Physics-based modeling for partial slip behavior of spherical contacts

M. Eriten a, A.A. Polycarpou a,∗, L.A. Bergman b

a Department of Mechanical Science and Engineering, University of Illinois, Urbana-Champaign, USA
b Department of Aerospace Engineering, University of Illinois, Urbana-Champaign, USA

ARTICLE INFO

Article history:
Received 10 December 2009
Received in revised form 6 April 2010
Available online 24 May 2010

Keywords:
Spherical contact
Partial slip
Fretting
Tangential stiffness

ABSTRACT

A physics-based modeling approach for partial slip behavior of a spherical contact is proposed. In this approach, elastic and elastic–plastic normal preload and preload-dependent friction coefficient models are integrated into the Cattaneo–Mindlin partial slip solution. Partial slip responses to cyclic tangential loading (fretting loops) obtained by this approach are favorably compared with experiments and finite element results from the literature. In addition to load-deformation curves, tangential stiffness of the contact and energy dissipation per fretting cycle predictions of the models are also provided. Finally, the critical assumptions of elastically similar bodies, smooth contact surface and negligible adhesion, and limitations of this physics-based modeling approach are discussed.

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1. Introduction

Contact of spheres is a large subset of axi-symmetric contacts seen in practice. Particle handling, sealing, electrical conductivity, MEMS and magnetic storage systems are only a few examples where spherical contact governs the interaction between different parts. Besides, the contact of nominally flat rough surfaces can be modeled as numerous spherical contacts assuming spherical asperity tips. Therefore, understanding the normal and tangential loading of spherical contacts is critical in modeling, design and control of many engineering systems, such as interference fits of rotating or vibrating assemblies, rolling bearings, wire ropes, turbine blades, and mechanical joints (bolted, lap, dovetail, etc.).

As fundamental as the combined loading of spherical contact is, it is complicated to analyze due to three major reasons. Firstly, although many engineering systems containing interfaces are brought into contact by a prescribed clamping load, both tangential and normal loading vary throughout system operation. Nevertheless, assuming a constant normal clamping load and proceeding with varying tangential loading simplifies the problem considerably while providing insight about the problem. Normal loading under different contact conditions has been analyzed extensively by many researchers [elastic frictionless contact (Hertz, 1881); elastic fully-adhered contact (Goodman, 1962; Spence, 1968); elastic–plastic frictionless contact (Chang et al., 1987; Kogut and Etsion, 2002); elastic–plastic fully-adhered contact (Brizmer et al., 2006)].

Secondly, even though the normal loading problem can be accurately solved, a physics-based mechanism relating known normal load to unknown tangential load in combined loading problems is difficult to solve. Local Coulomb friction law assumes an either tabulated or experimentally determined constant friction coefficient value in order to couple the normal and tangential tractions on a sliding contact (Naboulsi and Nicholas, 2003). Similarly, power law coupling needs experimentally determined non-physical coefficients (Oden and Martins, 1985). By treating the plastic yielding as the trigger for sliding, friction force and friction coefficient can be related to physical quantities; however, the results proposed by several researchers vary significantly depending on the contact modeling assumptions [elastic contact (Burwell and Rabinowicz, 1953; Chang et al., 1988); elastic–plastic contact with partial slip, (Kogut and Etsion, 2003a); fully-adhered elastic–plastic contact (Brizmer et al., 2007)].

Thirdly, the tangential loading of spherical contacts is not as well understood as normal loading. This is because the contact condition (fully-adhered, lubricated, sliding, etc.) used in modeling affects the results. The first modeling attempts by Cattaneo (1938), and independently by Mindlin (1949), assumed fully-adhered purely elastic contact and showed that tangential loading would produce unreasonably high tangential tractions toward the edge of the spherical contact. For relaxation of this growing stress, they suggested “slip” and assumed that tangential tractions inside the slip region would obey the local Coulomb friction law. This contact condition, for which a slipping annular region encapsulates a sticking core in the contact patch, is named “partial slip”. Mindlin et al. (1952) expanded the same approach to solve the oscillating tangential loading response of spherical contact under partial slip. The partial slip contact under oscillating tangential loading is also...
referred to as fretting contact. Mindlin’s elastic partial slip model has been shown to agree with fretting experiments of steel ball and flat contact under moderate normal loads (Johnson, 1955). However, the energy dissipation measured during each unload/reloading (fretting) cycle deviates considerably from the theoretical calculation. In addition, Johnson (1955) points at four basic assumptions of Mindlin’s elastic model to be approached critically: (1) Contacting bodies are perfectly elastic, (2) Contacting surfaces are smooth and thus the contact is continuous, (3) Infinite shear stress due to the fully stick contact condition is relieved by relative slipping over an annulus (partial slip), and (4) Shear stress over the slip annulus obeys the local Coulomb friction law ($q = \mu p$). Ödfalk and Vingsbo (1992) challenged the perfect elasticity assumption and suggested that plastic displacements should be added to the elastic displacements in order to account for plasticity. Specifically, they suggested a parallelogram-shaped load–displacement curve for plastic regime loading. However, their formulation needs two experimentally-determined parameters, namely “fretting yield point” and “fretting hardening coefficient”, which in turn compromises the physical basis of predictive fretting modeling.

The purpose of this paper is to challenge the first and fourth assumptions listed above by combining elastic and elastic–plastic normal loading and preload-dependent friction coefficient models with the Cattaneo–Mindlin elastic solution to obtain physics-based partial slip responses for spherical contact. The models developed by this approach are compared with data existing in the literature. Furthermore, the limitations and assumptions – elastic similarity, smoothness, adhesion and plasticity – of this modeling approach are discussed.

2. Spherical contact under combined normal and tangential loading

We review elastic and elastic–plastic spherical contact (incomplete) models in this section. In doing so, we mention only normal and tangential loading/deformation responses. For more details about flat-on-flat fretting contact (complete), contact stresses, surface tractions, and their evolution during loading; the reader is referred elsewhere, e.g. Hills and Nowell (1994), Hills et al. (1992), Halling (1978), Johnson (1987), Comninou and Barber (1983) and Sackfield et al. (2002).

2.1. Normal loading

When two elastic spheres are pressed to each other (see Fig. 1), the contact is enclosed within a circle whose radius is dependent on the applied load, $P$ (incomplete contact). With the assumption that the contact radius is considerably smaller than the radii of the spheres, Hertz (1881) offered the first analytical solution to the load-penetration problem of elastic spheres under frictionless (perfect slip) contact. Subsequently researchers relaxed the assumptions needed by the Hertzian solution. Sneddon (1965) relaxed the small contact radius assumption by solving the problem of a rigid punch pressed into an elastic half-space under frictionless contact. Spence (1968, 1975) relaxed the frictionless contact assumption and solved first the fully-adhered rigid punch problem and then the same problem with finite Coulomb friction. Noting that plastic deformations either on or below the contact surface are inevitable, especially for heavier loading conditions and stress intensification due to geometry, Chang et al. (1987)—CEB Model—proposed an elastic–plastic solution to the frictionless sphere-on-flat contact by assuming volume conservation during plastic deformations. To solve this problem with minimal assumptions, Kogut and Etsion (2002)—KE Model—utilized finite element analysis (FEA), and Brizmer et al. (2006)—BKE Model—investigated the effects of contact conditions (fully-adhered or frictionless) and material properties on the normal contact, also by FEA.

We assume small contact radius throughout this study, so the Sneddon and Spence solutions will only serve as extreme cases. Hence, we summarize the load-penetration ($P - \omega$) relationships offered by Hertz, CEB, KE and BKE models in Eqs. (1)–(4), respectively as

\[
\text{Hertz: } P^* = \omega^{3/2},
\]

\[
\text{CEB: } P^* = \begin{cases} 
\omega^{3/2} & \omega^* \leq 1 \\
3(\omega^* - 0.5) & \omega^* > 1 
\end{cases},
\]

\[
\text{KE: } P^* = \begin{cases} 
\omega^{2/3} & \omega^* \leq 1 \\
1.03\omega^{1.425} & 1 < \omega^* \leq 6 \\
1.4\omega^{1.263} & 6 < \omega^* \leq 110 
\end{cases},
\]

\[
\text{BKE: } P^* = \frac{k}{\delta_c^{2/3}} \begin{cases} 
\omega^{3/2} & \omega^* \leq \delta_c \\
\omega^{3/2}(1 - \exp\left(\frac{\delta_c}{\omega^*}\right)) & \omega^* > \delta_c 
\end{cases},
\]

**Fig. 1.** Spheres in contact under combined normal and tangential loading (a) and schematic of a typical loading history (b). The contact occurs in a circle of radius $a$ and the stick region constitutes the core region of that contact with radius $(c)$. 
where \( P^* = P/P_c \) and \( \omega^* = \omega/\omega_c \) are the normalized load and penetration. Chang et al. (1987) defines the critical interference, \( \omega_c \), contact radius, \( a_c \), and normal load, \( P_c \). In the inception of plastic deformation as functions of Poisson's ratio, \( \nu \); hardness factor, \( K = 0.454 + 0.41\nu \); hardness of the softer material in contact, \( H \); combined Young's modulus, \( E^* = ((1 - \nu^2)/(E_1 + (1 - \nu^2)/E_2))^{-1} \); and combined radius of curvature, \( R = (1/R_1 + 1/R_2)^{-1} \) as

\[
\omega_c = \left(\frac{\pi KH}{2E^*}\right)^2 R, \\
a_c = \sqrt{\omega_c R} = \frac{\pi KH}{2E^*}, \quad (5)
\]

Note that \( L_c = 8.88\nu - 10.13(\nu^2 + 0.089), \tilde{H} = 6.82\nu - 7.83\nu^2 + 0.0586, \) and \( \tilde{\nu} = 0.174 + 0.08\nu \) are given in Brizmer et al. (2006).

Fig. 2 depicts the normalized loads, \( P^* \), given by Eqs. (1)–(4) as a function of normalized penetration, \( \omega^* \), for \( \nu = 0.3 \). As seen in Fig. 2, both CEB (Eq. (2)) and KE (Eq. (3)) models exhibit discontinuities at the critical interference penetration values, which is not physical. For instance, the CEB model calculates normalized load values of 1 and 1.5 and normalized contact stiffness values of 1.5 and 3 before and after yielding (\( \omega^* = 1 \)). This non-physical transition is due to the fact that the CEB model assumes that the volume of the sphere is conserved while plastically deforming, although elastic contact is not volume-conserving. Everseev et al. (1991) experimentally studied the elastic–plastic deformation of a spherical contact and recommended a general model to fix the discontinuity observed in the CEB model. Zhao et al. (2000), on the other hand, utilized mathematical smoothening to express a continuous transition in the CEB formulation. Since we use CEB’s normal load model in conjunction with CEB’s static friction model with vanishing friction coefficient at \( \omega = 1 \), the partial slip model developed is not affected from this discontinuity. Therefore, we used the original CEB formulation. The discontinuities in the KE model appears in two transition penetration values, \( \omega^* = 1.6 \), both of which are associated with curve-fitting of the FEA results. These artificial discontinuities are bounded within 3% for normalized load and 10% for normalized contact stiffness. Therefore, the use of the original KE formulation does not significantly affect the partial slip response either.

Note that, for a wide range of practical normal penetration values, i.e. \( \omega^* < 5 \), the elastic–plastic normal loading results given by the frictionless KE and fully-adhered BKE models remain very close to each other and within 10% of the purely elastic response (Hertz). Furthermore, the fully-adhered BKE solution deviates from the frictionless Hertz solution only by 3% within the elastic loading regime. These observations lead us to an early remark: friction's effect on the load-penetration response of the spherical contact is negligible. This remark has been shown to hold by Spence (1968). Spence's analysis of a fully-adhered rigid punch pressed into an elastic half-space led to a 5% increase in load compared to the frictionless punch solution of Sneddon with a Poisson's ratio of 0.3. This increase is more than the BKE/Hertz yields, due to the fact that both BKE and Hertz (CEB and KE, as well) assume small penetrations and, thus, small contact radii. Larger penetrations result in stronger surface tractions, and, hence, the ratio of loads under different contact conditions increases. Nevertheless, it is reasonable to conclude that the contact condition under normal loading has negligible effect on the load-penetration response.

### 2.2. Tangential loading

Subsequent application of a tangential load, \( Q \), on a spherical contact normally preloaded by a constant load, \( P \), disturbs the axisymmetric nature of the problem. Additionally, tangential loading causes physical complications such as junction growth and change in normal tractions, and the nature of contact (frictionless, fully-adhered or finite friction) affects the tangential load–displacement response (even though we do not study them, the contact and bulk stresses also depend on the contact conditions). Starting from fully-adhered elastic contact and continuing with finite friction (friction coefficient, \( \mu \)), Cattaneo (1938) and Mindlin (1949) independently proposed the same solution to the elastic spherical contact under constant normal and increasing tangential loading. Furthermore, Mindlin et al. (1952) proposed a solution to cyclic tangential loading (unloading/reloading) of spherical contacts under partial slip. A maximum tangential load, \( Q_m \), which is not sufficient to cause gross sliding, is applied to the contacting bodies repeatedly (see Fig. 1) and superposition is used to obtain solutions for the tangential load–displacement, \( Q - \delta \), while unloading/reloading. Eqs. (6)–(9) summarize these findings.

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**Fig. 2.** Dimensionless normal load vs. interference for Hertz (elastic, frictionless); CEB and KE (elastic–plastic, perfect slip) and BKE (elastic–plastic, full stick) models.
crease, Ödfalk and Vingsbo (1992) relaxed the elastic assumption of additional tangential displacement without a considerable force in-

relation, \( Q \), with Mindlin’s elastic solution, \( Q \):

\[
\text{Plastic Slip (cyclic) } Q : \delta_p = \beta\left(\frac{Q + Q_m}{k_p} + \frac{Q_m - Q_y}{2k_p}\right).
\]

where \( G = (2 - \nu_1)/G_1 + (2 - \nu_2)/G_2 \) is the combined shear modulus and \( a \) is the contact radius. Observing that plastic flow causes additional tangential displacement without a considerable force in increase, Ödfalk and Vingsbo (1992) relaxed the elastic assumption of Mindlin’s tangential unloading/reloading solution. They suggested a simple superposition of a parallelogram-shaped load–displacement relation, \( Q \), with Mindlin’s elastic solution, \( Q \):

\[
\text{Plastic Slip (cyclic) } Q : \delta_p = \beta\left(\frac{Q + Q_m}{k_p} + \frac{Q_m - Q_y}{2k_p}\right).
\]

where \( \beta = \begin{cases} 1 & \text{when } Q_m > Q_y \\ 0 & \text{when } Q_m \leq Q_y \end{cases} \) and

\( \gamma = \begin{cases} 1 & \text{when } Q > Q_y \text{ and } \frac{dQ}{dQ} > 0 \\ 0 & \text{else} \end{cases} \).

There are two critical parameters to be determined in this model, namely “fretting yield point”, \( Q_y \), and “fretting hardening coefficient”, \( k_p \). The fretting yield point is described as the load needed for the inception of plastic yield due to increasing tangential load. This parameter depends on the contact geometry, material properties of the contacting bodies, and the normal preload. The fretting hardening coefficient depends on the mean strain rate and, hence, on the fretting frequency. Both parameters are obtained experimentally and could not be generalized. In addition, both Mindlin and Ödfalk and Vingsbo use a local Coulomb friction law, with a pre-determined, constant friction coefficient. Therefore, one could argue that neither the Mindlin nor the Ödfalk and Vingsbo model is entirely physics-based.

The dashed curve in Fig. 3 shows Mindlin’s partial slip response (hysteresis curve or fretting loop) under cyclic tangential loading. The dynamical behavior of the contact can be completely characterized by the fretting loop. For instance, the slope of the hysteresis curve at the beginning of the loading and unloading regimes suggests a measure of the tangential contact stiffness. Essentially, both machine support and contact stiffnesses contribute to this slope. Provided that a careful experimentalist isolates the contact stiffness, through use of a relatively rigid attachment to the machine support, the slope coincides with the tangential contact stiffness. Mathematically, this can be described as

\[
\frac{1}{K_T} = \frac{\partial \delta_p}{\partial Q} = \frac{\partial Q}{\partial Q} = \frac{\partial Q}{\partial Q} = \frac{1}{8G a}.
\]

Fig. 3 also shows a purely plastic fretting loop (dotted curve) representing the Ödfalk and Vingsbo partial slip model. The elastic–plastic fretting loop (solid curve) is obtained by summing the elastic and plastic displacements calculated for the same tangential load, and, hence, this model shows a larger range of tangential motion when compared to Mindlin’s purely elastic solution (dashed curve). However, the tangential contact stiffness remains the same as Mindlin’s prediction.

Another critical phenomenon which can be observed from fretting loops is damping. Even though Mindlin’s model assumes purely elastic contact, the reloading response does not follow the unloading response in Fig. 3 (hysteresis behavior). The energy needed to reload the contact is more than the energy released by unloading because of the frictional losses incurred by slippage toward the contact edges. The energy dissipated during one cycle of unloading/reloading is given by the area inside the fretting loop. This area can be calculated via the difference of the work done by reloading (Eq. (9)) and unloading (Eq. (8)) as

\[
\Delta W = \int_{Q}^{Q_m} \left(1 - \left(1 - \frac{Q_m}{\mu P}\right)^{2/3} \right)\left(1 - \frac{Q_m}{\mu P}\right) \, dQ - \int_{Q}^{Q_m} \left(1 - \left(1 - \frac{Q_m}{\mu P}\right)^{2/3} \right)\left(1 - \frac{Q_m}{\mu P}\right) \, dQ.
\]

The elastic–plastic model shown in Fig. 3 indicates a broader fretting loop and thus more energy dissipation per cycle compared to the purely elastic model. This is due to the additional energy dissipation incurred by plasticity.

2.3. Friction coefficient models

As mentioned in Section 2.2, both the elastic Mindlin and elastic–plastic Ödfalk and Vingsbo models assume a local Coulomb friction law. Accordingly, the partial slip region enlarges by increasing tangential load, and, once the tangential load reaches a pre-determined fraction of the normal load, gross sliding occurs. This pre-determined fraction is referred to as the static friction coefficient and is usually available from engineering handbooks. Rabinowicz (1966) lists time and speed of sliding, loading conditions, and degree of vacuum as factors affecting the friction coefficient values obtained through experiments. Experimentally-measured friction coefficient values tabulated in different sources vary drastically, and their practical significance thus diminishes. To exemplify, let’s study the friction coefficients tabulated for unlubricated (dry) steel on steel contact. Rabinowicz (1966) tabulates values for metal-on-metal contact as a function of the
metallurgical compatibility of the metals. For identical metals, the friction coefficient is 0.8, whereas for incompatible metals, 0.35. Similarly, Tabor (1973) documents 0.8 for mild and 0.4 for tool steel. On the other hand, Gieck and Gieck (1997) provides a range of values for static (0.15–0.3) and sliding friction coefficients (0.1–0.3). Concise Metals Data Handbook by Davis (1997) tabulates 0.31 for the static friction coefficient of stainless steel 1032 and twice that value (0.62) for mild steel. Clearly, there is a large uncertainty and variation in friction coefficient values pre-determined from experiments, and that uncertainty influences the partial slip model responses significantly.

An alternative approach to pre-determined friction coefficient values starts by attributing yielding mechanisms to the origins of sliding inception. Since both sliding inception and plastic yielding are physical phenomena, the static friction coefficient, defined as the ratio of the maximum tangential load that a junction can carry (static friction force) to the normal preload, can be found directly from a physics-based methodology. A simple example of this approach is achieved by assigning average shear, $\tau_{av}$, and normal compressive strengths, $H$, to the junction. Since the maximum loads in the shear and normal directions are related to the shear and normal strengths by the real contact area, $A$, the ratio of the loads, and thus friction coefficient, is the ratio of shear and normal strengths of the junction. It is customary to assume that the shear and normal strengths of the junction cannot exceed those of the weaker bulk material in contact (Burwell and Rabinowicz, 1953). Thus, this simple approach yields a constant friction coefficient (CFC) equal to the ratio of the shear strength, $s$, and hardness, $H$, of the softer material in contact (Bowden and Tabor, 2001).

$$\text{CFC} : \mu = \frac{Q_{\text{max}}}{P_{\text{max}}} = \frac{\tau_{av}}{H} \approx \frac{s}{H} = \mu_0$$  \hspace{1cm} (13)

This approach assumes that each shear and normal loading independently causes plastic yielding of the junction. In reality, some invariant combination of shear and normal stresses (von Mises, Tresca, etc.) is related to the plastic yielding. Therefore, the friction coefficient estimate given in Eq. (13) does not hold for most engineering applications.

To alleviate the above-mentioned limitation of this physics-based approach, recent works used Hamilton’s sub-surface stress calculations and numerical methods (FEA) to calculate the maximum static friction force (and hence friction coefficient) at sliding inception. Chang et al. (1988) treated sliding inception as the first occurrence of plastic yield either on or below the contact area. In this model, the von Mises yielding criterion is used as the equivalent stress needed for plastic yield. The effect of normal tractions is subtracted from the equivalent stress, and the remaining stress is assumed to be caused by tangential loading only, without any interaction between normal and tangential loading. Therefore, this model assumes that tangential loading following a constant preload has no effect on the normal contact stresses, the shape, and dimensions of the contact area. This assumption infers that tangential loading does not cause junction growth, or a change in the plasticity inception location (this point is shown to be on the axis of symmetry, $x, y = 0$, for purely elastic normal loading and slightly deviating from that point toward the direction of tangential loading for normally loaded sliding contacts, by Hamilton (1983)).

Since, the model treats the first occurrence of plasticity, whether in the contacting bodies or on the contacting surface, as sliding inception, and additional tangential loading has no effect on the preceding normal loading, the Hertz solution is safely used for normal loading response. The stress field under the contact is taken from Hamilton’s formulation for sliding contact, and the maximum tangential load needed for the inception of plastic yield and, thus, sliding is calculated. In contrast to Chang et al. (1988), Kogut and Etsion (2003a) combined an analytical solution with FEA results and calculated sliding inception of a spherical contact under both purely elastic and elastic–plastic loading. This model assumes that the normal preload is not sufficient to produce any plastic yielding, and, thus, the additional tangential force causes the first yield to occur at the contact interface because the shear stress due to the fact that tangential loading is higher on the contact surface. Employing a similar methodology as Chang et al. (1988), and stress distributions obtained from FEA, the authors proposed two relationships for the friction coefficient, one for elastic and the other for elastic–plastic contact conditions.

Brizmer et al. (2007) studied via FEA elastic–plastic fully-adhered contact of a deformable sphere with a rigid flat under combined normal and tangential loading. The authors relaxed the assumptions of Kogut and Etsion (2003a) about constant interference, contact pressure and area due to normal loading by imposing fully-adhered contact conditions (thus allowing junction growth and further interference due to tangential loading). According to this model, sliding is assumed to initiate when the contact cannot carry any additional tangential force; i.e. when the tangential stiffness approaches zero. Using this criterion, the FEA results were curve-fitted to a nonlinear function to obtain the friction coefficient. Eqs. (14)–(16) summarize the results of the physics-based friction coefficient modeling efforts as

$$\text{CEB} : \mu = \min \left( \frac{0.2045}{K(C_1)} \left( \frac{1}{\mu^*} - 1 \right)^{1/2} - C_4 + \frac{C_4^2 - 4C_4C_5}{2C_5} \right), \hspace{1cm} (14)$$

$$\text{KE} : \mu = \begin{cases} \frac{0.536\mu^* - 0.0186\mu^{0.5} \mu^* \leq 1}{-0.0075\mu^{3} + 0.085\mu^{0.5}} \\ -0.389\mu^{0.5} + 0.822 \hspace{1cm} 1 \leq \mu^* \leq 6.2 \end{cases} \hspace{1cm} (15)$$

$$\text{BKE} : \mu = 0.26\text{coth}(0.27(\delta x^{0.46})) \hspace{1cm} (16)$$

where $c_1 = 1 + \frac{1}{2}\tan^{-1}(1/\zeta) - \frac{\zeta}{2(1+\zeta)}$, $c_2 = (1 + \nu)(\zeta\tan^{-1}(1/\zeta) - 1) + \frac{3}{2(1+\zeta)}$, $c_3 = \frac{8\pi^2}{(2 - \nu + \nu \zeta^2)}$, $c_4 = \frac{2\pi^2}{(1 - 2\nu)(1 - \nu/2)}$, and $c_5 = \frac{1}{2}(1 - 2\nu)^2 - \frac{1}{5\pi^2}$. Note that $\zeta$ is the normal direction location of the plastic yield normalized by the contact radius and is approximately found to be 0.48 with Poisson’s ratio of 0.3.

Fig. 4 shows the friction coefficient of AISI 304 N stainless steel ($s = 186$ MPa and $H = 655$ MPa from MatWeb (2010) material property database) found as $\mu = 0.284$ by Eq. (13) and from penetration-dependent models defined in Eqs. (14)–(16). Unlike the constant friction coefficient, since the penetration-dependent models treat the sliding inception as a failure mechanism of the contact, i.e. plastic yielding, the friction coefficient values reduce as the normal penetration increases. CEB and KE models use the stress field presented by Hamilton (1983) in the elastic loading regime to compute the additional tangential load needed for plastic yield. The difference between the models in the elastic loading regime, i.e. $\mu^* < 1$, stems from the modeling assumptions. The CEB model assumes that plastic yielding, even at a single point beneath or on the contacting surface, would cause the sliding inception; however, the KE model necessitates that all the points in the contacting region should reach plastic yield. Therefore, the KE model predicts higher static friction force and coefficient values than the CEB model. In practice, the CEB model’s assumption that no additional tangential load can be carried after the plastic yielding of a single point beneath or on the surface leads to a significant underestimation of the static friction force and coefficient because the plastically-yielded point is still surrounded by a large elastic region, which is capable of carrying additional tangential load. Accordingly, the CEB model unrealistically predicts zero friction coefficient due to normal...
preload, whereas the KE model predicts nonzero values up to \( \omega^* = 6.2 \). At \( \omega^* = 6.2 \), the central elastic core is surrounded by a plastic region, and, thus, any additional tangential load causes that elastic island to float rather than showing resistance. On the other hand, the BKE model predicts higher friction coefficient values than the KE model, and the friction coefficient never vanishes (instead approaching an asymptotic value of 0.26). This occurs because the inception of sliding inception under the full-adhered contact condition is caused by a vanishing tangential stiffness. It is important to note that the BKE model relaxes many assumptions of the CEB and KE models by assuming a fully-adhered contact condition. However, in practice, the fully-adhered condition is difficult to establish and maintain (slip occurs due to surface contamination, wear debris, etc.), and, hence, the friction coefficient values predicted by this model should generally serve as an upper bound for the actual values.

In developing the plasticity-based friction coefficient models, it is important to discuss the assumptions about the scale of the contact and material properties. While introducing the plasticity-driven formulation of the CFC model, Burwell and Rabinowicz (1953) assumed macroscopic contact and provided an estimate for the friction coefficient of a dry clean contact. The CEB, KE and BKE models, in contrast, are developed by analyzing a smooth, dry and clean spherical contact at the asperity scale. To carry the results of these models to the macroscopic contact is a difficult task because real contacting surfaces contain asperities, contamination and third-bodies (wear debris). One needs to verify each assumption about the contact conditions carefully, and, contact conditions are particularly complicated to control in macroscopic applications. That is why micro and nano-scale experiments are chosen for comparison in Section 4. Even so, the assumptions of smoothness and the effect of adhesion in each experiment are checked before proceeding with the conclusions. In addition, plastic response of the material will affect the comparisons. The CEB model uses the von Mises yielding criterion as the limit for static friction. Therefore, material response after yielding is not needed in its formulation. However, the KE and BKE models develop friction coefficient formulations involving elastic–plastic responses of the softer material beyond the von Mises yielding surface. Both models inherently assume that the harder material behaves elastically during contact. However, this assumption fails to hold when two identical materials are in contact. The KE model treats the softer material as an elastic-perfectly plastic material with identical response in tension and compression, whereas the BKE model uses elastic linear isotropic hardening with a tangent modulus of 2% of the Young modulus. Note that the materials referred to in Section 4, bearing steel and silicon, show different responses in plastic deformation. Many steel grades show work-hardening behavior, whereas silicon exhibits almost no plastic deformation at room temperature. Therefore, one needs to be careful in using the KE and BKE models for these materials under plastic deformation. The experiments in Section 4 involve only elastic deformation, and, hence, the validity of the comparison with the models presented in this section.

3. Proposed partial slip models

Cattaneo–Mindlin’s approach and formulation of tangential displacement-load relationships (Eq. (7) for increasing and Eq. (8) for oscillating tangential loads) and energy dissipation per cycle (Eq. (12)) under partially slipping contact are combined with elastic–plastic normal loading and preload-dependent friction coefficient models (presented in Section 2) to obtain physics-based models for partial slip. The portion of the models involving plasticity violates the elasticity assumption of Cattaneo–Mindlin. Therefore, only the elastic portion can safely be tested against experimental results. Nevertheless, the Cattaneo–Mindlin formulation qualitatively represents the weakening of the junction due to plasticity (softening spring behavior) and, hence, can be used for qualitative comparison.

It is worth mentioning the fact that the proposed partial slip models are no different than the Cattaneo–Mindlin function except for the physics-based formulation of the friction coefficient (Section 2). In fact, the presented friction models predict a constant friction coefficient value for each set of material properties, sphere radius and loading condition. Moreover, the normal preload and penetration responses of each model are shown to agree well with the Hertzian solution for small penetration values (see Fig. 2). Therefore, the contact pressure is safely assumed to follow the Hertzian pressure. As a result, the proposed models conform to the Cattaneo–Mindlin formulation of the superposed surface tractions and the stick–slip regions in the elastic loading regime. Under plastic loading, the Hertzian pressure assumption fails to hold, and since the Cattaneo–Mindlin formulation is valid only for elastic deformations, the proposed approach has no physical meaning.
The proposed procedure to obtain the partial slip response is summarized in the flowchart of Fig. 5. For instance, via the proposed methodology shown in Fig. 5, we obtain Eqs. (17)–(20) for the tangential load–displacement response to increasing tangential loading (see the Appendix A for expressions of cyclic loading and energy dissipation per cycle):

\[
\text{CFC: } \delta^* = 0.284\omega^* \left(1 - \left(1 - \frac{3.52Q'}{\omega^*^{3/2}}\right)^{2/3}\right),
\]

\[
\text{CEB: } \omega^* \leq 0.95 \Rightarrow \delta^* = \omega^* \left(-0.112 + 0.047\sqrt{-4.5 + \frac{72}{\omega^*}}\right) \times \left(1 - \left(1 + \frac{Q'}{0.112\omega^*^{3/2} - 0.047\omega^*\sqrt{-4.5\omega^* + 72}}\right)^{2/3}\right),
\]

\[
\omega^* > 0.95 \Rightarrow \delta^* = 1.245\omega^* \sqrt{-1 + \frac{1}{\omega^*} \left(1 - \frac{0.803Q'}{\omega^*\sqrt{1 - \omega^*}}\right)^{2/3}},
\]

\[
\text{KE: } \delta^* = (0.536\omega^*^{1/2} - 0.0186\omega^*^{3/2}) \times \left(1 - \left(1 - \frac{1.87Q'}{\omega^* (1 - 0.035\omega^*)}\right)^{2/3}\right),
\]

\[
\text{BKE: } \delta^* = 0.267\omega^* \coth(0.306\omega^*^{0.46}) \times \left(1 - \left(1 - \frac{3.75Q'}{\omega^*^{3/2}\tanh(0.306\omega^*^{0.46})}\right)^{2/3}\right),
\]

where \(\delta^* = 16G_a\sigma_b/(3P_c)\), \(Q' = Q/P_c\). The response to cyclic loading and energy dissipation per cycle are expressed in the Appendix A in a similar way. Note that the constant friction coefficient model uses \(\mu = 0.284\), i.e. the friction coefficient calculated for AISI 304 N stainless steel in Section 2. The expressions in Eqs. (17)–(21) correspond to elastic loading conditions only. The elastic–plastic formulation is too untidy to be shown in compact form; however, it can be obtained by the same procedure outlined in Fig. 5 with conditional statements on the normalized penetration.

### 4. Results

In this section, the responses resulting from the proposed models are summarized and in Section 5 compared to the experimental and FEA data existing in the literature. The procedure outlined in Fig. 5 is used to generate the results depicted in Figs. 6–8. Fig. 6 shows the normalized responses of the proposed models to increasing tangential loading up to sliding inception with a normalized penetration of \(\omega^* = 0.374\) (this value is specifically selected to match the first set of experiments shown in Section 5). Note that each model predicts similar tangential loading for small displacements and, hence, the same tangential contact stiffness. However, the maximum loading and deformation responses vary significantly due to the variation in predicted friction coefficients. Sliding inception occurs at the highest tangential load–displacement for the BKE model and the lowest for the CFC.

In Fig. 7, the cyclic tangential loading responses given by Eqs. (A.1)–(A.5) are shown for the same maximum normalized tangential displacement, \(\delta_m = 0.173\) (corresponding to the loading condition for the first set of experiments; i.e. \(\delta_m = 1.5\mu m\)). The normalized penetration is again set to \(\omega^* = 0.374\). All fretting loops except for CFC showed partial slip only, whereas CFC predicted mixed gross and partial slip behavior. This is mainly because CFC uses a constant friction coefficient value of 0.284, and this friction coefficient is not sufficient to prevent gross slip for higher tangential loads. Another observation is that the higher the friction coefficient value, the more the fretting behavior approaches the full-stick model. In other words, the fretting loop predicted by BKE has the smallest enclosed area; i.e. the contact is almost fully-adhered.

To analyze the energy dissipation (damping) predictions of the proposed models, we vary the maximum normalized tangential displacements and obtain the normalized energy dissipation, \(\Delta W^* = C a_0\Delta W/P_c^2\) at each fretting cycle. The results (Fig. 8) show
Fig. 6. Normalized tangential load vs. displacement obtained by the proposed models up to the onset of sliding.

Fig. 7. Fretting loops obtained by the proposed models.

Fig. 8. Energy dissipation vs. maximum imposed tangential displacement for each partial slip model.
5. Comparison with experiments/FEA and discussion

Next, the partial slip models proposed in Section 3 and illustrated in Section 4 are tested against two sets of experimental results reported in Varenberg et al. (2004). The first set includes a spherical contact with mm-scale geometry and μm-scale deformations, whereas the second set involves μm-scale geometry and nm-scale deformations. The experiments are referred to by the scale of deformations occurring, and, thus, the first set is denoted “micro-scale” and the second set “nano-scale”. In addition to experiments, FEA results from the literature are also used to assess the “BKE” model.

5.1. Micro-scale experiments

In the experiments, a standard bearing-steel ball specimen is fretted against an AISI 52100 flat steel specimen hardened to 63-67 HRC. The exact material properties are not specified by the authors. The values tabulated in Table 1 are found from the literature (MatWeb, 2010). Combined radius and shear and Young’s moduli are computed as described in Section 2 whereas softer material properties are used as the combined Possion’s ratio and hardness.

For the first experiment, the normal preload was set to 35 N and the maximum tangential displacement to 1.5 μm. Using Eq. (5) with the material properties and geometry given above, the critical contact radius, penetration and load are found to be 0.136 mm, 7.41 μm and 152.8 N, respectively. The normal contact is elastic since the penetration for all the models is estimated to be 2.77 μm, which is nearly 1/3 of the critical value for plastic yielding (see Table 2). The average roughness of the specimens used was documented to be 0.04–0.05 μm. Roughness-to-penetration ratio, $R_d/\alpha$, is less than 2%, and, therefore, it is safe to assume that smooth sphere-on-flat contact analysis would apply to model the experiments (according to Johnson, 1987 the threshold is at 5%). Moreover, adhesion is assumed to be insignificant in preloading the contact because the intermolecular separation-to-critical penetration ratio, $\xi/\alpha_d$, is very small (less than 0.01% for a typical intermolecular separation value of 0.4 nm Kogut and Etsion, 2003b).

In the second set of experiments, normal loading was set at 23 N and maximum tangential displacement to 10 μm. That means normal preload did not cause any plastic yielding (since the preload was even less than the preload used in the previous elastic experiments). The penetration is estimated to be 2.1 μm for this case, and, hence, the roughness-to-penetration ratio validates the smooth spherical contact assumption. The adhesion force is negligible as in the first experiment, since the intermolecular distance-to-critical penetration ratio does not depend on applied load but rather on material properties and radius of the sphere. Under the given loading conditions, friction coefficient values predicted by the CEB, KE and BKE models are also tabulated in Table 2.

Fig. 9a and b provide a comparison of the models proposed in Section 3 and the micro-scale experiments 1 and 2, respectively. As clearly seen, CEB matches the broadness (the width of the hysteresis loops) of the experimental results, whereas the maximum tangential loads obtained by this model are almost 15% less than those from the experiments. In addition, the tangential stiffness predicted by the experiments (17.1 N/μm) compares well with the value predicted by the Cattaneo–Mindlin solution (Eq. (11)); i.e. 16.17 N/μm for the first set of experiments and 14.06 N/μm for the second. Given that commercial load transducers possess stiffness values on the order of 1000 N/μm and the contact usually appears in a series configuration to the force transducer and machine support, the tangential contact stiffness should be 2–5% higher than the measured tangential stiffness (corresponds to 17.44–17.96 N/μm). Overall, the stiffness values obtained by the Cattaneo–Mindlin solution are within 20% error.

The proposed models other than CEB cannot replicate the behavior observed in the experiments due to several modeling limitations. CFC overestimates the broadness of the loop and underestimates the maximum tangential loading, whereas BKE does the reverse. The KE model predicts the maximum tangential load quantitatively as CEB, but fails to predict the broadness of the experimental fretting loops.

The differences discussed above essentially stem from the assumptions each model employs in order to determine the friction coefficient. Since CFC uses a lower friction coefficient, sliding occurs at lighter loads. In contrast, BKE uses the BKE friction model for a fully-adhered contact condition and, hence, predicts considerably larger tangential loads before sliding. This observation can be attributed to difficulties in maintaining a full-stick contact condition under laboratory conditions. At a relative humidity reported as 47% by Varenberg et al. (2004), contamination might induce slip nonlinear relationships between the energy dissipation and the maximum tangential displacements for each model under partial slip regimes and a linear relationship after gross slip is reached. The energy dissipation for small tangential displacements decreases with increasing friction coefficient; however, the opposite is true for large displacements and tangential loads. Note that the tangential load remains constant at $\mu P$ in the gross slip regime and further increase in tangential displacements $\Delta \alpha$ increase the energy dissipation by $\mu P \Delta \alpha$, and, hence, the linear slope equals to $\mu P$ in the gross slip regime. Since the preload is fixed, higher friction coefficients result in steeper slopes in the gross slip regime. This is essentially why CFC and BKE, respectively predict the highest and lowest energy losses (and thus damping) per fretting cycle in the partial slip regime, but the roles interchange for gross slip.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Mechanical properties and geometry of contacting materials used in micro-scale experiments documented in Varenberg et al. (2004). Combined values are listed for reference to calculations done in this paper.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Poisson’s ratio ($\nu$)</td>
</tr>
<tr>
<td>Flat steel (hardened)</td>
<td>0.3</td>
</tr>
<tr>
<td>Bearing steel (SAE 52100)</td>
<td>0.3</td>
</tr>
<tr>
<td>Flat steel (hardened)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Model parameters derived from the micro-scale experiments documented in Varenberg et al. (2004).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiments</td>
<td>$P$ (N)</td>
</tr>
<tr>
<td>Experiment 1</td>
<td>35</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>23</td>
</tr>
</tbody>
</table>
within the contact area. The assumptions used in CEB and KE friction models seem more appropriate in matching the experimental conditions due to the fact that they both allow partial slip contact in model development and employ the sliding contact sub-surface stress field found analytically.

5.2. Nano-scale experiments

In addition to micro-scale experiments, Varenberg et al. (2004) present results from nano-scale fretting experiments. In these experiments silica microspheres of radius 1.55 μm are first glued to the end of an AFM cantilevers and fretted against silicon (100) flat specimens. The material and geometry values for contacting bodies are tabulated in Table 3 (MatWeb, 2010). With these parameters, the critical penetration, contact radius, and load are calculated as 12.67 nm, 0.14 μm and 105.9 μN, respectively.

Three main differences should be kept in mind before applying the previously proposed models to nano-scale experiments. First of all, the elastic models presented in Sections 2 and 3 assume the contact of elastically similar materials, and, thus, the normal tractions do not cause tangential relative motion or shear tractions. In the micro-scale experiments, a bearing-steel ball was fretted against hardened flat steel; thus, the shear modulus and Poisson’s ratios of the contacting materials were the same. Therefore, the elastic similarity assumption is met, and the normal and tangential tractions can be assumed to be safely decoupled. Unlike these experiments, the elastic mismatch between silica microspheres and a silicon flat specimen is large; hence, the normal and tangen-

![Fig. 9. Comparison of the proposed models and micro-scale experiment 1 (a) and experiment 2 (b).](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Poisson’s ratio, ν</th>
<th>Shear Modulus, G(GPa)</th>
<th>Young’s Modulus, E(GPa)</th>
<th>Hardness, H(GPa)</th>
<th>Geometry (Radius, μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat silicon (100)</td>
<td>0.28</td>
<td>43.9</td>
<td>112.4</td>
<td>11.3</td>
<td>Flat (∞)</td>
</tr>
<tr>
<td>Silica microspheres, SiO₂</td>
<td>0.19</td>
<td>28.0</td>
<td>68.0</td>
<td>4.8</td>
<td>Sphere (1.55)</td>
</tr>
<tr>
<td>Combined</td>
<td>0.19</td>
<td>9.6</td>
<td>44.7</td>
<td>4.8</td>
<td>Flat-on-sphere (1.55)</td>
</tr>
</tbody>
</table>
tial tractions are highly coupled. Effects of elastic mismatch and fretting models accounting for it can be found elsewhere (Hutson et al., 2006; Nowell et al., 1988; Rajeev and Farris, 2002). Secondly, the silicon flat specimen is documented to have an average roughness of 0.5–0.7 nm. Roughness-to-penetration ratio calculated for both experiments 1 and 2 (0.99 and 0.94, see Table 4) is an order of magnitude higher than the critical value of 0.05 documented in Johnson (1987). As a consequence, we cannot safely apply the partial slip models for smooth spherical contact. Instead, models accounting for roughness on sphere or flat can be used, as done in Greenwood and Tripp (1967); however, it is beyond the scope of this work. The third complication stems from growing adhesion effects as the size of the spherical contacts decrease. In this set of experiments, the intermolecular separation-to-critical penetration ratio is 3.2% which is considerably higher than the critical ratio of 0.01% for a typical intermolecular separation value of 0.4 nm. The adhesion force obtained by the Kogut and Etsion (2003b) adhesion model reaches three times the applied load for each nano-scale experiment. In that case, the effect of adhesion on friction, and hence the fretting characteristics, cannot be ignored. It is evident that the partial slip models proposed in this work are not applicable to all experimental or practical situations where fretting occurs.

Three main assumptions elastic similarity, smoothness, and non-adhesive contact, play important roles in the development of the models presented and should therefore be met before application of these models to experiments.

Fig. 10 depict the proposed models and results from the nano-scale experiments 1 and 2 of Varenberg et al. (2004) respectively.

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$P$ (μN)</th>
<th>$\delta_d$ (nm)</th>
<th>$\omega$</th>
<th>$R_g/\omega$</th>
<th>$\varepsilon/\omega$</th>
<th>$\rho$ (CFC, CEB, KE, BKE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>1.1</td>
<td>5</td>
<td>0.048</td>
<td>0.99</td>
<td>3.2e – 2</td>
<td>(0.259, 1.769, 2.498, 3.077)</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>1.2</td>
<td>15</td>
<td>0.051</td>
<td>0.94</td>
<td>3.2e – 2</td>
<td>(0.259, 1.714, 2.424, 2.996)</td>
</tr>
</tbody>
</table>

Table 4 Model parameters derived from the nano-scale experiments documented in Varenberg et al. (2004).
Although the loading conditions for both experiments are within elastic limits, the models match neither the broadness nor maximum tangential load results from the experiments.

The friction coefficient for experiment 1 cannot be predicted directly from the data since gross sliding is not reached. However, the value should be greater than 1 since the maximum tangential load to applied preload ratio is almost 1 under the partial slip regime, seen in Fig. 10a. The friction coefficient for experiment 2 is 0.583. The CFC friction model gives $\mu = 0.259$ with the shear strength and hardness values of silica found in the literature, and CEB, KE and BKE models predict friction coefficient values greater than 1.7 for both experiments (Table 4). These high values explain the discrepancy at high maximum tangential loads obtained by the models. The tangential stiffness for both experiments (203–539 N/m) is smaller than model predictions (2356–2426 N/m). This disagreement might stem from the low stiffness values of AFM cantilevers dominating the overall tangential stiffness. As a conclusion, the experimental conditions should match modeling assumptions of the proposed models for direct comparison and correlation between models and experiments.

### 5.3. FEA: fully-adhered spherical contacts

As mentioned in Section 2, the tangential loading response is greatly influenced by the contact condition. According to Cattaneo–Mindlin, the partial slip contact condition occurs immediately after a tangential load is applied to a normally preloaded spherical contact as a stress relaxation mechanism. Although the CEB and KE models agree with this, the BKE model instead assumes that the contact region formed by the preload is under full-stick condition and remains so throughout subsequent tangential loading. This assumption is associated with highly-adhesive contacts, which is difficult to justify for metallic interfaces under light and moderate loading. The FEA results of Brizmer et al. (2007) for tangential loading of a sphere-on-flat contact under full-stick condition show a softening spring behavior of the interface as the tangential loading curve of Cattaneo–Mindlin’s partial slip solution demonstrated in Fig. 3 (the material behavior is assumed as elastic linear isotropic hardening with a 2% tangent modulus of the Young’s modulus). The physical basis behind this softening behavior is a weakening junction due to increasing plasticity rather than interfacial slip. Despite that fundamental difference, the proposed BKE model combines the BKE friction coefficient model with Cattaneo–Mindlin partial slip model to account for the weakening junction.

Fig. 11 shows the BKE model predictions for a tangential loading history with various preloads, Mindlin stick model, as well as Brizmer et al. (2007)’s FEA results. The partial slip curves labeled with normalized preloads 1, 10 and 100 compare reasonably well with the FEA curves. In addition to the weakening junction behavior, the dependence on normal preload is well mimicked by this model. Note that, as the normal preload decreases, the tangential loading curves approach Mindlin stick model. This occurs because the model response with lower normal preloads stays within the elastic limit for wider ranges of tangential displacements (for instance, for $P = 0.1$, only 10% of the critical load is reached before tangential loading, and the contact behaves elastically for larger imposed tangential displacements).

It is important to note that the formulation of the BKE model is achieved by combining the elastic Cattaneo–Mindlin approach with the friction coefficient and preload values which were obtained from fully-adhered elastic–plastic contact analyses. There are two inconsistencies in this approach, namely elastic–plastic formulation and contact conditions. Despite these physical inconsistencies, the BKE model results compare well with the FEA results. There are two possible reasons for this: First, the BKE friction model is originally developed from the same FEA results, and hence, the tangential loads at the onset of sliding inevitably match with the ones obtained through FEA. Secondly, either because of partial slip or plastic deformation, the junction uniformly weakens while loading (softening spring behavior). In addition, both Cattaneo–Mindlin partial slip approach and elastic–plastic FEA should give the same tangential contact stiffness as the Mindlin stick model at the initial stages of loading and zero tangential stiffness at the onset of sliding. Ultimately, the problem of expressing the FEA results of BKE simplifies to a problem of finding a uniformly-behaving softening spring with certain stiffness values at specified forces ($K_T = 8G$ at 0 and $Q_n$ at $Q = 0$ and $Q_n$). Obviously, this problem cannot be solved uniquely without further physical arguments. However, having the same derivatives as the FEA results at specified values, BKE was a natural candidate of such solutions. As can be seen in Fig. 11, the junction behavior is mimicked quite well except for differences in the maximum tangential displacement. As a conclusion, better

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**Fig. 11.** Dimensionless tangential load vs. dimensionless tangential displacement predicted by BKE model for different normal preloads. FEA results reproduced from Brizmer et al. (2007).
physics-based models employing elasticity and plasticity theory to solve the tangential loading of spherical contact are still to be developed.

6. Conclusion

In this work, purely elastic and elastic–plastic behavior of spherical contacts under combined normal and tangential loading are studied, and a physics-based partial slip modeling approach is proposed and compared with both experiments and FEA results. The proposed modeling approach essentially incorporates elastic-plastic normal loading response and preload-dependent friction coefficient models into the elastic Cattaneo–Mindlin solution. Although very promising, Odfalk and Vingsbo’s elastic–plastic partial slip model is briefly discussed but not used in this study, primarily because this model uses experimentally determined parameters (phenomenological model). The proposed models do not utilize any empirical data or curve-fit coefficients. As we employ physics-based parameters and variables, we call this approach “physics-based partial slip modeling”. The physics-based partial slip models—CFC, CEB, KE, and BKE—and the tangential stiffness values have been tested against experimental results existing in the literature, and the CEB has been shown to correlate reasonably well with micro-scale experiments. These experiments correspond to the elastic loading regime, and, thus, the plastic loading portion of the CEB and KE models is not tested in this work. Additionally, tangential contact stiffness predicted by the models compared reasonably well with available data at the micro-scale. However, large differences between model predictions and experiments at the nano-scale are observed. Although the deformations in these nano-scale experiments are calculated to be just above breakdown of continuum theory, the modeling assumptions effectively contribute to the discrepancies between experimental data and modeling results. Three major modeling simplifications (elastic similarity, smoothness and non-adhesive contact) used in our approach are shown to be critical assumptions before application of these models for comparison with experimental results. The nano-scale experiments, where silica microspheres were fretted against silicon flat surfaces, essentially violated all three assumptions and, hence, the proposed models cannot be used to simulate nano-scale fretting behavior.

Finally, the BKE model proposed in this work is tested against FEA results. The comparison shows that the BKE model developed by our approach matches reasonably well with FEA results in mimicking the softening spring behavior of the junction during loading. Therefore, the “non-physical” approach of combining the purely elastic Cattaneo–Mindlin partial slip solution with elastic–plastic models seems to replicate full-stick contact weakening due to plasticity.

Acknowledgment

This material is based in part upon work supported by the National Science Foundation under Grant No. CMMI 08-00208.

Appendix A

Cyclic tangential loading responses are quantified by the following equations:

CFC

\[
\delta^* \leq 0.95 \Rightarrow \delta^* = \left(0.112 \omega^* - 0.047 \sqrt{\omega^* - 4.5 \omega^*} \right) \times \left[1 + \left(1 - \frac{0.112 \omega^{1.2} - 0.047 \omega^* \sqrt{72 - 4.5 \omega^*}}{0.112 \omega^{1.2} - 0.047 \omega^* \sqrt{72 - 4.5 \omega^*}} \right)^{2/3} \right],
\]

\[\omega^* > 0.95 \Rightarrow \delta^* = \left[1 - \frac{0.803 Q_m^*}{\omega^* \sqrt{1 - \omega^*}} \right]^{2/3} + 2 \left[1 - \frac{0.402 (Q_m^* - Q^*)}{\omega^* \sqrt{1 - \omega^*}} \right]^{2/3} \]  

CEB

\[
\delta^* \leq 0.95 \Rightarrow \delta^* = \left(0.112 \omega^* - 0.047 \sqrt{4.5 \omega^*} \right) \times \left[1 + \left(1 + \frac{Q_m^*}{0.112 \omega^{1.2} - 0.047 \omega^* \sqrt{72 - 4.5 \omega^*}} \right)^{2/3} \right],
\]

\[\omega^* > 0.95 \Rightarrow \delta^* = \left[1 - \frac{0.803 Q_m^*}{\omega^* \sqrt{1 - \omega^*}} \right]^{2/3} + 2 \left[1 - \frac{0.402 (Q_m^* - Q^*)}{\omega^* \sqrt{1 - \omega^*}} \right]^{2/3} \]  

KE

\[
\delta^* = \sqrt{\omega^* (0.536 - 0.0186 \omega^*)} \times \left[1 - \frac{1.87 Q_m^*}{\omega^* (1 - 0.035 \omega^*)} \right]^{2/3} + 2 \left[1 - \frac{0.94 (Q_m^* - Q^*)}{\omega^* (1 - 0.035 \omega^*)} \right]^{2/3} \]  

BKE

\[
\delta^* = 0.267 \omega^* \coth (0.306 \omega^* \omega^*) \times \left[1 - \left(1 - \frac{3.75 Q_m^* \tanh (0.306 \omega^* \omega^*)}{\omega^* (1 - 0.035 \omega^*)} \right)^{2/3} + 2 \left[1 - \frac{1.88 (Q_m^* - Q^*) \tanh (0.306 \omega^* \omega^*)}{\omega^* (1 - 0.035 \omega^*)} \right]^{2/3} \right].
\]

Energy dissipation per fretting cycle responses are given as follows:

CFC

\[
\Delta W^* = \omega^* \delta^* \left[1 - \left(1 - \frac{3.52 Q_m^*}{\omega^* \omega^* \omega^* \omega^*} \right)^{5/3} \right] - 2.935 Q_m^* \omega^* \left[1 + \left(1 - \frac{3.52 Q_m^*}{\omega^* \omega^* \omega^* \omega^*} \right)^{2/3} \right] \]  

CEB

\[
\Delta W^* = 0.9 \omega^* \left[0.112 - 0.047 \sqrt{\frac{72}{\omega^*} - 4.5} \right]^2 \times \left[1 + \left(1 - \frac{Q_m^*}{0.112 - 0.047 \sqrt{72/\omega^*} - 4.5} \right)^{2/3} \right] - 6 \left(0.112 + 0.047 \sqrt{72/\omega^*} - 4.5 \right)^{2/3} \left[1 + \left(1 + \frac{Q_m^*}{0.112 + 0.047 \sqrt{72/\omega^*} - 4.5} \right)^{2/3} \right] \]  

\[\omega^* \leq 0.95 \Rightarrow \Delta W^* = 1.4 \left[1 + \frac{1}{\omega^*} \right] \left[1 - \frac{0.803 Q_m^*}{\omega^* \sqrt{1 - \omega^*}} \right]^{5/3} \]  

\[\omega^* > 0.95 \Rightarrow \Delta W^* = 1.4 \left[1 + \frac{1}{\omega^*} \right] \left[1 - \frac{0.803 Q_m^*}{\omega^* \sqrt{1 - \omega^*}} \right]^{5/3} \]  

\[\omega^* > 0.95 \Rightarrow \Delta W^* = 1.4 \left[1 + \frac{1}{\omega^*} \right] \left[1 - \frac{0.803 Q_m^*}{\omega^* \sqrt{1 - \omega^*}} \right]^{5/3} \]
\[ KE = 0.9(0.536 - 0.019(\sigma')^2(\sigma')^{-3/2}) \times \left[ 1 - \left(1 - \frac{Q_m^*}{\sigma' - 0.035(\sigma')^2} \right)^{5/3} - \frac{10Q_m^*}{6(\sigma' - 0.035(\sigma')^2)} \left(1 + \left(1 - \frac{Q_m^*}{\sigma' - 0.035(\sigma')^2}\right)^{2/3}\right)^{5/3} \right] \]  \hspace{1cm} (A.9)

\[ BKE = 0.064\sigma'^{5/2}(\coth(0.306(\sigma')^{0.46}))^2 \times \left[ 1 - \left(1 - \frac{3.748Q_m^* \tanh(0.306(\sigma')^{0.46})}{\sigma'^{2/2}} \right)^{5/3} - \frac{3.123Q_m^* \tanh(0.306(\sigma')^{0.46})}{\sigma'^{2/2}} \left(1 + \left(1 - \frac{3.748Q_m^* \tanh(0.306(\sigma')^{0.46})}{\sigma'^{2/2}}\right)^{2/3}\right)^{5/3} \right] \]  \hspace{1cm} (A.10)

References


