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USp(32) string as spontaneously supersymmetry broken theory

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Abstract

It was suggested by Sugimoto that there is a new supersymmetry breaking mechanism by an orientifold plane which is oppositely charged as the usual one. Here we prove the trace formula for this system to show that the supersymmetry is broken not explicitly but spontaneously. We also discuss the possibility of interpreting the orientifold plane as an intrinsic object of the superstring theory.

1. Introduction and summary

Since our world has no explicit supersymmetry, if we try to identify the superstring theory as the unique theory of our world, the supersymmetry in the superstring theory should be broken spontaneously. The supersymmetry breaking mechanism using both D-branes and anti-D-branes or non-BPS D-branes has been fully investigated [1–3]. However, we still have little knowledge for a system whose supersymmetry is broken by the orientifold plane.

Recently, it was suggested by Sugimoto [4] that there is a new supersymmetry breaking mechanism. In his seminal paper he showed that it is possible to consider an orientifold plane which is oppositely charged as the usual one. In this case we should add 32 anti-D9-branes, instead of 32 D9-branes, for cancellation of the Ramond–Ramond (RR) D9-brane charge. The resulting gauge group is USp(32) instead of SO(32) and the supersymmetry in this system is completely broken. The anomaly cancellation mechanism of this system is studied in [4,5].

In the present Letter we shall prove explicitly the trace formula [3,6,7]

$$\mathrm{Tr}_{\mathrm{NS-R}}\, m^2 = 0, \tag{1}$$

for this system. Here the masses squared m^2 are summed over all the physical Hilbert space with those of particles in the NS-sector (spacetime bosons) contributing as they are and those of the R-sector particles (spacetime fermions) with an extra negative sign. In supersymmetric field theoretical models the degree of freedom of bosons and that of fermions have to be balanced at every mass level. If the supersymmetry is broken spontaneously, the degrees of freedom are no longer balanced at every mass level. Instead, the degrees of freedom are balanced as a whole in the sense of the trace formula. Hence, our result that the trace formula holds for this USp(32) string theory suggests that the supersymmetry is broken spontaneously. The trace formula for the system with both D-branes and anti-D-branes is first proved in [3]. Here we shall fol-

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low all the technics in it. See also [8] for general structures of the trace formula. [1]

This result is consistent with [9]. In the paper the low energy effective supergravity theory for this system is constructed as the type I theory [10]. Although the closed string sector has explicit supersymmetry, the supersymmetry of the open string sector is broken spontaneously and realized non-linearly.

The motivation of the present Letter is as follows. Since we all believe that all the five perturbative superstring theories are realized as particular configurations of the unique M-theory, conceptually the orientifold plane in the type I theory should also be realized as an intrinsic object in the off-shell formalism of the superstring theory. Although there are of course no off-shell formalisms to construct the orientifold configuration explicitly, we would like to give some evidences for this suggestion by proving the trace formula for a non-supersymmetric system with the orientifold. We shall return to this question in the final section.

The contents of the present paper are as follows. In the next section we shall first review the spectrum of the USp(32) string theory and then proceed to prove the trace formula for the system. The final section is devoted to conclusions and discussions.

2. Trace formula in the USp(32) string theory

In this section, we shall examine the trace formula for the USp(32) string theory. For this purpose, we shall first briefly review the open string spectrum of the USp(32) theory [4]. Since the USp(32) theory is defined by reversing the orientifold charge of the type I theory, let us begin by recalling the spectrum of the type I theory. The type I theory is obtained by projecting out worldsheet orientation. The RR charge is cancelled only when we add 32 D9-branes. The open string spectrum of the type I theory is summarized in the partition function

$$Z = \mathrm{Tr}_{\mathrm{NS\text{–}R}}\, q^{-H}(1+\Omega)\big(1+(-)^F\big), \tag{2}$$

with $H = \alpha' m^2$ defined as

$$H_{\mathrm{NS}} = \sum_{m=1}^{\infty} \alpha_{-m}\cdot\alpha_m + \sum_{r=1/2}^{\infty} r b_{-r}\cdot b_r - \frac{1}{2}, \tag{3}$$

$$H_{\mathrm{R}} = \sum_{m=1}^{\infty} \alpha_{-m}\cdot\alpha_m + \sum_{m=1}^{\infty} m d_m\cdot d_m, \tag{4}$$

for the NS-sector and the R-sector, respectively. Hereafter we shall split the partition function into the bosonic part Z_{NS} and the fermionic part Z_{R}: $Z = Z_{\mathrm{NS}} - Z_{\mathrm{R}}$. Each part is given as follows [11]

$$\begin{aligned} Z_{\mathrm{NS}} = (32)^2\cdot\frac{\vartheta_0^0(it)^8}{\eta(it)^8} - (32)^2\cdot\frac{\vartheta_1^0(it)^8}{\eta(it)^8} \\ - 32\cdot\frac{\vartheta_1^0(2it)^8\vartheta_0^1(2it)^8}{\eta(2it)^8\vartheta_0^0(2it)^8}, \end{aligned} \tag{5}$$

$$Z_{\mathrm{R}} = (32)^2\cdot\frac{\vartheta_0^1(it)^8}{\eta(it)^8} - 32\cdot\frac{\vartheta_1^0(2it)^8\vartheta_0^1(2it)^8}{\eta(2it)^8\vartheta_0^0(2it)^8}, \tag{6}$$

where the eta function $\eta(it)$ and the theta functions $\vartheta_\beta^\alpha(it)$ are defined as

$$\eta(it) = q^{1/24}\prod_{m=1}^{\infty}\big(1-q^m\big), \tag{7}$$

$$\vartheta_0^0(it) = q^{-1/48}\prod_{m=1}^{\infty}\big(1+q^{m-1/2}\big), \tag{8}$$

$$\vartheta_1^0(it) = q^{-1/48}\prod_{m=1}^{\infty}\big(1-q^{m-1/2}\big), \tag{9}$$

$$\vartheta_0^1(it) = \sqrt{2}\,q^{1/24}\prod_{m=1}^{\infty}\big(1+q^m\big), \tag{10}$$

with $q = e^{-2\pi t}$. Here the first term and the second term in Z_{NS} comes from $\mathrm{Tr}\, q^{-H}$ and $\mathrm{Tr}\, q^{-H}(-)^F$, respectively. The first term in Z_{R} comes from $\mathrm{Tr}\, q^{-H}$. The final term in both Z_{NS} and Z_{R} are due to $\mathrm{Tr}\, q^{-H}\Omega(1+(-)^F)$. Note also that the final term in each part contributes as the NSNS source in Z_{NS} but as the RR source in Z_{R}. Using the Jacobi's abstruse formula

$$\vartheta_0^0(it)^8 - \vartheta_1^0(it)^8 - \vartheta_0^1(it)^8 = 0, \tag{11}$$

we find Z vanishes totally, as expected for a supersymmetric theory.

[1] The author is grateful to his colleagues for calling his attention to the work. The work contains the trace formula for a wide class of consistent string theories. Here, however, we shall discuss more specific properties of the orientifold plane.

The USp(32) theory is defined as reversing the charge of the orientifold plane. From a discussion of the action of Ω on open string states, we find that the gauge group is restricted only to the SO type and the USp type and they are related by reversing the orientifold charge [11]. Note that this discussion holds generally and we have to reverse both the NSNS charge and the RR charge of the orientifold charge. Correspondingly, we have to add 32 anti-D9-branes for cancellation of the RR charge.

Thus, if we would like to read off the spectrum of the USp(32) string theory from the type I theory, we have only to reverse the sign of the last term in Z_{NS} (5), because this term corresponds to exchange of NSNS mode between the orientifold plane and anti-D9-branes and only the orientifold charge is reversed.

$$Z_{\mathrm{NS}} = (32)^2 \cdot \frac{\vartheta_0^0(it)^8}{\eta(it)^8} - (32)^2 \cdot \frac{\vartheta_1^0(it)^8}{\eta(it)^8} + 32 \cdot \frac{\vartheta_1^0(2it)^8 \vartheta_0^1(2it)^8}{\eta(2it)^8 \vartheta_0^0(2it)^8}, \tag{12}$$

$$Z_{\mathrm{R}} = (32)^2 \cdot \frac{\vartheta_0^1(it)^8}{\eta(it)^8} - 32 \cdot \frac{\vartheta_1^0(2it)^8 \vartheta_0^1(2it)^8}{\eta(2it)^8 \vartheta_0^0(2it)^8}. \tag{13}$$

Now we have acquired enough information to analyze the trace formula. All we have to do is to calculate $(\partial/\partial q) Z|_{q=1}$. Using the Jacobi's abstruse formula (11), we find

$$Z = 2 \cdot 32 \cdot \frac{\vartheta_1^0(2it)^8 \vartheta_0^1(2it)^8}{\eta(2it)^8 \vartheta_0^0(2it)^8}. \tag{14}$$

Since the modular forms are made of infinite polynomials, to evaluate the trace formula, we have to use the modular transformation of the eta function and the theta functions:

$$\eta(it) = \frac{1}{\sqrt{t}} \eta(i/t), \tag{15}$$

$$\vartheta_\beta^\alpha(it) = \vartheta_\alpha^\beta(i/t), \tag{16}$$

to transform the partition function Z into the sum of finite polynomials and infinite non-perturbative effects like $e^{-2\pi/t}$. Using the modular transformation we find

$$Z = 2 \cdot 32 \cdot (\sqrt{2t})^8 \frac{\vartheta_1^0(i/2t)^8 \vartheta_0^1(i/2t)^8}{\eta(i/2t)^8 \vartheta_0^0(i/2t)^8} = 1024\, t^4 + O\big(e^{-2\pi/2t}\big). \tag{17}$$

Especially, the coefficient of t in the partition function Z is zero. This shows that the trace formula holds and implies that in the USp(32) theory the supersymmetry is broken spontaneously. Note that the trace formula also holds for any power except 4. This is the same situation as the D-brane–anti-D-brane system [3].

3. Conclusion and discussion

In this work we showed that the trace formula also holds in the USp(32) string theory. This suggests that the supersymmetry is broken spontaneously. Since the supersymmetry of this system is broken by the orientifold plane and anti-D9-branes, we expect that the orientifold plane and anti-D9-branes are intrinsic objects of the theory.

Although this interpretation seems plausible, we cannot regard our results as a rigorous evidence for it. Here we shall discuss the possibility of interpreting the orientifold plane as an intrinsic object by counting the goldstinos which are expected when the supersymmetry is broken spontaneously.[2]

Let us first consider the type I theory. The type I theory is defined by adding to the type II theory an orientifold plane and D9-branes to break half the supersymmetries. If the orientifold plane and D9-branes are both intrinsic objects, one should regard the supersymmetry as broken spontaneously and expect the goldstino to appear. However, we cannot find a massless fermion singlet to be regarded as the goldstino.

In the case of the USp(32) string theory the supersymmetries are completely broken by both the orientifold plane and anti-D9-branes. Since we all know that anti-D9-branes break only half the supersymmetries, the rest of the supersymmetries should be broken by the orientifold plane. There is a massless fermion singlet to be identified as the goldstino. This, however, is not enough, since originally we are considering the type II theory.

Note also that the absence of the goldstinos does not mean directly that the orientifold plane is not an intrinsic object. In fact when the broken symmetry is

[2] We are grateful to S. Sugimoto for a valuable discussion on this point.

a local symmetry, the goldstino may be eaten up by some fields. For example, when the superstring theory is compactified on the Calabi–Yau manifold to construct a realistic model, we does not expect to find a goldstino.

Therefore, one might possibly stick to considering that the orientifold plane breaks half the supersymmetries explicitly and regard our proof of the trace formula in this paper only as a sign of the fact that D-branes and anti-D-branes are intrinsic objects. To avoid this possibility it remains to clarify the mechanism how the goldstino for the orientifold plane is eaten up by other fields. This is an interesting future problem.

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