



Gauge coupling unification in the exceptional supersymmetric Standard Model

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Abstract

We consider the renormalisation group flow of gauge couplings within the so-called exceptional supersymmetric Standard Model (E_6 SSM) based on the low-energy matter content of 27-dimensional representations of the gauge group E_6 , together with two additional non-Higgs doublets. The two-loop beta functions are computed, and the threshold corrections are studied in the E_6 SSM. Our results show that gauge coupling unification in the E_6 SSM can be achieved for phenomenologically acceptable values of $\alpha_3(M_Z)$, consistent with the central measured low-energy value, unlike in the minimal supersymmetric Standard Model (MSSM) which, ignoring the effects of high energy threshold corrections, requires significantly higher values of $\alpha_3(M_Z)$, well above the experimentally measured central value.

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1. Introduction

Unification of gauge couplings is probably one of the most appealing features of supersymmetric (SUSY) extensions of the Standard Model (SM). More than fifteen years ago it was found that the electroweak (EW) and strong gauge couplings extracted from LEP data (hence at the EW scale) and extrapolated to high energies using the renormalisation group equation (RGE) evolution do not meet within the SM but converge to a common value at some high energy scale (within $\alpha_3(M_Z)$ uncertainties) after the inclusion of supersymmetry, e.g. in the framework of the minimal SUSY Standard Model (MSSM) [1]. This allows one to embed SUSY extensions of the SM into Grand Unified Theories (GUTs) (and superstring ones) that make possible partial unification of gauge interactions with gravity. Simultaneously, the incorporation of weak and strong gauge interactions within GUTs permits to explain the peculiar assignment

of $U(1)_Y$ charges postulated in the SM and to address the observed mass hierarchy of quarks and leptons.

Due to the lack of direct evidence verifying or falsifying the presence of superparticles at low energies, gauge coupling unification remains the main motivation for low-energy supersymmetry based on experimental data. But since 1990 the uncertainty in the determination of $\alpha_3(M_Z)$ has reduced significantly and the analysis of the two-loop RG flow of gauge couplings performed in [2–4] revealed that it is rather problematic to achieve exact unification of gauge couplings within the MSSM. This is also demonstrated in Fig. 1(a), where we plot the running of the gauge couplings from the EW (M_Z) scale to the GUT (M_X) scale. Fig. 1(b) shows a blow-up of the crucial region in the vicinity of $M_X = 3 \times 10^{16}$ GeV. To ensure the correct breakdown of the EW symmetry requires an effective SUSY threshold scale around 250 GeV, which corresponds to a SUSY Higgs mass parameter $\mu \simeq 1.5$ TeV. Dotted lines show the interval of variations of gauge couplings caused by 1σ deviations of $\alpha_3(M_Z)$ around its average value, i.e. $\alpha_3(M_Z) \simeq 0.118 \pm 0.002$ [5]. From Fig. 1(b) it is clear that exact gauge coupling unification in the MSSM cannot be attained even within 2σ deviations from the current average value of $\alpha_3(M_Z)$. Recently, it was argued that it is possible to get the

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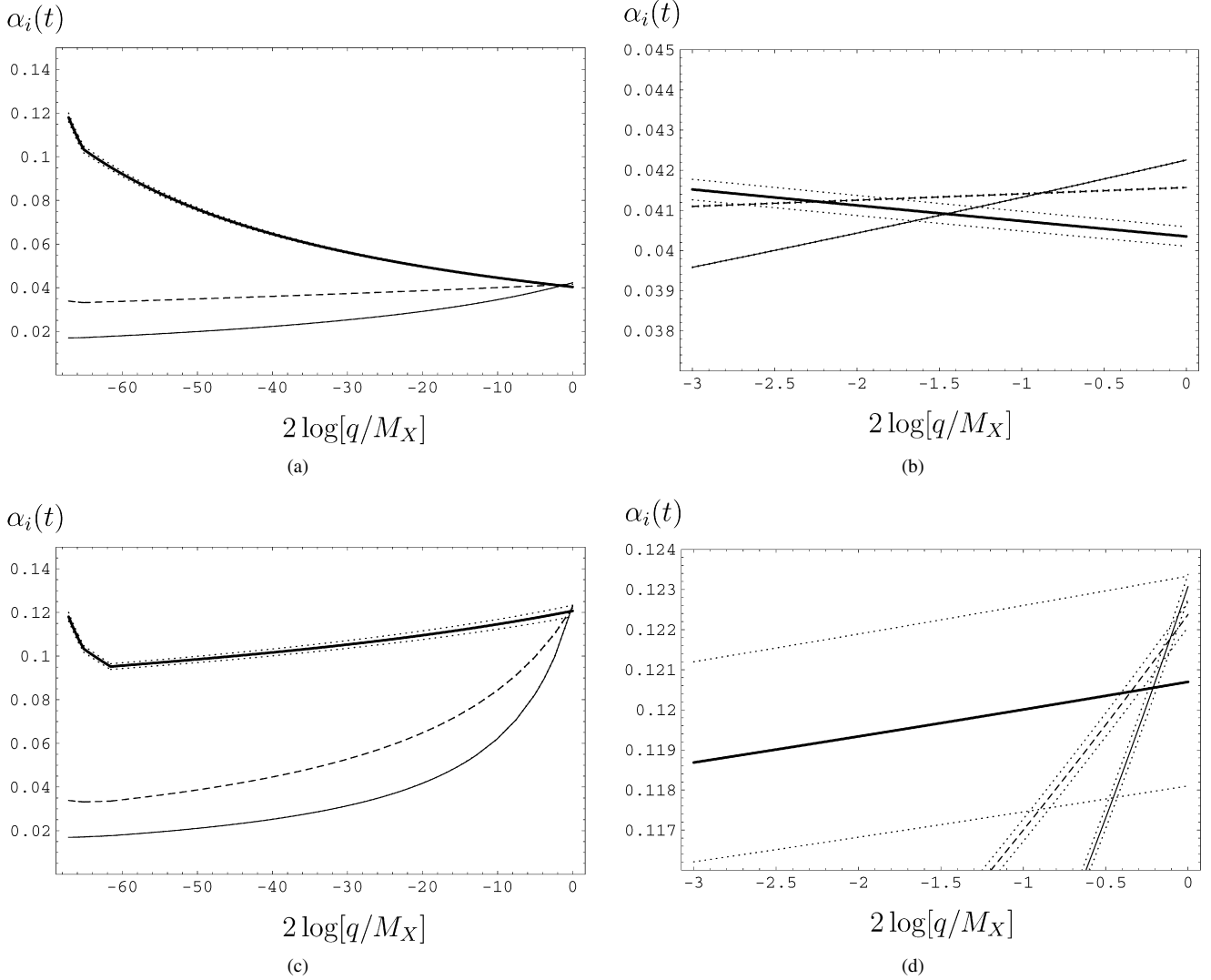


Fig. 1. Two-loop RG flow of gauge couplings: (a) evolution of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings from EW to GUT scale M_X in the MSSM; (b) running of SM gauge couplings near the scale M_X in the MSSM; (c) RG flow of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings from M_Z to M_X in the E_6 SSM; (d) running of SM gauge couplings in the vicinity of M_X in the E_6 SSM. Thick, dashed and solid lines correspond to the running of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings respectively. We used $\tan\beta = 10$, an effective SUSY threshold scale $M_S = 250$ GeV, $M_{Z'} = 1.5$ TeV, $\kappa(M'_Z) = \kappa_1(M'_Z) = \kappa_2(M'_Z) = \lambda(M_{Z'}) = \lambda_1(M_{Z'}) = \lambda_2(M_{Z'}) = g'_1(M_{Z'})$, $g'^2_1(M_{Z'}) = 0.2271$, $g_{11}(M_{Z'}) = 0.02024$, $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$ and $\sin^2\theta_W = 0.231$. The dotted lines represent the uncertainty in $\alpha_i(t)$ caused by the variation of the strong gauge coupling from 0.116 to 0.120 at the EW scale.

unification of gauge couplings in the minimal SUSY model for $\alpha_3(M_Z) = 0.123$ [6].

The above observation is in fact true for a whole class of GUTs that break to the SM gauge group in one step and which predict a so-called “grand desert” between the EW and GUT scales. This conclusion must be qualified, however, by the fact that in general there are non-negligible high energy GUT/string threshold corrections to the running of the couplings associated with heavy particle thresholds and higher dimension operator effects which we shall not consider here. Furthermore, in this Letter, we restrict our considerations to the minimal scenario for GUT symmetry group breakdown—the aforementioned one-step GUTs—as this allows one to get a stringent prediction for $\alpha_s(M_Z)$. In particular, we examine gauge coupling unification within an E_6 inspired extension of the MSSM, the exceptional supersymmetric Standard Model (E_6 SSM) of Refs. [7,8] in

which the E_6 symmetry breaking proceeds uniquely at a single step through the $SU(5)$ breaking direction. This results in a low-energy SM gauge group augmented by a unique $U(1)_N$ gauge group under which right-handed neutrinos have zero charge, allowing them to be superheavy, shedding light on the origin of the mass hierarchy in the lepton sector and providing a mechanism for the generation of the lepton and baryon asymmetry of the Universe. The μ problem of the MSSM is solved within the E_6 SSM in a similar way to the NMSSM, but without the accompanying problems of singlet tadpoles or domain walls. Thus the E_6 SSM is a low-energy alternative to the MSSM or NMSSM.

In this Letter we calculate the two-loop beta functions of the gauge couplings in the E_6 SSM, and then apply them to the question of gauge coupling unification, including the important effects of low-energy threshold corrections. The structure of the two-loop contributions to the corresponding beta functions is

such that the EW and strong couplings meet at some high energy scale for an $\alpha_3(M_Z)$ value which is just slightly higher than the experimentally measured central value, with the low-energy threshold effects pushing it further towards the central measured value. As the results in Fig. 1(c), (d) will show, the unification of gauge couplings in the E_6 SSM is achieved for values of $\alpha_3(M_Z)$ consistent with the measured central value, unlike in the MSSM which, ignoring the effects of high energy threshold corrections, requires significantly higher values of $\alpha_3(M_Z)$, well above the experimentally measured central value.

The layout of the remainder of the Letter is as follows. In Section 2 we present an analytical approach to the solution of the RGEs for the gauge couplings that allows one to examine the unification of forces in SUSY models and we specialise to the MSSM case in Section 3. In Section 4 we briefly review the E_6 SSM and in Section 5 we discuss the two-loop RG flow of the gauge couplings within this model, including the low-energy threshold corrections, leading to the stated results. Section 6 concludes the Letter.

2. RG flow of gauge couplings in SUSY models

In SUSY models the running of the SM gauge couplings is described by a system of RGEs which can be written in the following form:

$$\frac{d\alpha_i}{dt} = \frac{\beta_i \alpha_i^2}{(2\pi)}, \quad \beta_i = b_i + \frac{\tilde{b}_i}{4\pi}, \quad (1)$$

where b_i and \tilde{b}_i are one-loop and two-loop contributions to the beta functions [9,10], $t = \ln(\mu/M_Z)$, μ is a renormalisation scale, with the index i running from 1 to 3 corresponding to $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$ interactions, respectively. One can obtain an approximate solution of the RGEs in Eq. (1) that at high energies can be written as [3]

$$\frac{1}{\alpha_i(t)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}t - \frac{C_i}{12\pi} - \Theta_i(t) + \frac{b_i - b_i^{\text{SM}}}{2\pi} \ln \frac{T_i}{M_Z}, \quad (2)$$

where the third term in the right-hand side of Eq. (2) is the $\overline{\text{MS}} \rightarrow \overline{\text{DR}}$ conversion factor with $C_1 = 0$, $C_2 = 2$, $C_3 = 3$ [11],

$$\Theta_i(t) = \frac{1}{8\pi^2} \int_0^t \tilde{b}_i d\tau, \quad T_i = \prod_{k=1}^N (m_k) \frac{\Delta b_i^k}{b_i - b_i^{\text{SM}}} \quad (3)$$

and b_i^{SM} are the coefficients of the one-loop beta functions in the SM, while m_k and Δb_i^k are masses and contributions to the beta functions due to new particles appearing in the considered SUSY models. Because the two-loop corrections to the running of the gauge couplings $\Theta_i(t)$ are considerably smaller than the leading terms, the gauge and Yukawa couplings in \tilde{b}_i are usually replaced by the corresponding solutions of the RGEs obtained in the one-loop approximation. The threshold corrections associated with the last terms in Eq. (2) are of the same order as

or even less than $\Theta_i(t)$. Therefore in Eqs. (2)–(3) only leading one-loop threshold effects are taken into account.

Relying on the approximate solution of the RGEs in Eqs. (2) and (3) one can establish the relationships between the values of the gauge couplings at the EW and GUT scales, for any general SUSY model. Then by using the expressions describing the RG flow of $\alpha_1(t)$ and $\alpha_2(t)$ it is rather easy to find the scale M_X where $\alpha_1(M_X) = \alpha_2(M_X) = \alpha_0$ and the value of the overall gauge coupling α_0 at this scale. Substituting M_X and α_0 into the solution of the RGE for the strong gauge coupling one finds the value of $\alpha_3(M_Z)$ for which exact gauge coupling unification takes place [12]:

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{b_1 - b_2} \left[\frac{b_1 - b_3}{\alpha_2(M_Z)} - \frac{b_2 - b_3}{\alpha_1(M_Z)} \right] - \frac{1}{28\pi} + \Theta_s - \Delta_s, \\ \Theta_s = \left(\frac{b_2 - b_3}{b_1 - b_2} \Theta_1 - \frac{b_1 - b_3}{b_1 - b_2} \Theta_2 + \Theta_3 \right), \\ \Theta_i = \Theta_i(M_X), \quad (4)$$

where Δ_s are combined threshold corrections whose precise form depends on the model under consideration.

3. MSSM

In this section we apply the results of the previous section to the MSSM. In the MSSM Δ_s takes the form [2,3,12,13]:

$$\Delta_s = -\frac{19}{28\pi} \ln \frac{M_S}{M_Z}, \quad M_S = \frac{T_2^{100/19}}{T_1^{25/19} T_3^{56/19}}. \quad (5)$$

For example, assuming for simplicity that superpartners of all quarks are degenerate, i.e. their masses are equal to $m_{\tilde{q}}$, and all sleptons have a common mass $m_{\tilde{l}}$, we find:

$$M_S \simeq \mu \left(\frac{m_A}{\mu} \right)^{3/19} \left(\frac{M_2}{\mu} \right)^{4/19} \left(\frac{M_2}{M_3} \right)^{28/19} \left(\frac{m_{\tilde{l}}}{m_{\tilde{q}}} \right)^{3/19}. \quad (6)$$

In Eq. (6) M_3 and M_2 are masses of gluinos and winos (superpartners of $SU(2)_W$ gauge bosons), whereas μ and m_A are μ -term and masses of heavy Higgs states respectively. In general T_1 , T_2 and T_3 , obtained from Eq. (3), can be quite different.

We now perform a simplified numerical discussion of the previous results in order to illustrate the effect of threshold corrections on gauge unification in the MSSM. In our simplified discussion we shall assume the effective threshold scales T_i be equal to each other, $T_1 = T_2 = T_3 = M_S$, where the last equality follows from Eq. (5). From Eqs. (4)–(5) and Table 2 it follows that, in order to achieve gauge coupling unification in the MSSM with $\alpha_s(M_Z) \simeq 0.118$, the effective threshold scale must be around $M_S \simeq 1$ TeV. However the correct pattern of EW symmetry breaking (EWSB) requires μ to lie within the 1–2 TeV range, while from Eq. (6) it follows that $M_S \simeq \mu/6$, which implies that the effective threshold scale should be $M_S < 200$ –300 GeV [2,3,12–14]. For such small values of the scale M_S exact gauge coupling unification can be obtained only for large values of $\alpha_3(M_Z) \gtrsim 0.123$, which are disfavoured by the recent fit to experimental data. Put it another way, assuming that the low-energy QCD coupling is at its central value $\alpha_s(M_Z) = 0.118$, and assuming $M_S = 250$ GeV, the

gauge couplings fail to meet exactly at the GUT scale, as shown in Fig. 1(a)–(b). As we shall show, this situation is improved dramatically in the E_6 SSM.

4. E_6 SSM—a brief review

In this section, in order to make the Letter self-contained, we give a brief review of the E_6 SSM which was proposed recently in [7,8]. The E_6 SSM involves an additional low-energy gauged $U(1)_N$ not present in the MSSM, and in order to ensure anomaly cancellation the particle content of the E_6 SSM is also extended to include three complete fundamental 27 representations of E_6 at low energies. These multiplets decompose under the $SU(5) \times U(1)_N$ subgroup of E_6 as follows [15]:

$$27_i \rightarrow \left(10, \frac{1}{\sqrt{40}}\right)_i + \left(5^*, \frac{2}{\sqrt{40}}\right)_i + \left(5^*, -\frac{3}{\sqrt{40}}\right)_i + \left(5, -\frac{2}{\sqrt{40}}\right)_i + \left(1, \frac{5}{\sqrt{40}}\right)_i + (1, 0)_i. \quad (7)$$

The first and second quantities in the brackets are the $SU(5)$ representation and extra $U(1)_N$ charge while i is a family index that runs from 1 to 3. An ordinary SM family which contains the doublets of left-handed quarks Q_i and leptons L_i , right-handed up- and down-quarks (u_i^c and d_i^c) as well as right-handed charged leptons, is assigned to $(10, \frac{1}{\sqrt{40}})_i + (5^*, \frac{2}{\sqrt{40}})_i$. Right-handed neutrinos N_i^c should be associated with the last term in Eq. (7), $(1, 0)_i$. The next-to-last term in Eq. (7), $(1, \frac{5}{\sqrt{40}})_i$, represents SM-type singlet fields S_i which carry non-zero $U(1)_N$ charges and therefore survive down to the EW scale. The pair of $SU(2)_W$ -doublets (H_{1i} and H_{2i}) that are contained in $(5^*, -\frac{3}{\sqrt{40}})_i$ and $(5, -\frac{2}{\sqrt{40}})_i$ have the quantum numbers of Higgs doublets. So they form either Higgs or non-Higgs $SU(2)_W$ multiplets. Other components of these $SU(5)$ multiplets form colour triplets of exotic quarks \bar{D}_i and D_i with electric charges $-1/3$ and $+1/3$ respectively. However these exotic quark states carry a $B - L$ charge ($\pm \frac{2}{3}$) twice larger than that of ordinary ones. Therefore in phenomenologically viable E_6 inspired models they can be either diquarks or leptoquarks. In addition to the complete 27_i multiplets the low-energy particle spectrum of the E_6 SSM is supplemented by $SU(2)_W$ doublet H' and anti-doublet \bar{H}' states from the extra $27'$ and $\bar{27}'$ to preserve gauge coupling unification. Thus, in addition to a Z' corresponding to the $U(1)_N$ symmetry, the E_6 SSM involves extra matter beyond the MSSM that forms three $5 + 5^*$ representations of $SU(5)$ plus three $SU(5)$ singlets with $U(1)_N$ charges. The presence of a Z' boson and exotic quarks predicted by the E_6 SSM provides spectacular new physics signals at the LHC which were discussed in [7,8,16].

The superpotential in E_6 inspired models involves a lot of new Yukawa couplings in comparison to the SM. In general these new interactions induce non-diagonal flavour transitions. To avoid a flavour changing neutral current (FCNC) problem an extra Z_2^H symmetry is postulated in the E_6 SSM. Under this symmetry all superfields except one pair of H_{1i} and H_{2i} (say $H_d \equiv H_{13}$ and $H_u \equiv H_{23}$) and one SM-type singlet field

($S \equiv S_3$) are odd. The Z_2^H symmetry reduces the structure of the Yukawa interactions to:

$$W_{\text{ESSM}} \simeq \lambda_i S(H_{1i}H_{2i}) + \kappa_i S(D_i\bar{D}_i) + f_{\alpha\beta} S_\alpha(H_d H_{2\beta}) + \tilde{f}_{\alpha\beta} S_\alpha(H_{1\beta} H_u) + \mu'(H'\bar{H}') + g_i e_i^c(H_d H') + W_{\text{MSSM}}(\mu = 0), \quad (8)$$

where $\alpha, \beta = 1, 2$ and $i = 1, 2, 3$. The $SU(2)_W$ doublets H_u and H_d play the role of Higgs fields generating the masses of quarks and leptons after EWSB. Therefore it is natural to assume that only S , H_u and H_d acquire non-zero vacuum expectation values (VEVs). The VEV of the SM-type singlet field S breaks the extra $U(1)_N$ symmetry thereby providing an effective μ term as well as the necessary exotic fermion masses and also inducing that of the Z' boson. To guarantee that only H_u , H_d and S develop VEVs in the E_6 SSM a certain hierarchy between the Yukawa couplings is imposed, i.e. $\lambda_3 \gtrsim \lambda_{1,2} \gg f_{\alpha\beta}, \tilde{f}_{\alpha\beta}, g_i$.

However the Z_2^H symmetry can only be an approximate one because it forbids all Yukawa interactions that would allow the exotic quarks to decay. Since models with stable charged exotic particles are ruled out by different experiments [17] the Z_2^H symmetry has to be broken. At the same time the breakdown of Z_2^H should not give rise to operators leading to rapid proton decay. There are two ways to overcome this problem. The resulting Lagrangian has to be invariant with respect to either a Z_2^L symmetry, under which all superfields except lepton ones are even, or a Z_2^B discrete symmetry, which implies that exotic quark and lepton superfields are odd whereas the others remain even. Because Z_2^H symmetry violating operators may also give an appreciable contribution to the amplitude of $K^0 - \bar{K}^0$ oscillations and give rise to new muon decay channels like $\mu \rightarrow e^- e^+ e^-$ the corresponding Yukawa couplings are expected to be small. Therefore Z_2^H symmetry violating Yukawa couplings are irrelevant for the analysis of the RG flow of gauge couplings considered here.

It is worth to emphasise that all the discrete symmetries Z_2^H , Z_2^L and Z_2^B that we use here to prevent rapid proton decay break E_6 because different components of the fundamental 27 representation transform differently under these symmetries. Another manifestation of the breakdown of the E_6 symmetry is the presence of the $SU(2)_W$ doublet H' and anti-doublet \bar{H}' in the low-energy particle spectrum of the E_6 SSM that comes from the splitting of extra $27'$ and $\bar{27}'$. Because the splitting of 27-plets is a necessary ingredient of the considered model, as it is required in order to attain gauge coupling unification, it seems to be very attractive to reduce all origins of the E_6 symmetry breakdown (including postulated discrete symmetries) to the splitting of different E_6 multiplets. The splitting of GUT multiplets can be naturally achieved in the framework of orbifold GUTs [18].

The E_6 GUT model whose incomplete multiplets form the particle content of the E_6 SSM at low energies involves at least eight 27 and one $\bar{27}$ multiplets. One 27-plet Φ_0 includes only five components that survive down to the EW scale and compose the Higgs sector of the E_6 SSM, namely S , H_u , H_d . Such E_6 GUT model should also have three pairs of 27-plets Φ_i and Φ_i^L which accommodate three generations of quarks and lep-

Table 1
Transformation properties of E_6 multiplets under Z_2^H and Z_2' discrete symmetries

	Φ_0	Φ_i	Φ_i^L	Φ'	$\bar{\Phi}'$	Σ
Z_2^H	+	−	−	−	−	−
Z_2'	+	+	−	−	−	+

tons, where i is a family index. The E_6 multiplets Φ_i^L contain left-handed and right-handed lepton superfields (L_i, e_i^c, N_i^c) while the Φ_i 's involve all quark superfields (Q_i, u_i^c, d_i^c) as well as non-Higgs and SM singlet fields. The only exception is Φ_3 , that does not include either non-Higgs or SM-type singlet fields. Exotic quarks \bar{D}_i and D_i belong either to Φ_i^L (if they are leptoquarks) or to Φ_i (if they are diquarks). Finally extra $27'$ and $\bar{27}'$ (Φ' and $\bar{\Phi}'$) contain only two light components each that form the $SU(2)_W$ doublet H' and anti-doublet \bar{H}' with quantum numbers of left-handed lepton fields.

In order to get a suitable pattern of Yukawa couplings postulated above we impose the invariance of the Lagrangian of the considered E_6 GUT model under the $Z_2^H \otimes Z_2'$ symmetry. As before all E_6 multiplets except Φ_0 are odd under the Z_2^H symmetry transformations. The Z_2' symmetry is equivalent to either the Z_2^L symmetry when exotic quarks are diquarks or the Z_2^B symmetry if exotic quarks are leptoquarks. The transformation properties of E_6 multiplets under the Z_2^H and Z_2' symmetries are summarised in Table 1. Here we also introduce the singlet field Σ that does not participate in the E_6 gauge interactions. Just as other E_6 multiplets, Σ is odd under Z_2^H .

The most general superpotential which is invariant under E_6 and $Z_2^H \otimes Z_2'$ symmetry transformations is given by

$$\begin{aligned}
W = & \lambda \Phi_0^3 + \sigma_{ij} \Phi_0 \Phi_i \Phi_j + \tilde{\sigma}_{ij} \Phi_0 \Phi_i^L \Phi_j^L \\
& + \frac{\Sigma}{M_{\text{Pl}}} (\eta_i \Phi_0^2 \Phi_i + \zeta_{ijk} \Phi_i \Phi_j \Phi_k + \tilde{\zeta}_{ijk} \Phi_i \Phi_j^L \Phi_k^L) \\
& + \mu_X \Sigma^2 + \xi \frac{\Sigma^4}{M_{\text{Pl}}} + \dots
\end{aligned} \quad (9)$$

In the superpotential (9) we omit higher order terms that are suppressed as $1/M_{\text{Pl}}^2$ or even stronger. If $\mu_X \ll M_{\text{Pl}}$ the singlet field Σ may acquire vacuum expectation value which is many orders of magnitude smaller than the Planck scale. Non-zero vacuum expectation value of Σ breaks the Z_2^H symmetry spontaneously. Then the first three terms in Eq. (9) result in the Z_2^H symmetric part of the superpotential of the E_6 SSM at low energies while the next three terms give rise to couplings that violate the Z_2^H symmetry explicitly. In this case the effective Yukawa couplings which are induced after the breakdown of the Z_2^H symmetry are naturally suppressed by the small ratio $\frac{\langle \Sigma \rangle}{M_{\text{Pl}}}$ leading to the desirable hierarchical structure of Yukawa interactions postulated in the E_6 SSM.

5. Gauge coupling unification in the E_6 SSM

We now turn to the central issue of this Letter, that of gauge coupling unification in the E_6 SSM. We first present our results for the two-loop beta functions in this model, before going on

to consider the question of gauge coupling unification in the presence of low-energy threshold effects. The running of gauge couplings in the E_6 SSM is affected by a kinetic term mixing [7,19]. As a result the RGEs can be written as follows:

$$\frac{dG}{dt} = G \times B, \quad \frac{dg_2}{dt} = \frac{\beta_2 g_2^3}{(4\pi)^2}, \quad \frac{dg_3}{dt} = \frac{\beta_3 g_3^3}{(4\pi)^2}, \quad (10)$$

where B and G are 2×2 matrices

$$\begin{aligned}
G &= \begin{pmatrix} g_1 & g_{11} \\ 0 & g_1' \end{pmatrix}, \\
B &= \frac{1}{(4\pi)^2} \begin{pmatrix} \beta_1 g_1^2 & 2g_1 g_1' \beta_{11} + 2g_1 g_{11} \beta_1 \\ 0 & g_1'^2 \beta_1' + 2g_1' g_{11} \beta_{11} + g_{11}^2 \beta_1 \end{pmatrix}.
\end{aligned} \quad (11)$$

As always the two-loop diagonal β_i and off-diagonal β_{11} beta functions may be presented as a sum of one-loop and two-loop contributions (see Eq. (1)). In the one-loop approximation the beta functions are given by

$$\begin{aligned}
b_1 &= \frac{3}{5} + 3N_g, & b_1' &= \frac{2}{5} + 3N_g, & b_{11} &= \frac{\sqrt{6}}{5}, \\
b_2 &= -5 + 3N_g, & b_3 &= -9 + 3N_g.
\end{aligned} \quad (12)$$

The parameter N_g appearing in Eq. (12) is the number of generations in the E_6 SSM forming complete E_6 fundamental representations at low energies ($E \ll M_X$). As one can easily see $N_g = 3$ is the critical value for the one-loop beta function of the strong interactions. Since by construction three complete 27 -plets survive to low energies in the E_6 SSM \tilde{b}_3 is equal to zero in our case and in the one-loop approximation the $SU(3)_C$ gauge coupling remains constant everywhere from M_Z to M_X . Because of this any reliable analysis of gauge coupling unification requires the inclusion of two-loop corrections to the beta functions of gauge couplings. Here we calculate the two-loop contributions to the diagonal beta functions only. Using the results of the computation of two-loop beta functions in a general softly broken $N = 1$ SUSY model [10] we find the following two-loop beta functions for the E_6 SSM:

$$\begin{aligned}
\tilde{b}_1 &= 8N_g \alpha_3 + \left(\frac{9}{5} + 3N_g \right) \alpha_2 + \left(\frac{9}{25} + 3N_g \right) \alpha_1 \\
&+ \left(\frac{6}{25} + N_g \right) \alpha_1' - \frac{26}{5} y_t - \frac{14}{5} y_b - \frac{18}{5} y_\tau \\
&- \frac{6}{5} \Sigma_\lambda - \frac{4}{5} \Sigma_\kappa,
\end{aligned}$$

$$\begin{aligned}
\tilde{b}_1' &= 8N_g \alpha_3 + \left(\frac{6}{5} + 3N_g \right) \alpha_2 + \left(\frac{6}{25} + N_g \right) \alpha_1 \\
&+ \left(\frac{4}{25} + 3N_g \right) \alpha_1' - \frac{9}{5} y_t - \frac{21}{5} y_b - \frac{7}{5} y_\tau \\
&- \frac{19}{5} \Sigma_\lambda - \frac{57}{10} \Sigma_\kappa,
\end{aligned}$$

$$\begin{aligned}
\tilde{b}_2 &= 8N_g \alpha_3 + (-17 + 21N_g) \alpha_2 + \left(\frac{3}{5} + N_g \right) \alpha_1 \\
&+ \left(\frac{2}{5} + N_g \right) \alpha_1' - 6y_t - 6y_b - 2y_\tau - 2\Sigma_\lambda,
\end{aligned}$$

$$\tilde{b}_3 = \alpha_3(-54 + 34N_g) + 3N_g \alpha_2 + N_g \alpha_1$$

$$+ N_g \alpha'_1 - 4y_t - 4y_b - 2\Sigma_\kappa,$$

$$\Sigma_\lambda = y_{\lambda_1} + y_{\lambda_2} + y_{\lambda_3}, \quad \Sigma_\kappa = y_{\kappa_1} + y_{\kappa_2} + y_{\kappa_3}, \quad (13)$$

where $y_t = \frac{h_t^2}{4\pi}$, $y_b = \frac{h_b^2}{4\pi}$, $y_\tau = \frac{h_\tau^2}{4\pi}$, $y_{\lambda_i} = \frac{\lambda_i^2}{4\pi}$ and $y_{\kappa_i} = \frac{\kappa_i^2}{4\pi}$. Because our previous analysis performed in [7] revealed that an off-diagonal gauge coupling g_{11} being set to zero at the scale M_X remains very small at any other scale below M_X we neglect two-loop corrections to the off-diagonal beta function β_{11} .

The results of our numerical studies of gauge coupling unification in this model are summarised in Fig. 1(c)–(d) where the two-loop RG flow of gauge couplings in the E₆SSM is shown. As before we fix the effective SUSY threshold scale to be equal to 250 GeV, that on the one hand results in appreciable threshold corrections to the RG running of the gauge couplings but on the other hand does not spoil the breakdown of the EW symmetry. We also assume that the masses of the Z' and all exotic fermions and bosons predicted by the E₆SSM are degenerate around 1.5 TeV. Thus we use the SM beta functions to describe the running of gauge couplings between M_Z and M_S , then we apply the two-loop RGEs of the MSSM to compute the flow of $g_i(t)$ from M_S to $M_{Z'}$ and the two-loop RGEs of the E₆SSM to calculate the evolution of $g_i(t)$ between $M_{Z'}$ and M_X which is equal to 3.5×10^{16} GeV in the case of the E₆SSM. Again dotted lines in Fig. 1(c)–(d) represent the changes of the evolution of gauge couplings induced by the variations of $\alpha_3(M_Z)$ within 1σ around its average value. From Fig. 1(a)–(d) one can easily see that the interval of variations of $\alpha_3(t)$ is always considerably wider than the ones for $\alpha_1(t)$ and $\alpha_2(t)$. However one may expect that the dependence of $\alpha_1(t)$ and $\alpha_2(t)$ on the value of the strong gauge coupling at the EW scale should be relatively weak because $\alpha_3(t)$ appears only in the two-loop contributions to the corresponding beta functions.

It is also worth to notice that at high energies the uncertainty in $\alpha_3(t)$ caused by the variations of $\alpha_3(M_Z)$ is much bigger in the E₆SSM than in the MSSM. This happens because in the MSSM the strong gauge coupling reduces with increasing renormalisation scale. Therefore the interval of variations of $\alpha_3(t)$ near the scale M_X shrinks drastically. This focusing effect of the errors in the MSSM can be readily understood by examining the one-loop solution for $\alpha_3(t)$ in the MSSM. In the E₆SSM the strong gauge coupling has a zero one-loop beta function whereas at two-loop level the coupling has a mild growth as the renormalisation scale increases. This implies that the uncertainty in the high energy value of $\alpha_3(t)$ in the E₆SSM is thus approximately equal to the low-energy uncertainty in $\alpha_3(t)$. The relatively large uncertainty in $\alpha_3(M_X)$ in the E₆SSM, compared to the MSSM, allows one to achieve exact unification of gauge couplings even within 1σ deviation of $\alpha_3(M_Z)$ from its average value.

The RG flow of gauge couplings in the E₆SSM can be also analysed using the analytical approach for $g_i(t)$ presented in Section 2. Substituting the derived two-loop beta functions from Eqs. (12)–(13) into Eqs. (2)–(3) we find the approximate solution for the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings in the E₆SSM. The effective threshold scales \tilde{T}_1 , \tilde{T}_2 and \tilde{T}_3 in

Table 2

The corrections to $1/\alpha_i(M_X)$ and $1/\alpha_3(M_Z)$ induced by the two-loop contributions to the beta functions in the MSSM and E₆SSM for $\alpha(M_Z) = 1/127.9$, $\sin^2\theta_W = 0.231$, $\alpha_3(M_Z) = 0.118$ and $\tan\beta = 10$. In the case of the E₆SSM we consider three cases: the scenario E₆SSM I corresponds to $\kappa(M_{Z'}) = \kappa_1(M_{Z'}) = \kappa_2(M_{Z'}) = \lambda(M_{Z'}) = \lambda_1(M_{Z'}) = \lambda_2(M_{Z'}) = g'_1(M_{Z'})$, $g_1'^2(M_{Z'}) = 0.227$, $g_{11}(M_{Z'}) = 0.0202$; in the scenario E₆SSM II we fix $\kappa_i = \lambda_i = 0$, $g_1'^2(M_{Z'}) = 0.227$, $g_{11}(M_{Z'}) = 0.0202$; in the scenario E₆SSM III we ignore all Yukawa and $U(1)_N$ gauge couplings. Note that in all versions of the E₆SSM the large individual contributions Θ_i conspire to partially cancel when forming the quantity Θ_s which describes the effect of the two-loop corrections to determining the low-energy value of $\alpha(M_Z)$

	Θ_1	Θ_2	Θ_3	Θ_s
MSSM	0.556	0.953	0.473	−0.764
E ₆ SSM I	1.558	2.322	2.618	−0.250
E ₆ SSM II	1.604	2.385	2.638	−0.305
E ₆ SSM III	1.602	2.389	2.627	−0.326

such a model can be expressed in terms of the MSSM ones, i.e.:

$$\begin{aligned} \tilde{T}_1 &= T_1^{5/11} m_{H_\alpha}^{4/55} \mu_{\tilde{H}_\alpha}^{8/55} m_{\tilde{D}_i}^{4/55} \mu_{D_i}^{8/55} m_{H'}^{2/55} \mu_{\tilde{H}'}^{4/55}, \\ \tilde{T}_2 &= T_2^{25/43} m_{H_\alpha}^{4/43} \mu_{\tilde{H}_\alpha}^{8/43} m_{H'}^{2/43} \mu_{\tilde{H}'}^{4/43}, \\ \tilde{T}_3 &= T_3^{4/7} m_{\tilde{D}_i}^{1/7} \mu_{D_i}^{2/7}, \end{aligned} \quad (14)$$

where μ_{D_i} and $m_{\tilde{D}_i}$ are the masses of exotic quarks and their superpartners, m_{H_α} and $\mu_{\tilde{H}_\alpha}$ are the masses of non-Higgs and non-Higgsino fields of the first and second generation, while $m_{H'}$ and $\mu_{\tilde{H}'}$ are the masses of the scalar and fermion components of H' and \tilde{H}' .

The value of strong gauge coupling at the EW scale that results in the exact gauge coupling unification can be predicted anew. It is given by Eq. (4) where the E₆SSM beta functions and new threshold scales \tilde{T}_i should be substituted. Such replacement does not change the form of Eq. (4) because extra matter in the E₆SSM form complete $SU(5)$ representations which contribute equally to the one-loop beta functions of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ interactions. Due to this the differences of the coefficients of the one-loop beta functions $b_i - b_j$ remain intact. But the contributions of two-loop corrections to $\alpha_i(M_X)$ (Θ_i) and $\alpha_3(M_Z)$ (Θ_s) change. From Table 2 it becomes clear that the absolute value of Θ_s is considerably smaller in the E₆SSM than in the MSSM while the Θ_i 's are a few times larger in the former than in the latter. One can also see that the corresponding two-loop corrections depend rather weakly on the values of the Yukawa couplings and are almost independent of the extra $U(1)_N$ gauge coupling. The dominant contribution to these corrections give $SU(2)_W$ and $SU(3)_C$ gauge couplings which are considerably larger in the E₆SSM as compared with the MSSM case. This explains the large difference between the contributions of two-loop corrections to $\alpha_i(M_X)$ in the E₆SSM and MSSM. Conversely this is also a reason why one may expect that the absolute value of the corresponding correction to $\alpha_3(M_Z)$ should be at least twice larger in the exceptional SUSY model than in the minimal one leading to the greater value of $\alpha_3(M_Z)$ at which exact gauge coupling unification takes place. But due to the remarkable cancellation of different two-loop corrections in Eq. (4), the absolute value of Θ_s is more than

three times smaller in the E₆SSM as compared with the MSSM (see Table 2). Such cancellation is caused by the structure of the two-loop corrections to the beta functions of the SM gauge couplings in the considered model. Because of the cancellation of two-loop contributions in Eq. (4) the prediction for $\alpha_3(M_Z)$ obtained in the E₆SSM is considerably lower than in the MSSM. It is quite close to the central value of the recent fit of experimental data even without the inclusion of threshold corrections, i.e. $\alpha_3(M_Z) \simeq 0.121$.

As in the MSSM the inclusion of threshold effects lowers the prediction for the value of the strong gauge coupling at the EW scale. The contribution of threshold corrections Δ_s to the value of $\alpha_3(M_Z)$, that results in the exact gauge coupling unification in the E₆SSM, can be parametrised in a manner which is very similar to what we had in the MSSM, i.e.

$$\tilde{\Delta}_s = -\frac{19}{28\pi} \ln \frac{\tilde{M}_S}{M_Z}, \quad \tilde{M}_S = \frac{\tilde{T}_2^{172/19}}{\tilde{T}_1^{55/19} \tilde{T}_3^{98/19}}. \quad (15)$$

In the limit when all squarks are degenerate, all sleptons are degenerate, all exotic quarks have the same mass μ_{D_i} and the masses of exotic squarks are universal ($m_{\tilde{D}_i}$) we find

$$\begin{aligned} \tilde{M}_S &= M_S \cdot \frac{m_{H_\alpha}^{12/19} \mu_{\tilde{H}_\alpha}^{24/19} \mu'^{18/19}}{m_{\tilde{D}_i}^{18/19} \mu_{D_i}^{36/19}} \\ &= \mu' \cdot \left(\frac{\mu}{\mu'}\right)^{1/19} \left(\frac{m_{H_\alpha}^{12/19} \mu_{\tilde{H}_\alpha}^{24/19} \mu^{18/19}}{m_{\tilde{D}_i}^{18/19} \mu_{D_i}^{36/19}}\right) \\ &\quad \times \left(\frac{m_A}{\mu}\right)^{3/19} \left(\frac{M_2}{\mu}\right)^{4/19} \left(\frac{M_2}{M_3}\right)^{28/19} \left(\frac{m_{\tilde{l}}}{m_{\tilde{q}}}\right)^{3/19} \\ &\simeq \mu' \cdot \left(\frac{M_{\text{weak}}}{M_{\text{colour}}}\right)^{4.5}. \end{aligned} \quad (16)$$

Here we also assume that non-Higgs fields of the first two generations have the same mass m_{H_α} and the masses of non-Higgsinos of the first and second generation are equal to $\mu_{\tilde{H}_\alpha}$ while the masses of scalar non-Higgs fields and their superpartners from H' and \tilde{H}' are degenerate around μ' . In Fig. 1(c)–(d) we keep the masses of all extra exotic particles appearing in the E₆SSM to be degenerate around 1.5 TeV. It means that $\tilde{M}_S = M_S$ in this particular case. However from Eq. (16) it is obvious that in contrast with the MSSM the effective threshold scale in the E₆SSM is set by μ' . The term $\mu' H' \tilde{H}'$ in the superpotential is not involved in the process of EW symmetry breaking. Therefore the parameter μ' remains arbitrary. Because the corresponding mass term is not suppressed by the E₆ symmetry the components of the doublet superfields H' and \tilde{H}' are expected to be heavy $\gtrsim 10$ TeV. As a consequence, although the effective threshold scale \tilde{M}_S may be considerably less than μ' , the corresponding mass parameter can be always chosen so that \tilde{M}_S lies in a few hundred GeV range that allows to get the exact unification of gauge couplings for any value of $\alpha_3(M_Z)$ which is in agreement with current data.

6. Conclusions

In this Letter we have presented the two-loop RGEs of the E₆SSM and examined gauge coupling unification in this model using both analytical and numerical techniques. We have seen that the running of the gauge couplings in the MSSM and E₆SSM are completely different. For example, in the E₆SSM, the strong gauge coupling grows with increasing renormalisation scale whereas in the MSSM it decreases at high energies. Therefore the interval of variation of $\alpha_3(M_X)$ caused by the uncertainty in $\alpha_3(M_Z)$ is considerably wider in the E₆SSM than in the MSSM. Because at any intermediate scale the gauge couplings in the E₆SSM are considerably larger, as compared to the ones in the MSSM, the two-loop corrections to $\alpha_i(M_X)$ are a few times bigger in the former than in the latter. At the same time the absolute value of the corresponding corrections to $\alpha_3(M_Z)$ at which exact gauge coupling unification takes place are much smaller in the E₆SSM than in the MSSM, as is demonstrated in Table 2. As a consequence the unification of gauge couplings in the E₆SSM can be achieved for significantly lower values of $\alpha_3(M_Z)$ than in the MSSM. The remarkable accidental cancellation of different two-loop contributions to the prediction for $\alpha_3(M_Z)$ is caused by the structure of the two-loop corrections to the beta functions of the gauge couplings in the E₆SSM. Thus the structure of the two-loop contributions to the beta functions of gauge couplings and large uncertainty in $\alpha_3(M_X)$ allow one to get exact unification of gauge couplings in the E₆SSM for values of $\alpha_3(M_Z)$ which are within one standard deviation of its measured central value.

Finally we emphasise that the effective threshold scale in the E₆SSM is set by the mass term of H' and \tilde{H}' from the extra 27' and $\overline{27}'$, which can in principle be very large, significantly enhancing the contribution of threshold corrections to the predictions for $\alpha_3(M_Z)$. Indeed, since the only purpose of the states H' and \tilde{H}' in this model is to achieve gauge coupling unification, their mass term can be arbitrarily adjusted to give exact gauge coupling unification in the E₆SSM for any phenomenologically reasonable value of $\alpha_3(M_Z)$. Future experimental measurements of the mass of these states, together with more accurate determinations of $\alpha_3(M_Z)$, will test the self-consistency of this framework.

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