time increases, the magnitude of the acceleration command increases. However, all the acceleration commands converge to small magnitudes in the terminal

Table 1 Simulation parameters.

Parameter	Value	Parameter	Value
(x_0, y_0) (m)	(0, 500)	a_t (m/s^2)	-0.2
θ_0 ($^\circ$)	30	n	2
(x_f, y_f) (m)	(10,000, 0)	p	1.5
V (m/s)	250	q	2.5
V_t (m/s)	10	N	3
θ_{t0} ($^\circ$)	135	Δ (s)	10

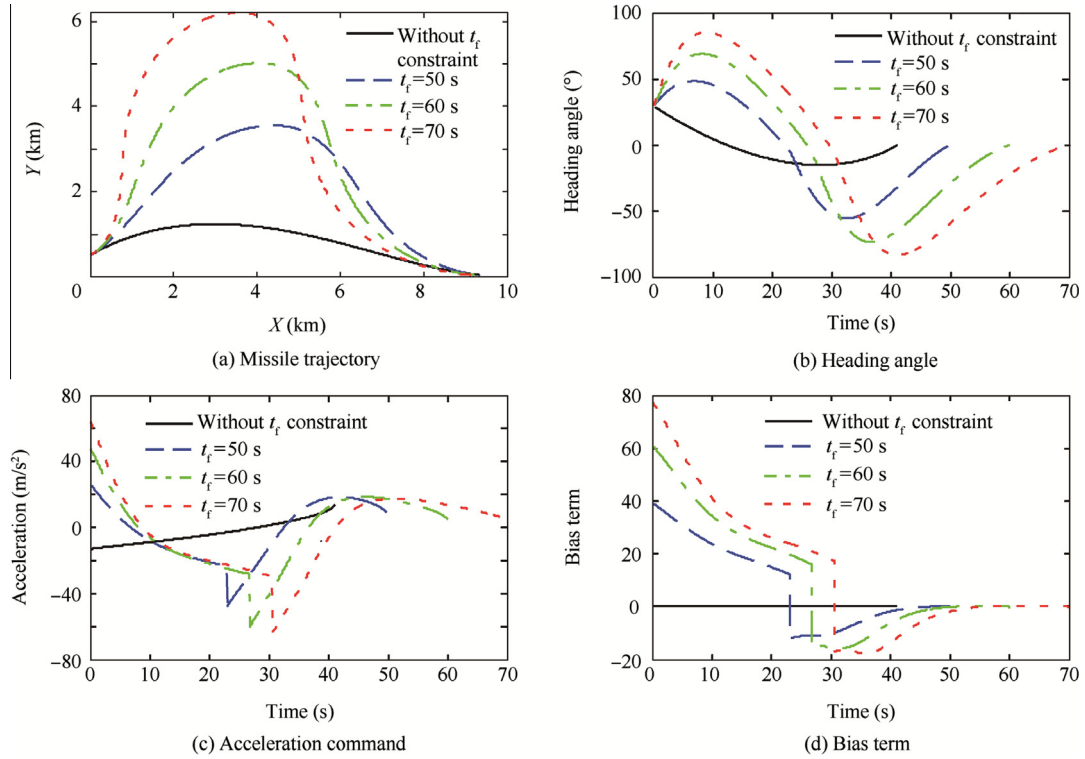


Fig. 4 Simulation results for different impact time.

Table 2 Impact time and angle errors for different impact time cases.

Desired impact time (s)	Impact time error (s)	Impact angle error (°)
Without t_f constraint	N/A	0.0090
50	0.0027	0.0042
60	0.0026	0.0026
70	0.0031	0.0022

homing phase, which ensures some operational margin for the guidance command to deal with potential disturbances. Note that the discontinuity of the acceleration command results from the switch of the solution of B . Fig. 4(d) presents the histories of the bias term $BV^2(p-q)x_{go}^{q-1}$, which is used to eliminate the impact time errors. As can be seen, the magnitude of the bias term also has the proportional relationship with the desired impact time, i.e., the larger t_f is, the larger the magnitude of the bias term will be. It can be further noted from Fig. 4(d) that the bias terms converge to zero as the missile approaches the target. It implies that the impact time error gradually vanishes and correspondingly the ITA guidance law reduces to the impact angle guidance law Eq. (27).

6.2. Case study for different launch angles

This set of simulations is conducted by choosing a specific impact time of 60 s and allowing the launch angle to vary among values of 0°, 60°, 120° and 180°. The results for these simulations are depicted in Fig. 5.

For each designated launch angle, Fig. 5(a) demonstrates that the guidance law is capable of generating a feasible trajectory which fulfills the impact angle requirement. From Fig. 5(b), it is revealed that the heading angle histories converge to zero at roughly 60 s, indicating that the impact time constraint is satisfied as well. The concrete values of impact time errors and impact angle errors are shown in Table 3. Further, from the results in Fig. 5(a) and (c), one can observe that a relatively large magnitude of lateral acceleration is required during the period when the missile changes its course, especially for the turning back maneuver. The simulation results in this subsection validate that the proposed ITA guidance law is able to achieve the desired terminal constraints even in the presence of large initial heading errors.

6.3. Case study for different impact angles

In this part, the simulations are performed with different impact angles of 0°, -30° and -60°. The desired impact time is set to be 50 s. The flight trajectories, heading angle histories and acceleration command histories are shown in Fig. 6.

It can be seen that successful interceptions are guaranteed for all the three cases. The impact angle errors and impact time errors for this set of simulations are shown in Table 4, which validates that the impact time and angle constraints are satisfied as well. Because different impact angle constraints are imposed, these flight trajectories are apparently different from each other from roughly 20 s onwards. The flight trajectory for the case of $\theta_f = 0^\circ$ is more curved than the others, and thus, the corresponding magnitude of the acceleration command is much larger. Also, it can be found that no sudden change occurs in guidance command for the case of $\theta_f = 60^\circ$, which

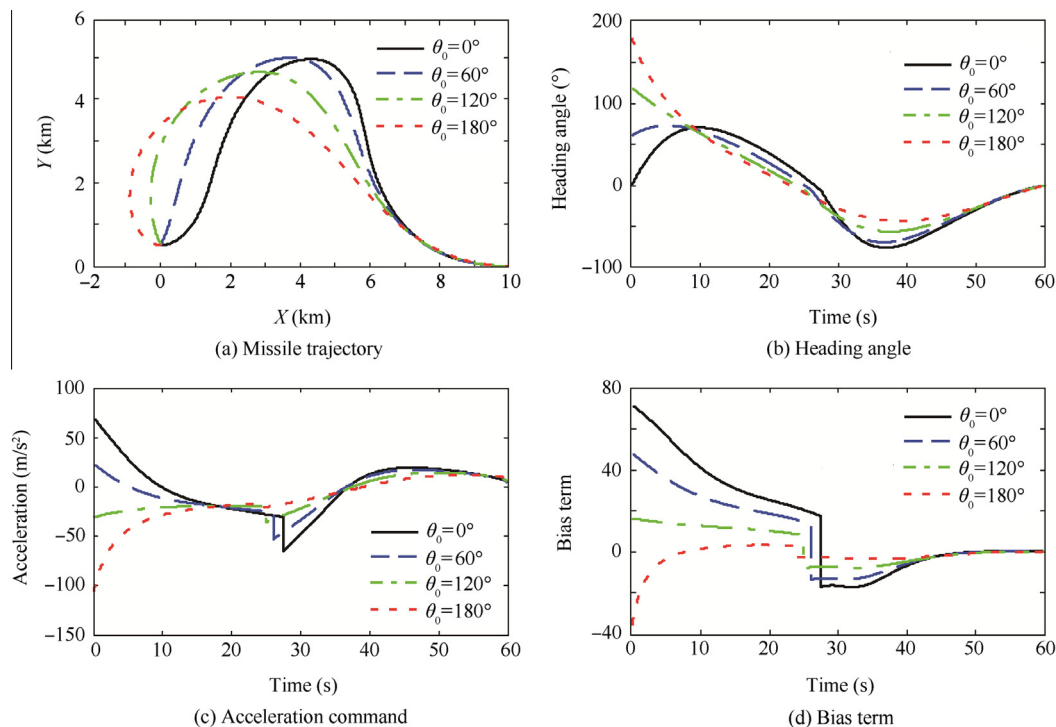


Fig. 5 Simulation results for different launch angles.

Table 3 Impact time and angle errors for different launch angle cases.

Launch angle (°)	Impact time error (s)	Impact angle error (°)
0	0.0026	0.0026
60	0.0023	0.0027
120	0.0021	0.0036
180	0.0027	0.0055

implies that the solution of B in this case would not switch during the entire flight.

6.4. Case study for comparative analysis

The simulations in this part aim at providing comparative analysis between the proposed ITA guidance law and the guidance law investigated in Ref.²³ (Lee's law). The desired impact time is selected as 50 s and the guidance gains for the proposed law are selected as $n = 1.7$, $p = 1.2$ and $q = 4.3$. For comparison, the optimal solutions are also obtained using the General Pseudospectral Optimization Software under the same engagement conditions. The associated performance index is defined as the integral of the square of the control effort, i.e., $J = 0.5 \int_{t_0}^{t_f} u(t)^2 dt$. It is worth noticing that the guidance law in Ref.²³ is also derived by employing the optimal control theory with jerk used as the guidance command. The simulation results for this case are shown in Fig. 7.

From Fig. 7(a) and (b), it can be seen that both the proposed guidance law and Lee's law are able to steer the missile to intercept the target at the desired impact time with the designated impact angle. However, the generated homing

trajectories and heading angle histories are different. As can be seen, the results obtained from the proposed ITA guidance law with $n = 1.7$, $p = 1.2$ and $q = 4.3$ are quite similar to the optimal solutions. Correspondingly, it can be observed from Fig. 7(c) that the proposed ITA guidance law has the similar acceleration history to the optimal solution and requires less acceleration capability than Lee's law. The values of the total control effort (i.e., J) for the three cases are calculated, and the results are given as follows:

$$J_{\text{optimal}} = 7062.9, J_{\text{proposed}} = 7543.2, J_{\text{Lee's}} = 10311.1$$

It can be found that the proposed ITA guidance law has similar control cost to the optimal solution, and it is lower than that of Lee's law.

6.5. Case study for a constant velocity target with system uncertainty

In realistic scenarios, the system response will be affected by system uncertainties or external disturbances. For example, the target's instantaneous position that is measured by the sensors will definitely contain noise, and thus, the information cannot be known accurately. In the previous simulations, the system uncertainty is not considered. In order to analyze the effects of system uncertainty on the guidance system, the simulations in this subsection are performed with addition of measurement noise in the target's instantaneous position. The target is assumed to be traveling at a constant velocity and the acceleration is zero, which implies that the trajectory of the target is a straight line. Without loss of generality, the noise is assumed to be Gaussian noise. It is worth noticing that noise in the target's position would also lead to noise in other system states such as the LOS angle, LOS range and time-to-go estimation.

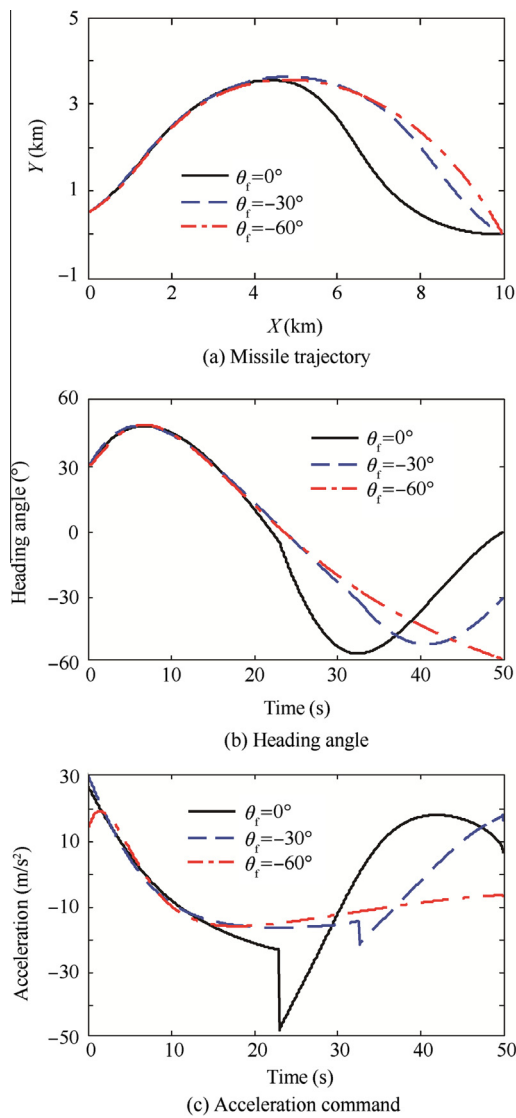


Fig. 6 Simulation results for different impact angles.

Table 4 Impact time and angle errors for different impact angle cases.

Impact angle (°)	Impact time error (s)	Impact angle error (°)
0	0.0028	0.0042
-30	0.0030	0.0117
-60	0.0033	0.0041

First, the 1 m standard deviation noise case is considered and the results are depicted in Fig. 8. From an inspection of Fig. 8, the desired terminal constraints are all fulfilled. The miss distance, impact angle error and impact time error are 0.0745 m, 0.0709° and 0.0521 s, respectively. Although the noise has small effects on the resulting flight trajectory and heading angle history, it does affect the acceleration command. As can be seen, chattering in acceleration command is induced near the time of interception. However, it should be noted that it is an expected result because the uncertainty caused by the

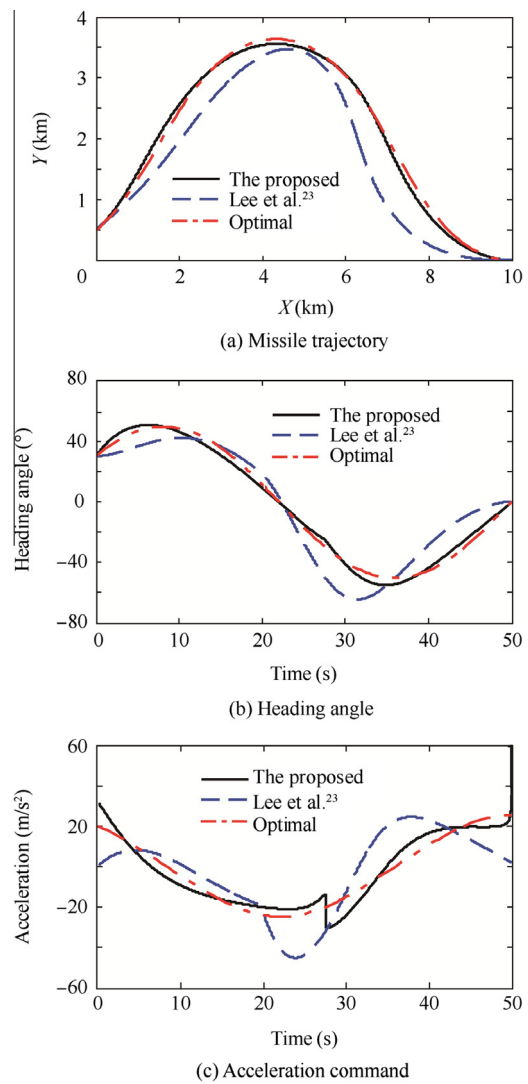


Fig. 7 Simulations results for comparative analysis.

noise in target’s position increases as the range between the missile and target decreases.

In the sequel, different noise levels are studied, i.e., 2 m standard deviation noise, 3 m standard deviation noise and 5 m standard deviation noise. These results are listed in Table 5. As expected, the miss distance, impact angle error and impact time error increase as the noise level increases. However, these results are still acceptable as long as the size of the target is much larger than the associated miss distance, e.g., a modern warfare ship. The simulation results in this part also verify the robustness of the proposed guidance law.

6.6. Case study for a maneuvering target with autopilot lag and acceleration limit

The final set of simulations is performed with the ITA guidance law applied to a maneuvering target scenario. Note that the influences of the autopilot lag and acceleration limit are also practically taken into consideration. The conditions for the target can be found in Table 1. The desired impact time is chosen as 60 s. With the assumption of a first-order lag system, the achieved missile acceleration u_a can be obtained as

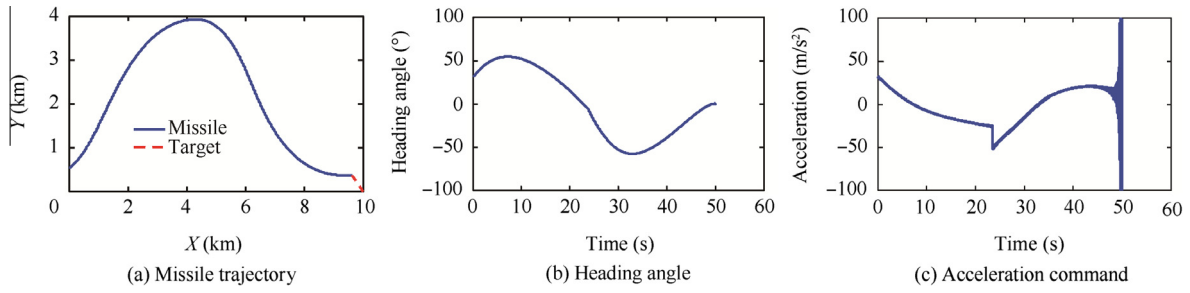


Fig. 8 Results for a constant velocity target with system uncertainty.

Table 5 Results with different noise levels.

Noise level (m)	Miss distance (m)	Impact angle error (°)	Impact time error (s)
2	0.1949	0.1375	0.1526
3	0.9587	0.2614	0.3258
5	2.4618	0.7503	0.7460

$$u_a = \frac{1}{\tau s + 1} u \quad (45)$$

where $\tau = 1/3$ s is the time constant. The maximum lateral acceleration that the missile can generate is bounded within $(-40, 40)$ m/s². The results for this set of simulations are represented in Fig. 9. From an inspection of this figure, it can be revealed that, even in the presence of autopilot lag and acceleration limit, the miss distance can be reduced in the vicinity of the maneuvering target while the exact impact time and angle are still preserved. Besides, it can be seen that the t_{go}

calculation method (i.e., Eq. (40)) could provide accurate results near the time of interception. These results also indicate that the change in the acceleration command would not produce any significant effect on the guidance performance in practical implementation.

7. Conclusions

In this paper, a new impact time and angle guidance law has been presented for homing missiles against both stationary and maneuvering targets. To achieve the desired terminal constraints, a trajectory reshaping process is introduced, and it results in defining a specific polynomial function with two unknown coefficients. These tuning coefficients can be analytically obtained, and thus, the closed-form guidance law for both impact time and impact angle control is derived. According to the selection of guidance gains, the homing trajectory can be shaped in relation to the operational conditions without violation of the desired terminal constraints. The simulations have revealed that the proposed guidance law is capable of

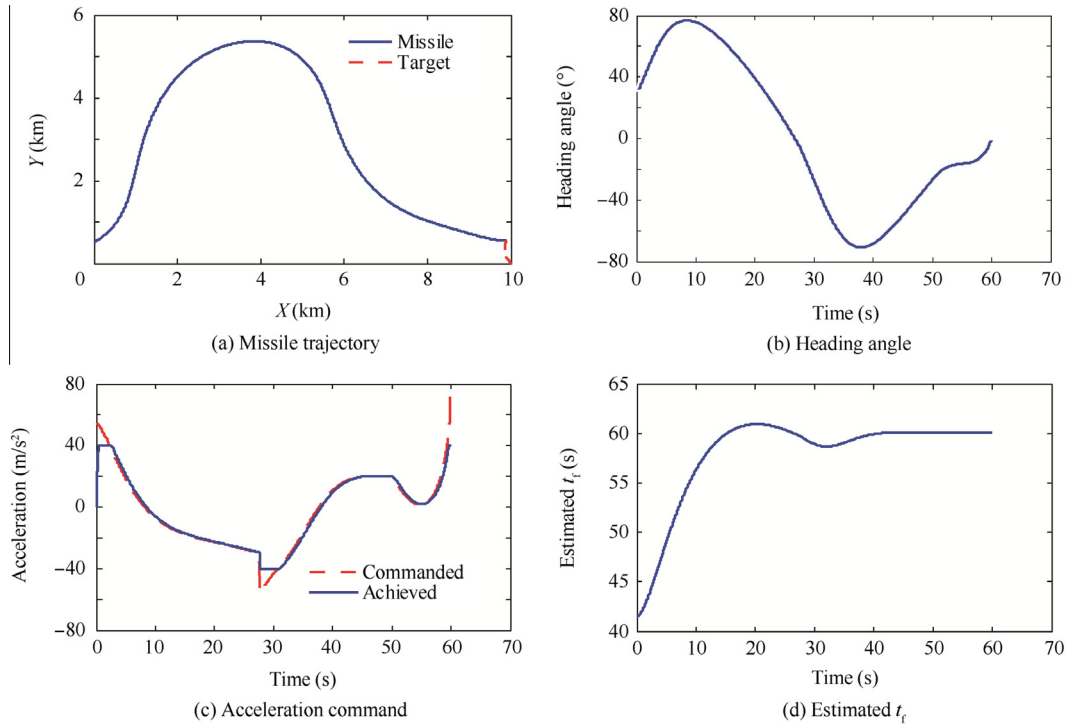


Fig. 9 Simulation results for a maneuvering target with autopilot lag and acceleration limit.

ensuring a successful interception with designated impact time and angle even in the presence of large initial heading errors, system lag and acceleration limit.

Acknowledgements

This study was supported by the National Natural Science Foundation of China (Nos. 11402020 and 51407011).

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