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MSSM via Pati–Salam from intersecting branes on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$

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Abstract

We construct an MSSM-like model via Pati–Salam from intersecting D-branes in type IIA theory on the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orientifold where the D-branes wrap rigid 3-cycles. Because the 3-cycles are rigid, there are no extra massless fields in the adjoint representation, arising as open-string moduli. The presence of these unobserved fields would create difficulties with asymptotic freedom as well as the prediction of gauge unification. The model constructed has four generations of MSSM matter plus right-handed neutrinos, as well as additional vector-like representations. In addition, we find that all of the required Yukawa couplings are allowed by global symmetries which arise from $U(1)$'s which become massive via a generalized Green–Schwarz mechanism. Furthermore, we find that the tree-level gauge couplings are unified at the string scale.

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1. Introduction

At the present, string theory is the only framework for realization of a unification of gravitation with gauge theory and quantum mechanics. In principle, it should be possible to derive all known physics from the string, as well as potentially provide something new and unexpected. This is the goal of string phenomenology. However, in spite of this there exist many solutions that may be derived from string, all of which are consistent vacua. One of these vacua should correspond to our universe, but then the question becomes why this particular vacuum is selected. One possible approach to this state of affairs is to statistically classify the possible vacua, in essence making a topographical map of the ‘landscape’. One then attempts to assess the likelihood that vacua with properties similar to ours will arise.¹ Another approach is to take the point of view that there are unknown dynamics, perhaps involving a departure from criticality, which determine the vacuum that corresponds to our universe. Regardless of the question of uniqueness, if string theory is correct then it should be possible to find a solution which corresponds *exactly* to our universe, at least in its low energy limit. Although there has been a great deal of progress in constructing semi-realistic models, this has not yet been achieved.

An elegant approach to model construction involving type I orientifold (type II) compactifications is where chiral fermions arise from strings stretching between D-branes intersecting at angles (type IIA picture) [7] or in its T-dual (type IIB) picture with magnetized D-branes [8]. Many consistent standard-like and grand unified theory (GUT) models have been constructed [9–12] using D-brane constructions. The first quasi-realistic supersymmetric models were constructed in type IIA theory on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [13,14]. Following this, models with standard-like, left–right symmetric (Pati–Salam), unflipped $SU(5)$ gauge groups

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were constructed based upon the same framework and systematically studied [15–18]. In addition, several different flipped $SU(5)$ [19–21] models have also been built using intersecting D-brane constructions [12,22–26].²

Although much progress has been made, none of these models have been completely satisfactory. Problems include extra chiral and non-chiral matter, and the lack of a complete set of Yukawa couplings, which are typically forbidden by global symmetries. In addition to the chiral matter which arises at brane intersections, D-brane constructions typically will have non-chiral open string states present in the low-energy spectrum associated with the D-brane position in the internal space and Wilson lines. This results in adjoint or additional matter in the symmetric and antisymmetric representations unless the open string moduli are completely frozen. These light scalars are not observed and are not present in the MSSM. While it is possible that these moduli will obtain mass after supersymmetry is broken, it would typically be of the TeV scale. While this would make them unobservable in present experiments, the successful gauge unification in the MSSM would be spoiled by their presence. While it may be possible to find some scenarios where the problems created by these fields are ameliorated, it is much simpler to eliminate these fields altogether. One way to do this is to construct intersecting D-brane models where the D-branes wrap rigid cycles.³ Another motivation for the absence of these adjoint states is that this is consistent with a $k = 1$ Kac–Moody algebra in models constructed from heterotic string, some of which may be dual.

In this Letter, we construct an intersecting D-brane model on the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orientifold background, also known as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with discrete torsion, where the D-branes wrap rigid cycles thus eliminating the extra adjoint matter. This Letter is organized as follows: First, we briefly review intersecting D-brane constructions on the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orientifold. We then proceed to construct a supersymmetric four-generation MSSM-like model obtained from a Pati–Salam model via spontaneous gauge symmetry breaking. All of the required Yukawa couplings are allowed by global symmetries present in the model. We find that the tree-level gauge couplings are unified at the string scale.

2. Intersecting branes on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with and without discrete torsion

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold has been the subject of extensive research, primarily because it is the simplest background space which allows supersymmetric vacua. We will essentially follow along with the development given in [30]. The first supersymmetric models based upon the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold were explored in [13–16]. In type IIA theory on the $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, the \mathbf{T}^6 is product of three two-tori and the two orbifold group generators θ , ω act on the complex coordinates (z_1, z_2, z_3) as

$$\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3), \quad \omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3) \quad (1)$$

while the antiholomorphic involution R acts as

$$R(z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3). \quad (2)$$

As it stands, the signs of the θ action in the ω sector and vice versa have not been specified, and the freedom to do so is referred to as the choice of discrete torsion. One choice of discrete torsion corresponds to the Hodge numbers $(h_{11}, h_{21}) = (3, 51)$ and the corresponding to $(h_{11}, h_{21}) = (51, 3)$. These two different choices are referred to as with discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}'_2$) and without discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}_2$), respectively. To date, most phenomenological models that have been constructed have been without discrete torsion. Consequently, all of these models have massless adjoint matter present since the D-branes do not wrap rigid 3-cycles. However, in the case of $\mathbb{Z}_2 \times \mathbb{Z}'_2$, the twisted homology contains collapsed 3-cycles, which allows for the construction of rigid 3-cycles.

D6-branes wrapping cycles are specified by their wrapping numbers (n^i, m^i) along the fundamental cycles $[a^i]$ and $[b^i]$ on each torus. However, cycles on the torus are, in general, different from the cycles defined on the orbifold space. In the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, all of the 3-cycles on the orbifold are inherited from the torus, which makes it particularly easy to work with. The $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orientifold contains 16 fixed points, from which arise 16 additional 2-cycles with the topology of $\mathbf{P}^1 \cong S^2$. As a result, there are 32 collapsed 3-cycles for each twisted sector. A D6-brane wrapping collapsed 3-cycles in each of the three twisted sectors will be unable to move away from a particular position on the covering space \mathbf{T}^6 , which means that the 3-cycle will be rigid.

A basis of twisted 3-cycles may be denoted as

$$[\alpha_{ij,n}^\theta] = 2[\epsilon_{ij}^\theta] \otimes [a^3], \quad [\alpha_{ij,m}^\theta] = 2[\epsilon_{ij}^\theta] \otimes [b^3], \quad (3)$$

$$[\alpha_{ij,n}^\omega] = 2[\epsilon_{ij}^\omega] \otimes [a^1], \quad [\alpha_{ij,m}^\omega] = 2[\epsilon_{ij}^\omega] \otimes [b^1], \quad (4)$$

$$[\alpha_{ij,n}^{\theta\omega}] = 2[\epsilon_{ij}^{\theta\omega}] \otimes [a^2], \quad [\alpha_{ij,m}^{\theta\omega}] = 2[\epsilon_{ij}^{\theta\omega}] \otimes [b^2], \quad (5)$$

where $[\epsilon_{ij}^\theta]$, $[\epsilon_{ij}^\omega]$, and $[\epsilon_{ij}^{\theta\omega}]$ denote the 16 fixed points on $\mathbf{T}^2 \times \mathbf{T}^2$, where $i, j \in 1, 2, 3, 4$.

² For excellent reviews, see [27] and [28].

³ This possibility was first explored in [29] and [30].

A fractional D-brane wrapping both a bulk cycle as well as the collapsed cycles may be written in the form

$$\Pi_a^F = \frac{1}{4}\Pi^B + \frac{1}{4}\left(\sum_{i,j \in S_\theta^a} \epsilon_{a,ij}^\theta \Pi_{ij,a}^\theta\right) + \frac{1}{4}\left(\sum_{j,k \in S_\omega^a} \epsilon_{a,jk}^\omega \Pi_{jk,a}^\omega\right) + \frac{1}{4}\left(\sum_{i,k \in S_{\theta\omega}^a} \epsilon_{a,ik}^{\theta\omega} \Pi_{ik,a}^{\theta\omega}\right), \tag{6}$$

where the D6-brane is required to run through the four fixed points for each of the twisted sectors. The set of four fixed points may be denoted as S^g for the twisted sector g . The constants $\epsilon_{a,ij}^\theta$, $\epsilon_{a,jk}^\omega$ and $\epsilon_{a,ki}^{\theta\omega}$ denote the sign of the charge of the fractional brane with respect to the fields which are present at the orbifold fixed points. These signs, as well as the set of fixed points, must satisfy consistency conditions. However, they may be chosen differently for each stack.

A bulk cycle on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold space consists of the toroidal cycle wrapped by the brane D_a and its three orbifold images:

$$[\Pi_a^B] = (1 + \theta + \omega + \theta\omega)\Pi_a^{T^6}. \tag{7}$$

Each of these orbifold images is homologically identical to the original cycle, thus

$$[\Pi_a^B] = 4[\Pi_a^{T^6}]. \tag{8}$$

If we calculate the intersection number between two branes, we will find

$$[\Pi_a^B] \circ [\Pi_b^B] = 4[\Pi_a^{T^6}] \circ [\Pi_b^{T^6}] \tag{9}$$

which indicates that the bulk cycles $[\Pi_a^B]$ do not expand a unimodular basis for the homology lattice $H_3(M, \mathbb{Z})$. Thus, we must normalize these purely bulk cycles as $[\Pi_a^o] = \frac{1}{2}[\Pi_a^B]$ [27,30]. So, in terms of the cycles defined on the torus, the normalized purely bulk cycles of the orbifold are given by

$$[\Pi_a^o] = \frac{1}{2}(1 + \theta + \omega + \theta\omega)[\Pi_a^{T^6}] = 2[\Pi_a^{T^6}]. \tag{10}$$

Due to this normalization, a stack of N D6-branes wrapping a purely bulk cycle will have a $U(N/2)$ gauge group in its world-volume. However, this does not apply to a brane wrapping collapsed cycles, so that a stack of N branes wrapping fractional cycles as in Eq. (6) will have in its world-volume a gauge group $U(N)$.

Since we will have D6-branes which are wrapping fractional cycles with a bulk component as well as twisted cycles, we will need to be able to calculate the intersection numbers between pairs of twisted 3-cycles. For the intersection number between two twisted 3-cycles of the form $[\Pi_{ij,a}^g] = n_a^{I_g}[\alpha_{ij,n}] + m_a^{I_g}[\alpha_{ij,m}]$ and $[\Pi_{kl,b}^h] = n_b^{I_h}[\alpha_{kl,n}] + m_b^{I_h}[\alpha_{kl,m}]$ we have

$$[\Pi_{ij,a}^g] \circ [\Pi_{kl,b}^h] = 4\delta_{ik}\delta_{jl}\delta^{gh}(n_a^{I_g}m_b^{I_g} - m_a^{I_g}n_b^{I_g}), \tag{11}$$

where I_g corresponds to the torus left invariant by the action of the orbifold generator g ; specifically $I_\theta = 3$, $I_\omega = 1$, and $I_{\theta\omega} = 2$.

Putting everything together, we will find for the intersection number between a brane a and brane b wrapping fractional cycles we will have

$$\begin{aligned} \Pi_a^F \circ \Pi_b^F &= \frac{1}{16}\left[\Pi_a^B \circ \Pi_b^B + 4(n_a^3 m_b^3 - m_a^3 n_b^3) \sum_{i_a j_a \in S_\theta^a} \sum_{i_b j_b \in S_\theta^b} \epsilon_{a,i_a j_a}^\theta \epsilon_{b,i_b j_b}^\theta \delta_{i_a i_b} \delta_{j_a j_b} \right. \\ &\quad + 4(n_a^1 m_b^1 - m_a^1 n_b^1) \sum_{j_a k_a \in S_\omega^a} \sum_{j_b k_b \in S_\omega^b} \epsilon_{a,j_a k_a}^\omega \epsilon_{b,j_b k_b}^\omega \delta_{j_a j_b} \delta_{k_a k_b} \\ &\quad \left. + 4(n_a^2 m_b^2 - m_a^2 n_b^2) \sum_{i_a k_a \in S_{\theta\omega}^a} \sum_{i_b k_b \in S_{\theta\omega}^b} \epsilon_{a,i_a k_a}^{\theta\omega} \epsilon_{b,i_b k_b}^{\theta\omega} \delta_{i_a i_b} \delta_{k_a k_b} \right]. \tag{12} \end{aligned}$$

The 3-cycle wrapped by the O6-planes is given by

$$2q_{\Omega R}[a^1][a^2][a^3] - 2q_{\Omega R\theta}[b^1][b^2][a^3] - 2q_{\Omega R\omega}[a^1][b^2][b^3] - 2q_{\Omega R\theta\omega}[b^1][a^2][b^3], \tag{13}$$

where the cross-cap charges $q_{\Omega Rg}$ give the RR charge and tension of a given orientifold plane g , of which there are two types, $O6^{(-,-)}$ and $O6^{(+,+)}$. In this case, $q_{\Omega Rg} = +1$ indicates an $O6^{(-,-)}$ plane, while $q_{\Omega Rg} = -1$ indicates an $O6^{(+,+)}$ while the choice of discrete torsion is indicated by the product

$$q = \prod_g q_{\Omega Rg}. \tag{14}$$

The choice of no discrete torsion is given by $q = 1$, while for $q = -1$ is the case of discrete torsion, for which an odd number of $O^{(+,+)}$ must be present.

Table 1
Net chiral matter spectrum in terms of three-cycles

Representation	Multiplicity
\square	$\frac{1}{2}([\Pi_a^o] \circ [\Pi_a^o] + [\Pi_{O6}] \circ [\Pi_a^o])$
\square	$\frac{1}{2}([\Pi_a^o] \circ [\Pi_a^o] - [\Pi_{O6}] \circ [\Pi_a^o])$
(\square_a, \square_b)	$[\Pi_a^o] \circ [\Pi_b^o]$
(\square_a, \square_b)	$[\Pi_a^o] \circ [\Pi_b^o]$

The action of ΩR on the bulk cycles is the same in either case, and is essentially just changes the signs of the wrapping numbers as $n_a^i \rightarrow n_a^i$ and $m_a^i \rightarrow -m_a^i$. However, in addition, there is an action on the twisted 3-cycle as

$$\alpha_{ij,n}^g \rightarrow -q_{\Omega R} q_{\Omega R g} \alpha_{ij,n}^g, \quad \alpha_{ij,m}^g \rightarrow q_{\Omega R} q_{\Omega R g} \alpha_{ij,m}^g. \quad (15)$$

Using these relations, one can work out the intersection number of a fractional cycle with it's ΩR image, we have

$$\Pi_a^F \circ \Pi_a^F = q_{\Omega R} \left(2q_{\Omega R} \prod_I n_a^I m_a^I - 2q_{\Omega R \theta} n_a^3 m_a^3 - 2q_{\Omega R \omega} n_a^1 m_a^1 - 2q_{\Omega R \theta \omega} n_a^2 m_a^2 \right) \quad (16)$$

while the intersection number with the orientifold planes is given by

$$\Pi_{O6} \circ \Pi_a^F = 2q_{\Omega R} \prod_I m_a^I - 2q_{\Omega R \theta} n_a^1 n_a^2 m_a^3 - 2q_{\Omega R \omega} m_a^1 n_a^2 n_a^3 - 2q_{\Omega R \theta \omega} n_a^1 m_a^2 n_a^3. \quad (17)$$

A generic expression for the *net* number of chiral fermions in bifundamental, symmetric, and antisymmetric representations consistent with the vanishing of RR tadpoles can be given in terms of the three-cycles [31] which is shown in Table 1.

3. Consistency and SUSY conditions

Certain conditions must be applied to construct consistent, supersymmetric vacua which are free of anomalies, which we discuss in the following sections.

3.1. RR and torsion charge cancellation

With the choice of discrete torsion $q_{\Omega R} = -1$, $q_{\Omega R \theta} = q_{\Omega R \omega} = q_{\Omega R \theta \omega} = 1$, the condition for the cancellation of RR tadpoles becomes

$$\sum N_a n_a^1 n_a^2 n_a^3 = -16, \quad \sum N_a m_a^1 m_a^2 n_a^3 = -16, \quad \sum N_a m_a^1 n_a^2 m_a^3 = -16, \quad \sum N_a n_a^1 m_a^2 m_a^3 = -16 \quad (18)$$

whilst for the twisted charges to cancel, we require

$$\sum_{a,ij \in S^\omega} N_a n_a^1 \epsilon_{ij,a}^\omega = 0, \quad \sum_{a,jk \in S^{\theta\omega}} N_a n_a^2 \epsilon_{jk,a}^{\theta\omega} = 0, \quad \sum_{a,ki \in S^\theta} N_a n_a^3 \epsilon_{ki,a}^\theta = 0, \quad (19)$$

where the sum is over *each* each fixed point $[e_{ij}^g]$. As stated in Section 2, the signs $\epsilon_{ij,a}^\theta$, $\epsilon_{jk,a}^\omega$, and $\epsilon_{ki,a}^{\theta\omega}$ are not arbitrary as they must satisfy certain consistency conditions. In particular, they must satisfy the condition

$$\sum_{ij \in S^g} \epsilon_{a,ij}^g = 0 \pmod{4} \quad (20)$$

for each twisted sector. Additionally, the signs for different twisted sectors must satisfy

$$\epsilon_{a,ij}^\theta \epsilon_{a,jk}^\omega \epsilon_{a,ik}^{\theta\omega} = 1, \quad \epsilon_{a,ij}^\theta \epsilon_{a,jk}^\omega = \text{const} \quad \forall j. \quad (21)$$

Note that we may choose the set of signs differently for each stack provided that they satisfy the consistency conditions. A trivial choice of signs which satisfies the constraints placed on them is just to have them all set to +1,

$$\epsilon_{a,ij}^\theta = 1 \quad \forall ij, \quad \epsilon_{a,jk}^\omega = 1 \quad \forall jk, \quad \epsilon_{a,ki}^{\theta\omega} = 1 \quad \forall ki. \quad (22)$$

Another possible non-trivial choice of signs consistent with the constraints is given by

$$\epsilon_{a,ij}^\theta = -1 \quad \forall ij, \quad \epsilon_{a,jk}^\omega = -1 \quad \forall jk, \quad \epsilon_{a,ki}^{\theta\omega} = 1 \quad \forall ki. \quad (23)$$

Note that a fractional three cycle is invariant under the transformations

$$(n_a^1, m_a^1) \otimes (n_a^2, m_a^2) \otimes (n_a^3, m_a^3) \Rightarrow (-n_a^1, -m_a^1) \otimes (n_a^2, m_a^2) \otimes (-n_a^3, -m_a^3), \quad \epsilon_{a,ij}^\theta \Rightarrow -\epsilon_{a,ij}^\theta, \quad \epsilon_{a,jk}^\omega \Rightarrow -\epsilon_{a,jk}^\omega. \quad (24)$$

More general sets of these signs may be found in [30].

3.2. Conditions for preserving $N = 1$ supersymmetry

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [7,13]. Essentially, this becomes a constraint on the angles made by each stack of branes with respect to the orientifold planes, viz. $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \pmod{2\pi}$, or equivalently $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1$. Applying simple trigonometry, these angles may be expressed in terms of the wrapping numbers as

$$\tan \theta_a^i = \frac{m_a^i R_2^i}{n_a^i R_1^i}, \quad (25)$$

where R_2^i and R_1^i are the radii of the i th torus. We may translate these conditions into restrictions on the wrapping numbers as

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0, \quad A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0, \quad (26)$$

where we have made the definitions

$$\tilde{A}_a = -m_a^1 m_a^2 m_a^3, \quad \tilde{B}_a = n_a^1 n_a^2 m_a^3, \quad \tilde{C}_a = m_a^1 n_a^2 n_a^3, \quad \tilde{D}_a = n_a^1 m_a^2 n_a^3, \quad (27)$$

$$A_a = -n_a^1 n_a^2 n_a^3, \quad B_a = m_a^1 m_a^1 n_a^3, \quad C_a = n_a^1 m_a^1 m_a^3, \quad D_a = m_a^1 n_a^1 m_a^3, \quad (28)$$

and the structure parameters related to the complex structure moduli are

$$x_a = \lambda, \quad x_b = \frac{\lambda}{\chi_2 \cdot \chi_3}, \quad x_c = \frac{\lambda}{\chi_1 \cdot \chi_3}, \quad \frac{\lambda}{\chi_1 \cdot \chi_2}, \quad (29)$$

where λ is a positive constant. One may invert the above expressions to find values for the complex structure moduli as

$$\chi_1 = \lambda, \quad \chi_2 = \frac{x_c}{x_b} \cdot \chi_1, \quad \chi_3 = \frac{x_d}{x_b} \cdot \chi_1. \quad (30)$$

3.3. The Green–Schwarz mechanism

Although the total non-Abelian anomaly cancels automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero [13]. These anomalies are cancelled by a generalized Green–Schwarz (G–S) mechanism which involves untwisted Ramond–Ramond forms. Integrating the G–S couplings of the untwisted RR forms to the $U(1)$ field strength F_a over the untwisted cycles of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold, we find

$$\int_{D6_a^{\text{untw}}} C_5 \wedge \text{tr } F_a \sim N_a \sum_i r_{ai} \int_{M_4} B_2^i \wedge \text{tr } F_a, \quad (31)$$

where

$$B_2^i = \int_{[\Sigma_i]} C_5, \quad [H_a] = \sum_{i=1}^{b_3} r_{ai} [\Sigma_i], \quad (32)$$

and $[\Sigma_i]$ is the basis of homology 3-cycles, $b_3 = 8$. Under orientifold action only half survive. In other words, $\{r_{ai}\} = \{\tilde{B}_a, \tilde{C}_a, \tilde{D}_a, \tilde{A}_a\}$ in this definition. Thus the couplings of the four untwisted RR forms B_2^i to the $U(1)$ field strength F_a are [10]

$$N_a \tilde{B}_a \int_{M_4} B_2^1 \wedge \text{tr } F_a, \quad N_a \tilde{C}_a \int_{M_4} B_2^2 \wedge \text{tr } F_a, \quad N_a \tilde{D}_a \int_{M_4} B_2^3 \wedge \text{tr } F_a, \quad N_a \tilde{A}_a \int_{M_4} B_2^4 \wedge \text{tr } F_a. \quad (33)$$

Besides the contribution to G–S mechanism from untwisted 3-cycles, the contribution from the twisted cycles should be taken into account. As in the untwisted case we integrate the Chern–Simons coupling over the exceptional 3-cycles from the twisted sector. We choose the sizes of the 2-cycles on the topology of S^2 on the orbifold singularities to make the integrals on equal foot to those from the untwisted sector. Consider the twisted sector θ as an example,

$$\int_{D6_a^{\text{tw},\theta}} C_5 \wedge \text{tr } F_a \sim N_a \sum_{i,j \in S_\theta^a} \epsilon_{a,ij}^\theta m_a^3 \int_{M_4} B_2^{\theta ij} \wedge \text{tr } F_a, \quad (34)$$

where $B_2^{\theta ij} = \int_{[\alpha_{ij,m}^\theta]} C_5$, with orientifold action taken again. Although i, j can run through each run through $\{1-4\}$ for each of the four fixed points in each sector, these are constrained by the wrapping numbers from the untwisted sector so that only four

possibilities remain. A similar argument may be applied for ω and $\theta\omega$ twisted sectors:

$$\int_{D6_a^{tw,\omega}} C_5 \wedge \text{tr} F_a \sim N_a \sum_{j,k \in S_\omega^a} \epsilon_{a,jk}^\omega m_a^1 \int_{M_4} B_2^{\omega jk} \wedge \text{tr} F_a, \quad (35)$$

$$\int_{D6_a^{tw,\theta\omega}} C_5 \wedge \text{tr} F_a \sim N_a \sum_{i,j \in S_{\theta\omega}^a} \epsilon_{a,ik}^{\theta\omega} m_a^2 \int_{M_4} B_2^{\theta\omega ik} \wedge \text{tr} F_a. \quad (36)$$

In summary, there are twelve additional couplings of the Ramond–Ramond 2-forms B_2^i to the $U(1)$ field strength F_a from the twisted cycles, giving rise to massive $U(1)$'s. However from the consistency condition of the ϵ 's (see Section 3.1) related to the discrete Wilson lines they may be dependent or degenerate. So even including the couplings from the untwisted sector we still have an opportunity to find a linear combination for a massless $U(1)$ group. Let us write down these couplings of the twisted sector explicitly:

$$N_a \epsilon_{a,ij}^\theta m_a^3 \int_{M_4} B_2^{\theta ij} \wedge \text{tr} F_a, \quad N_a \epsilon_{a,jk}^\omega m_a^1 \int_{M_4} B_2^{\omega jk} \wedge \text{tr} F_a, \quad N_a \epsilon_{a,ik}^{\theta\omega} m_a^2 \int_{M_4} B_2^{\theta\omega ik} \wedge \text{tr} F_a. \quad (37)$$

Checking the mixed cubic anomaly by introducing the dual field of B_2^i in the diagram, we can find the contribution from both untwisted and twisted sectors having a intersection number form and which is cancelled by the RR-tadpole conditions mentioned. These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G–S mechanism. Thus, in constructing MSSM-like models, we must ensure that the gauge boson of the hypercharge $U(1)_Y$ group does not receive such a mass. In general, the hypercharge is a linear combination of the various $U(1)$ s generated from each stack:

$$U(1)_Y = \sum_a c_a U(1)_a. \quad (38)$$

The corresponding field strength must be orthogonal to those that acquire G–S mass. Thus we demand

$$\sum_a c_a N_a \epsilon_{a,jk}^\omega m_a^1 = 0, \quad \sum_a c_a N_a \epsilon_{a,ki}^{\theta\omega} m_a^2 = 0, \quad \sum_a c_a N_a \epsilon_{a,ij}^\theta m_a^3 = 0, \quad (39)$$

for the twisted couplings as well as

$$\sum_a c_a N_a \tilde{A}_a = 0, \quad \sum_a c_a N_a \tilde{B}_a = 0, \quad \sum_a c_a N_a \tilde{C}_a = 0, \quad \sum_a c_a N_a \tilde{D}_a = 0, \quad (40)$$

for the untwisted.

3.4. K-theory constraints

RR charges are not fully classified by homological data, but rather by K-theory. Thus, to cancel all charges including those visible by K-theory alone, we require the wrapping numbers to satisfy certain constraints. We will not state these constraints here, but we will refer the reader to [30] where they are given explicitly.

4. MSSM via Pati–Salam

We begin with the seven-stack configuration of D-branes with the bulk wrapping numbers shown in Table 2, which produce the intersection numbers shown in Tables 3–4. We make the choice of cross-cap charges $q_{\Omega R} = -1$, $q_{\Omega R\theta} = q_{\Omega R\omega} = q_{R\theta\omega} = 1$, and assume for simplicity that each stack passes through the same set of fixed points. The resulting gauge group is that of a four generation Pati–Salam model. The ‘observable’ matter spectrum is presented in Table 5.

For Pati–Salam models constructed from bulk D-branes wrapping non-rigid cycles, the gauge symmetry may be broken to the MSSM by the process of brane splitting, which corresponds to assigning a VEV to an adjoint scalar in the field theoretic description. However, this option is not available in the present construction since the adjoint fields have been eliminated due to the rigidization of the cycles.

Although the adjoint fields have been eliminated by splitting the bulk D-branes into their fractional constituents, light non-chiral matter in the bifundamental representation may still appear between pairs of fractional branes [30]. These non-chiral states smoothly connect the configuration of fractional D-branes to one consisting of non-rigid D-branes. In the present case, all of the fractional D-branes are wrapping bulk cycles which are homologically identical, but differ in their twisted cycles. As discussed in [30], one

Table 2

Stacks, wrapping numbers, and torsion charges for a Pati–Salam model. With the choice of structure parameters $x_a = \sqrt{3}$, $x_b = x_c = x_d = \sqrt{3}/3$, $N = 1$ SUSY will be preserved. The cycles wrapped by each of the stacks pass through the same set of fixed points

Stack	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	$\epsilon_{ij}^\theta \forall ij$	$\epsilon_{jk}^\omega \forall jk$	$\epsilon_{kl}^{\theta\omega} \forall kl$
α	4	(-1, -1)	(-1, -1)	(-1, -1)	-1	-1	1
β	2	(-1, -1)	(-1, -1)	(-1, -1)	1	1	1
γ	2	(1, 1)	(1, 1)	(-1, -1)	1	1	1
1	2	(1, 1)	(1, 1)	(-1, -1)	-1	-1	1
2	2	(-1, -1)	(1, 1)	(1, 1)	-1	-1	1
3	2	(1, -1)	(1, -1)	(-1, 1)	-1	-1	1
4	2	(1, -1)	(-1, 1)	(1, -1)	-1	-1	1

Table 3

Intersection numbers between different stacks giving rise to fermions in the bifundamental representation. The resulting gauge group and chiral matter content is that of a four-generation Pati–Salam model

	α	β	γ	1	2	3	4	α'	β'	γ'	1'	2'	3'	4'
α	0	0	0	0	0	0	0	-8	4	-4	0	0	0	0
β	-	0	0	0	0	-4	-4	0	-8	0	0	0	0	0
γ	-	-	0	0	0	4	-4	0	0	-8	0	0	0	0
1	-	-	-	0	0	-8	0	0	-4	4	-8	0	0	0
2	-	-	-	-	0	0	0	0	-4	-4	0	-8	0	0
3	-	-	-	-	-	0	0	0	0	0	0	0	8	0
4	-	-	-	-	-	-	0	0	0	0	0	0	0	8

Table 4

Intersection numbers between different stacks and their images giving rise to antisymmetric and symmetric representations for a Pati–Salam model

Stack	Antisymmetric		Symmetric
α	8		0
β	8		0
γ	8		0
1	8		0
2	8		0
3	-8		0
4	-8		0

Table 5

The ‘observable’ spectrum of $SU(4) \times SU(2)_L \times SU(2)_R \times [U(2)^4 \times U(1)^3]$. The $\star'd$ representations indicate light, non-chiral matter which is present between pairs of fractional branes which wrap homologically identical bulk cycles, but differ in their twisted cycles

Rep.	Multi.	$U(1)_\alpha$	$U(1)_\beta$	$U(1)_\gamma$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	Field
$(\mathbf{4}_{\alpha'}, \mathbf{2}_\gamma)$	4	1	0	1	0	0	0	0	Matter
$(\bar{\mathbf{4}}_{\alpha'}, \mathbf{2}_\beta)$	4	-1	-1	0	0	0	0	0	Matter
$(\mathbf{2}_\beta, \mathbf{2}_\gamma)^\star$	-	0	1	-1	0	0	0	0	EW Higgs
$(\bar{\mathbf{4}}_\alpha, \mathbf{2}_\gamma)^\star$	-	-1	0	1	0	0	0	0	GUT Higgs
$(\mathbf{4}_\alpha, \bar{\mathbf{2}}_\beta)^\star$	-	1	-1	0	0	0	0	0	GUT Higgs
$(\mathbf{6}_{\alpha'\alpha})$	8	2	0	0	0	0	0	0	-
$(\mathbf{1}_{\beta'\beta})$	8	0	2	0	0	0	0	0	$\phi_{\beta\beta}$
$(\mathbf{1}_{\gamma'\gamma})$	8	0	0	2	0	0	0	0	$\phi_{\gamma\gamma}$

may compute the overlap between two such boundary states:

$$\tilde{A}_{a_i a_j} = \int_0^\infty dl \langle a_i | e^{-2\pi l H_{cl}} | a_j \rangle + \int_0^\infty dl \langle a_j | e^{-2\pi l H_{cl}} | a_i \rangle. \tag{41}$$

Due to the different signs for the twisted sector, it is found that in the loop channel amplitude

$$A_{a_i a_j} = \int_0^\infty \frac{dl}{l} \text{Tr}_{ij+ji} \left(\frac{1 + \theta + \omega + \theta\omega}{4} e^{-2\pi l H_{cl}} \right) \tag{42}$$

one massless hypermultiplet appears. Thus, the required states to play the role of the Higgs fields are present in this non-chiral sector.

In principle, one should determine that there are flat directions that can give the necessary VEV's to these states. This process would correspond geometrically to a particular brane recombination, where the CFT techniques fail and only a field theory analysis of D- and F-flat directions is applicable. For instance, a configuration of fractional branes in which one of these states receives a VEV should smoothly connect this configuration to one in which there is a stack of bulk D-branes wrapping a non-rigid cycle that has been split by assigning a VEV to an adjoint scalar. Such computations are technically very involved and beyond the scope of the present work, and we defer this for later work.

In Tables 6–9, we present an MSSM model which is obtained from the above Pati–Salam model by separating the stacks as

$$\alpha \rightarrow \alpha_B + \alpha_L, \quad \beta \rightarrow \beta_{r1} + \beta_{r2}. \tag{43}$$

This does not mean that the stacks are located at different positions in the internal space. After-all, there are no adjoint scalars which may receive a VEV. Rather, this separation reflects that there has been a spontaneous breaking of the Pati–Salam gauge symmetry down to the MSSM by the Higgs mechanism, where we have identified the Higgs states with $(\mathbf{4}, \mathbf{2}, 1)$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ representations of $SU(4) \times SU(2)_L \times SU(2)_R$ present in the non-chiral sector. The resulting gauge group of the model is then given by $SU(3) \times SU(2)_L \times U(1)_Y \times SU(2)^4 \times U(1)^8$, and the MSSM hypercharge is found to be

$$Q_Y = \frac{1}{6}(U(1)_{\alpha_B} - 3U(1)_{\alpha_L} - 3U(1)_{\beta_{r1}} + 3U(1)_{\beta_{r2}}). \tag{44}$$

Table 6
Stacks, wrapping numbers, and torsion charges for an MSSM-like model. The three-cycles wrapped by each of the stacks pass through the same set of fixed points

Stack	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	$\epsilon_{ij}^\theta \forall ij$	$\epsilon_{jk}^\omega \forall jk$	$\epsilon_{ki}^{\theta\omega} \forall kl$
α_B	3	(-1, -1)	(-1, -1)	(-1, -1)	-1	-1	1
α_L	1	(-1, -1)	(-1, -1)	(-1, -1)	-1	-1	1
β_{r1}	1	(-1, -1)	(-1, -1)	(-1, -1)	1	1	1
β_{r2}	1	(-1, -1)	(-1, -1)	(-1, -1)	1	1	1
γ	2	(1, 1)	(1, 1)	(-1, -1)	1	1	1
1	2	(1, 1)	(1, 1)	(-1, -1)	-1	-1	1
2	2	(-1, -1)	(1, 1)	(1, 1)	-1	-1	1
3	2	(1, -1)	(1, -1)	(-1, 1)	-1	-1	1
4	2	(1, -1)	(-1, 1)	(1, -1)	-1	-1	1

Table 7
Intersection numbers between different stacks giving rise to fermions in the bifundamental representation. The resulting gauge group and chiral matter content is that of a four-generation MSSM-like model

	α_B	α_L	β_{r1}	β_{r2}	γ	1	2	3	4	α'_B	α'_L	β'_{r1}	β'_{r2}	γ'	1'	2'	3'	4'
α_B	0	0	0	0	0	0	0	0	0	-8	-8	4	4	-4	0	0	0	0
α_L	-	0	0	0	0	0	0	0	0	0	-8	4	4	-4	0	0	0	0
β_{r1}	-	-	0	0	0	0	0	-4	-4	0	0	-8	-8	0	0	0	0	0
β_{r2}	-	-	-	0	0	0	0	-4	-4	0	0	0	-8	0	0	0	0	0
γ	-	-	-	-	0	0	0	4	-4	0	0	0	0	-8	0	0	0	0
1	-	-	-	-	-	0	0	-8	0	0	0	-4	-4	4	-8	0	0	0
2	-	-	-	-	-	-	0	0	0	0	0	-4	-4	-4	0	-8	0	0
3	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	8	0
4	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	8

Table 8
Intersection numbers between different stacks and their images giving rise to antisymmetric and symmetric representations for an MSSM-like model

Stack	Antisymmetric	Symmetric
α_B	8	0
α_L	8	0
β_{r1}	8	0
β_{r2}	8	0
γ	8	0
1	8	0
2	8	0
3	-8	0
4	-8	0

Table 9

The ‘observable’ spectrum of $[SU(3) \times SU(2)_L \times U(1)_Y] \times U(2)^4 \times U(1)^4$. The \star/d representations indicate light, non-chiral matter which exist between pairs of fractional branes which wrap identical bulk cycles, but differ in their twisted cycles

Rep.	Multi.	$U(1)_{\alpha_B}$	$U(1)_{\alpha_L}$	$U(1)_{\beta_{r1}}$	$U(1)_{\beta_{r2}}$	$U(1)_\gamma$	Q_Y	$U(1)_A$	$U(1)_B$	$U(1)_C$	$U(1)_D$	Field
$(\mathbf{3}'_{\alpha'_B}, \mathbf{2}_\gamma)$	4	1	0	0	0	1	1/6	5	-1	1	-5	Q
$(\bar{\mathbf{3}}'_{\alpha'_B}, \mathbf{1}_{\beta_{r2}})$	4	-1	0	0	-1	0	-2/3	-2	4	-2	4	U^c
$(\bar{\mathbf{3}}'_{\alpha'_B}, \mathbf{1}_{\beta_{r1}})$	4	-1	0	-1	0	0	1/3	-2	4	-2	4	D^c
$(\mathbf{1}'_{\alpha'_L}, \mathbf{2}_\gamma)$	4	0	1	0	0	1	-1/2	3	1	-1	-3	L
$(\mathbf{1}'_{\alpha'_L}, \mathbf{1}_{\beta_{r1}})$	4	0	-1	-1	0	0	1	0	2	0	2	E^c
$(\mathbf{1}'_{\alpha'_L}, \mathbf{1}_{\beta_{r2}})$	4	0	-1	0	-1	0	0	0	2	0	2	N
$(\mathbf{1}_{\beta_{r1}}, \bar{\mathbf{2}}_\gamma)^\star$	-	0	0	1	0	-1	-1/2	-3	-3	1	1	H_d
$(\mathbf{2}_\gamma, \mathbf{1}_{\beta_{r2}})^\star$	-	0	0	0	1	-1	1/2	-3	-3	1	1	H_u
$(\mathbf{1}_{\gamma'\gamma})$	8	0	0	0	0	-2	0	-4	-4	4	2	$\phi_{\gamma\gamma}$
$(\mathbf{1}_{\beta'_{r1}}, \mathbf{1}_{\beta_{r2}})$	8	0	0	1	1	0	0	-2	-2	-2	-2	$\phi_{\beta_{r1r2}}$
$(\mathbf{3}'_{\alpha'_B}, \mathbf{1}_{\alpha_L})$	8	1	1	0	0	0	-1/3	4	-4	4	-4	D_1
$(\bar{\mathbf{3}}'_{\alpha'_B}, \alpha_B)$	8	2	0	0	0	0	1/3	6	-6	6	-6	D_2^c

Of course, this is just

$$Q_Y = \frac{Q_B - Q_L}{2} + Q_{I_{3R}}, \tag{45}$$

where Q_B and Q_L are baryon number and lepton number respectively, while $Q_{I_{3R}}$ is like the third component of right-handed weak isospin.

As discussed, up to twelve $U(1)$ factors may obtain a mass *via* the GS mechanism. In order for the hypercharge to remain massless, it must be orthogonal to each of these factors. In this case, there are only four due to the degeneracy of the stacks. These $U(1)$'s remain to all orders as global symmetries and are given by

$$\begin{aligned} U(1)_A &= 3U(1)_{\alpha_B} - U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} + 2U(1)_\gamma - 2U(1)_1 + 2U(1)_2 + 2U(1)_3 + 2U(1)_4, \\ U(1)_B &= -3U(1)_{\alpha_B} + U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} + 2U(1)_\gamma + 2U(1)_1 + 2U(1)_2 - 2U(1)_3 + 2U(1)_4, \\ U(1)_C &= 3U(1)_{\alpha_B} - U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} - 2U(1)_\gamma + 2U(1)_1 - 2U(1)_2 - 2U(1)_3 + 2U(1)_4, \\ U(1)_D &= -3U(1)_{\alpha_B} + U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} - 2U(1)_\gamma - 2U(1)_1 - 2U(1)_2 + 2U(1)_3 + 2U(1)_4. \end{aligned} \tag{46}$$

Note that the hypercharge orthogonal to each of these $U(1)$ factors and so will remain massless. The ‘observable’ sector basically consists of a four-generation MSSM plus right-handed neutrinos. The rest of the spectrum primarily consists of vector-like matter, many of which are singlets under the MSSM gauge group. Using the states listed in Table 9, we may construct all of the required MSSM Yukawa couplings,

$$W_Y = y_u H_u Q U^c + y_d H_d Q D^c + y_l H_d L E^c \tag{47}$$

keeping in mind that all of the MSSM fields are charged under the global symmetries defined in Eq. (45). Typically, this results in the forbidding of some if not all of the desired Yukawa couplings. In this case, all of the Yukawa couplings are allowed by the global symmetries including a trilinear Dirac mass term for neutrinos,

$$W_D = \lambda_\nu L N H_u. \tag{48}$$

By itself, such a term would imply neutrino masses of the order of the quarks and charged leptons. However, if in addition there exists a Majorana mass term for the right-handed neutrinos,

$$W_m = M_m N N, \tag{49}$$

a see-saw mechanism may be employed. Such a mass term may in principle be generated by E2 instanton effects [32,33]. This mechanism may also be employed to generate a μ -term of the order of the EW scale.

In addition to the matter spectrum charged under the MSSM gauge groups and total gauge singlets, there is additional vector-like matter transforming under the ‘hidden’ gauge group $U(2)_1 \otimes U(2)_2 \otimes U(2)_3 \otimes U(2)_4$. By choosing appropriate flat directions, we may deform the fractional cycles wrapped by these stacks into bulk cycles such that

$$U(2)_1 \otimes U(2)_2 \rightarrow U(1); \quad U(2)_3 \otimes U(2)_4 \rightarrow U(1). \tag{50}$$

Thus, matter transforming under these gauge groups becomes a total gauge singlet or becomes massive and disappears from the spectrum altogether. The remaining eight pairs of exotic color triplets present in the model resulting from the breaking $\mathbf{6} \rightarrow \mathbf{3} \oplus \bar{\mathbf{3}}$,

while not truly vector-like due to their different charges under the global symmetries, may in principle become massive via instanton effects in much the same way a μ -term may be generated.

5. Gauge coupling unification

The MSSM predicts the unification of the three gauge couplings at an energy $\sim 2 \times 10^{16}$ GeV. In intersecting D-brane models, the gauge groups arise from different stacks of branes, and so they will not generally have the same volume in the compactified space. Thus, the gauge couplings are not automatically unified.

The low-energy $N = 1$ supergravity action is basically determined by the Kähler potential K , the superpotential W and the gauge kinetic function f . All of these functions depend on the background space moduli fields. For branes wrapping cycles not invariant under ΩR , the holomorphic gauge kinetic function for a D6-brane stack P is given by [28]

$$f_P = \frac{1}{2\pi l_s} \left[e^\phi \int_{\Pi_P} \text{Re}(e^{-i\theta_P} \Omega_3) - i \int_{\Pi_P} C_3 \right] \quad (51)$$

from which it follows (with $\theta_P = 0$ for $\mathbb{Z}_2 \times \mathbb{Z}_2$)

$$f_P = (n_P^1 n_P^2 n_P^3 s - n_P^1 m_P^2 m_P^3 u^1 - n_P^2 m_P^1 m_P^3 u^2 - n_P^3 m_P^1 m_P^2 u^3), \quad (52)$$

where u^i and s are the complex structure moduli and dilaton in the field theory basis. The gauge coupling constant associated with a stack P is given by

$$g_{D6_P}^{-2} = |\text{Re}(f_P)|. \quad (53)$$

Thus, we identify the $SU(3)$ holomorphic gauge function with stack α_B , and the $SU(2)$ holomorphic gauge function with stack γ . The Q_Y holomorphic gauge function is then given by taking a linear combination of the holomorphic gauge functions from all the stacks.⁴ In this way, it is found [34] that

$$f_Y = \frac{1}{6} f_{\alpha_B} + \frac{1}{2} f_{\alpha_L} + \frac{1}{2} f_{\beta_{r1}} + \frac{1}{2} f_{\beta_{r2}}. \quad (54)$$

Thus, it follows that the tree-level MSSM gauge couplings will be unified at the string scale

$$g_s^2 = g_w^2 = \frac{5}{3} g_Y^2 \quad (55)$$

since each stack will have the same gauge kinetic function.

6. Conclusion

In this Letter, we have constructed an intersecting D-brane model on the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orientifold background, also known as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with discrete torsion, where the D-branes wrap rigid cycles, thus eliminating the extra adjoint matter. The model constructed is a supersymmetric four generation MSSM-like model obtained from a spontaneously broken Pati–Salam, with a minimum of extra matter. All of the required Yukawa couplings are allowed by global symmetries which arise via a generalized Green–Schwarz mechanism. In addition, we find that the tree-level gauge couplings are unified at the string scale with a canonical normalization.

The main drawback of this model is that there are four generations of MSSM matter. However, the existence of a possible fourth generation is rather tightly constrained, although it is not completely ruled out. Of course, the actual fermion masses await a detailed analysis of the Yukawa couplings. The emergence of three light generations may in fact be correlated with the existence of three twisted sectors. If there turns out to be a fourth generation, then it would almost certainly be discovered at LHC within the next few years. Another interesting possibility is that the presence of discrete torsion will complexify the Yukawa couplings and thereby introduce CP violation into the CKM matrix [35]. Clearly, there is much work to be done to work out the detailed phenomenology of this model and we plan to return to this topic in the near future. With the LHC era just around the corner, it would be nice to have testable string models in hand.

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⁴ Note that we have absorbed a factor of $1/2$ in the definition of Q_Y so that the electric charge is given by $Q_{em} = T_3 + Q_Y$.

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