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Microarticle

Potential relativistic dispersion in material medium

Md Rejwan Ali *, Mostafa Sadoqi

Department of Physics, St John's University, 8000 Utopia Parkway, Queens, NY 11439, United States

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ABSTRACT

Lorentz space-time transformation has been applied to the phase factor of a plane electromagnetic wave in linear material medium. The derivation shows in the limiting case for $v = c$, the phase velocity converges to its monochromatic value implying no such dispersion effect can exist in free space. However in linear material medium, wave speed may exceed the monochromatic phase velocity by a factor purely due to the relativistic consideration of the phase factor invariant under Lorentz transformation. The equation suggests such speed dispersion factor will be higher in the denser medium to its monochromatic material phase velocity. A critical cut-off number for the refractive index may exist to excite such mode in the material. The results can be interesting particularly for materials with high refractive index as well as for anisotropic space-time metric formulations in Transformation Optics.

Introduction

Since their inception for over 100 years now, Special and General Theory of Relativity have been the cornerstones of our scientific culture and endeavors to explain many counter intuitive phenomena contrast to classical thought in reasoning. Recent detection of Gravitational Wave will surely open up new frontiers of research based on the paradigm of General and Special Theory of Relativity. This experimental Tour de Force will bring the scientific community in broad and particularly its physics component back to the essence of these two monumental physical theories in scientific history. General Theory of Relativity is a grand universal picture of space-time, a geometry that may assume some intrinsic curvature due to presence of matter. Geodesic of light or massless photon is the natural choice to parameterize such space-time picture. Such definition of space time is very generalized and even can permeate materials as in recent application in Transformation Optics, space-time curvature is mimicked by the metric tensors related to medium magnetic and dielectric behavior. In such modeling of material refractive index has a key role in formulating light path as the geodesic of space-time [1,2].

Relativistic dispersion

However, in this brief report we consider non-inertial material medium in which space-time is flat and the fields, velocities and co-ordinates are transformed via the rules of Lorentz transformations. Initially we assume a plane electromagnetic wave propagating in a direction and represented by the electric field component **E**:

$$\vec{E} \exp i(kx - wt) \quad \text{(i)}$$

where the symbols have their standard meaning and the electric field has the associated transverse magnetic component.

For non-inertial isotropic linear medium the associated **D** and **H** fields are transformed as second rank 4-tensors under the Lorentz transformation and given by [3,4]:

$$\vec{D} = \varepsilon\vec{E} + \gamma^2\left(\varepsilon - \frac{1}{\mu}\right)\left[\beta^2\overrightarrow{E_\perp} + \vec{\beta} \times \vec{B}\right] \quad \text{(ii)}$$

$$\vec{H} = \frac{1}{\mu}\vec{B} + \gamma^2\left(\varepsilon - \frac{1}{\mu}\right)\left[\beta^2\overrightarrow{B_\perp} + \vec{\beta} \times \vec{E}\right] \quad \text{(iii)}$$

For the **E** field vibrating in a plane with its associated transverse magnetic field component, the magnetic contribution from the field transformation will be along the same plane of the electric field **E** resulting only in net linear change in **D** or an overall scaling of dielectric value of the medium to ε'. The magnetic field intensity will have similar linear change in its overall value of μ' coming from electric field component transformation. We can conclude that both for the electric and magnetic field no dispersion related factors are to be introduced due to the field transformations in the medium.

As the phase factor is an invariant quantity under the Lorentz transformation, a second Lorentz space-time transformation will

* Corresponding author. Fax: +1 718 990 5812.
E-mail address: alim@stjohns.edu (M.R. Ali).

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not alter the field representation by the wave equation in the medium, rather it will provide phase behavior information. The space-time phase factor of the wave transformed accordingly the Eq. (iv):

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -v\gamma/c & 0 & 0 \\ -v\gamma/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \quad \text{(iv)}$$

Now the phase factor under the above transformation becomes:

$$k\left(\frac{x - vt}{\sqrt{(1-\beta^2)}}\right) - \omega\left(t - \frac{t - \frac{vx}{c^2}}{\sqrt{(1-\beta^2)}}\right) \quad \text{(v)}$$

Regrouping space and time parts and substituting medium phase velocity as ω/k we have:

$$k\left(\frac{x}{\sqrt{(1-\beta^2)}} + \frac{\omega}{k}\frac{\frac{vx}{c^2}}{\sqrt{(1-\beta^2)}}\right) - \omega\left(\frac{t}{\sqrt{(1-\beta^2)}} + \frac{kvt}{\omega\sqrt{(1-\beta^2)}}\right)$$

which upon simplification gives:

$$kx\left(\frac{1+\beta^2}{\sqrt{(1-\beta^2)}}\right) - \omega t\left(\frac{2}{\sqrt{(1-\beta^2)}}\right) \quad \text{(vi)}$$

Writing the phase factor in the standard form we have

$k'x - \omega' t$ where

$$k' = k\left(\frac{1+\beta^2}{\sqrt{(1-\beta^2)}}\right) \text{ and } \omega' = \omega\left(\frac{2}{\sqrt{(1-\beta^2)}}\right) \quad \text{(vii)}$$

Finally we obtain an expression for the phase velocity in term of medium monochromatic velocity accordingly:

$$v = \frac{\omega}{k}\left(\frac{2}{1+\beta^2}\right) \quad \text{(viii)}$$

In the case of $v = c$, the factor in the equation reduces to one meaning such dispersed phase velocity component cannot exist in the free space. However, in the case for material medium the equation is likely to suggest a dispersive component that we can write in the form:

$$v = \frac{\omega}{k}\left(\frac{2n^2}{n^2+1}\right) \quad \text{(ix)}$$

where n = refractive index of the linear isotropic medium.

Conclusion

It would be interesting in view of experiment if such velocity really exists in a spatially isotropic material medium. In case for the charged particle in an inhomogeneous medium, the effect is non-inertial causing acceleration and such radiation has been detected for a phase component exceeding the speed of light in the medium [5]. However, even in such inhomogeneous medium, the upper equation can be the cut-off limit for any phase velocity component. Also there might be some lower cut-off values in refractive index regarding observing of such mode in material medium. However, materials with high refractive index are likely to carry such dispersion mode and will be of interest to detect. Such results can also be of interest to metric formulations in Transformation Optics.

References

[1] Leonhardt U, Philbin T. General relativity in electrical engineering. New J Phys 2006;8(10):247.
[2] Leonhardt U, Philbin T. Geometry and light: the science of invisibility. New York: Dover; 2010.
[3] Jackson JD. Classical electrodynamics. New York: Wiley; 1999.
[4] Landau D, Lifshitz EM, Pitaevskii LP. Electrodynamics of continuous media. Theoretical physics course, vol. 8. New York: Pergamon; 1984.
[5] Cerenkov PA. Visible radiation produced by electrons moving in a medium with velocities exceeding that of light. Phys Rev 1937;52:378–9.