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Hao Wei*, Xiao-Peng Ma, Hao-Yu Qi

School of Physics, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

In analogy to $f(R)$ theory, recently a new modified gravity theory, namely the so-called $f(T)$ theory, has been proposed to drive the current accelerated expansion without invoking dark energy. In the present work, by extending Bisabr's idea, we try to constrain $f(T)$ theories with the varying fine structure "constant", $\alpha \equiv e^2/\hbar c$. We find that the constraints on $f(T)$ theories from the observational $\Delta\alpha/\alpha$ data are very severe. In fact, they make $f(T)$ theories almost indistinguishable from Λ CDM model.

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1. Introduction

The current accelerated expansion of our universe [1] has been one of the most active fields in modern cosmology since its discovery in 1998. This mysterious phenomenon could be due to an unknown energy component (dark energy) or a modification to general relativity (modified gravity) [1–4]. The well-known modified gravity theories are, for examples, $f(R)$ theory, scalar-tensor theory (including Brans–Dicke theory), braneworld scenarios (such as DGP, RSI and RSII), $f(\mathcal{G})$ theory (\mathcal{G} is the Gauss–Bonnet term), Horava–Lifshitz theory, MOND and TeVeS theories. We refer to e.g. [1–4,49] for some reviews.

Recently, a new modified gravity theory, namely the so-called $f(T)$ theory, attracted much attention in the community, where T is the torsion scalar. It is a generalized version of the so-called teleparallel gravity originally proposed by Einstein [5,6]. In teleparallel gravity, the Weitzenböck connection is used, rather than the Levi-Civita connection which is used in general relativity. Following [7,8], here we briefly review the key ingredients of teleparallel gravity and $f(T)$ theory. We consider a spatially flat Friedmann–Robertson–Walker (FRW) universe whose spacetime is described by

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2, \quad (1)$$

where a is the scale factor. The orthonormal tetrad components $e_i(x^\mu)$ relate to the metric through

$$g_{\mu\nu} = \eta_{ij} e_\mu^i e_\nu^j, \quad (2)$$

where Latin i, j are indices running over 0, 1, 2, 3 for the tangent space of the manifold, and Greek μ, ν are the coordinate indices on the manifold, also running over 0, 1, 2, 3. In teleparallel gravity, the gravitational action is

$$S_T = \frac{1}{2\kappa^2} \int d^4x |e| T, \quad (3)$$

where $\kappa^2 \equiv 8\pi G$, and $|e| = \det(e_\mu^i) = \sqrt{-g}$. The torsion scalar T is given by

$$T \equiv S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \quad (4)$$

where

$$T^\rho{}_{\mu\nu} \equiv -e_i^\rho (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i), \quad (5)$$

$$K^{\mu\nu}{}_\rho \equiv -\frac{1}{2} (T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}), \quad (6)$$

$$S_\rho{}^{\mu\nu} \equiv \frac{1}{2} (K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\theta\nu}{}_\theta - \delta_\rho^\nu T^{\theta\mu}{}_\theta). \quad (7)$$

For a spatially flat FRW universe, from Eqs. (4) and (1), one has

$$T = -6H^2, \quad (8)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter (a dot denotes the derivative with respect to cosmic time t). So, one can use T and H interchangeably. In analogy to $f(R)$ theory, one can replace T in the gravitational action (3) by a function $f(T)$.¹ In $f(T)$ theory,

¹ In the literature, one can instead write this function as $T + f(T)$, rather than $f(T)$ used here. One should be aware of the correspondence between these two formalisms.

* Corresponding author.

E-mail address: haowei@bit.edu.cn (H. Wei).

the modified Friedmann equation and Raychaudhuri equation are given by [7,8]

$$12H^2 f_T + f = 16\pi G\rho, \quad (9)$$

$$48H^2 f_{TT} \dot{H} - f_T(12H^2 + 4\dot{H}) - f = 16\pi Gp, \quad (10)$$

where $f_T \equiv \partial f / \partial T$, and ρ , p are the total energy density and pressure, respectively. In an universe with only dust matter, $p = p_m = 0$ and $\rho = \rho_m$. From Eqs. (9) and (10), one can find that the effective dark energy density and pressure from torsion are given by [7–9]

$$\rho_{de} = \frac{1}{16\pi G}(6H^2 - f - 12H^2 f_T), \quad (11)$$

$$p_{de} = -\rho_{de} - \frac{1}{4\pi G}(12H^2 f_{TT} - f_T + 1)\dot{H}. \quad (12)$$

Obviously, if $f(T) = T + \text{const}$, $f(T)$ theory reduces to the well-known Λ CDM model.

In fact, $f(T)$ theory was firstly used to drive inflation by Ferraro and Fiorini [10,11]. Later, Bengochea and Ferraro [7], as well as Linder [8], proposed to use $f(T)$ theory to drive the current accelerated expansion without invoking dark energy. Soon, many works followed. For examples, Myrzakulov [12] and Yang [13] proposed some new $f(T)$ forms; Bengochea [14], Wu and Yu [15] considered the cosmological constraints on $f(T)$ theories by using the latest observational data; Wu and Yu [16] also considered the dynamical behavior of $f(T)$ theory; Dent et al. [17], Zheng and Huang [18] considered the cosmological perturbations and growth factor in $f(T)$ theories; Wu and Yu [19], Bamba and Geng [20] discussed the equation-of-state parameter (EoS) crossing the phantom divide in $f(T)$ theories; Zhang et al. [21] discussed the dynamical analysis of $f(T)$ theories; Li, Sotiriou and Barrow [22] considered the large-scale structure and local Lorentz invariance in $f(T)$ theory; Deliduman and Yapiskan [23] discussed the relativistic neutron star in $f(T)$ theory; Cai et al. [24] considered the matter bounce in $f(T)$ theory; Wang [25] discussed the static solutions with spherical symmetry in $f(T)$ theories. We further refer to e.g. [26] for some relevant works.

In the literature, the observational constraints on $f(T)$ theories [14,15] were obtained mainly by using the cosmological data, such as type Ia supernovae (SNIa), baryon acoustic oscillation (BAO), and cosmic microwave background (CMB). In the present work, we instead try to constrain $f(T)$ theories with the varying fine structure “constant”, $\alpha \equiv e^2/\hbar c$. In Section 2, we briefly review the observational constraints on the temporal variation of the fine structure “constant” α . In Section 3, we briefly introduce the idea to constrain $f(R)$ theories with the varying α , which was proposed by Bisabr [27]. In Section 4, we extend Bisabr’s idea to $f(T)$ theories, and consider the corresponding constraints from the temporal variation of the fine structure “constant”. Finally, some brief concluding remarks are given in Section 5.

2. Observational constraints on the temporal variation of the fine structure “constant”

Motivated by the well-known large number hypothesis of Dirac proposed in 1937 [28], the varying fundamental “constants” remain as one of the unfading subjects for decades. Among the fundamental “constants”, the most observationally sensitive one is the electromagnetic fine structure “constant”, $\alpha \equiv e^2/\hbar c$. Since about 12 years ago, this subject attracted many attentions, mainly due to the first observational evidence from the quasar absorption spectra that the fine structure “constant” might change with cosmological time [29,30].

Subsequently, many authors obtained various observational constraints on the temporal variation of the fine structure “constant” α . In the literature, it is convenient to introduce a quantity $\Delta\alpha/\alpha \equiv (\alpha - \alpha_0)/\alpha_0$, where the subscript “0” indicates the present value of the corresponding quantity. Obviously, $\Delta\alpha/\alpha$ is time-dependent. A brief summary of the observational constraints on $\Delta\alpha/\alpha$ can be found in e.g. [31]. The most ancient constraint comes from the Big Bang Nucleosynthesis (BBN) [32,33], namely, $|\Delta\alpha/\alpha| \lesssim 10^{-2}$, in the redshift range $z = 10^{10}$ – 10^8 . The next constraint comes from the power spectrum of anisotropy in the cosmic microwave background (CMB) [33], i.e., $|\Delta\alpha/\alpha| < 10^{-2}$, for redshift $z \simeq 10^3$. In the medium redshift range, the constraint comes from the absorption spectra of distant quasars [29,30,34,35]. Since the results in the literature are controversial, it is better to consider the conservative constraint $|\Delta\alpha/\alpha| \lesssim 10^{-6}$ [31], in the redshift range $z = 3$ – 0.4 . From the radioactive life-time of ^{187}Re derived from meteoritic studies [36], the constraint is given by $|\Delta\alpha/\alpha| \lesssim 10^{-7}$ for redshift $z = 0.45$. Finally, from the Oklo natural nuclear reactor [37], it is found that $|\Delta\alpha/\alpha| \lesssim 10^{-7}$ for redshift $z = 0.14$. For convenience, we summarize the above constraints in Table 1, which will be used in the followings.

3. The idea to constrain $f(R)$ theories with varying alpha

3.1. Varying alpha driven by a general scalar field

Noting that $\alpha = e^2/\hbar c$, a varying α might be due to a varying speed of light c [38–40], while Lorentz invariance is broken. The other possibility for a varying α is due to a varying electron charge e . In 1982, Bekenstein proposed such a varying α model [41], which preserves local gauge and Lorentz invariance, and is generally covariant. This model has been revived and generalized after the first observational evidence of varying α from the quasar absorption spectra [29,30]. This is a dilaton theory with coupling to the electromagnetic F^2 part of the Lagrangian, but not to the other gauge fields. Later, the Bekenstein-type varying α model has been generalized by replacing the dilaton with a cosmological scalar field. Further, the coupling between the scalar field and the electromagnetic field could also be generalized. In fact, the varying α models driven by quintessence have been extensively investigated in the literature (see e.g. [31,42–45]). The varying α driven by phantom has been considered in the SSBM model [46] while its model parameter ω is negative. The special case of varying α driven by k -essence whose Lagrangian $\mathcal{L}(X, \phi) = X^n - V(\phi)$ has been considered in e.g. [42]. The varying α driven by Dirac–Born–Infeld scalar field has also been discussed in [47].

Following [31,42,44], the relevant action in Einstein frame is generally given by

$$\mathcal{S} = \mathcal{S}_g + \int d^4x \sqrt{-g} \mathcal{L}_\phi - \frac{1}{4} \int d^4x \sqrt{-g} B_F(\phi) F_{\mu\nu} F^{\mu\nu} + \mathcal{S}_m, \quad (13)$$

where \mathcal{S}_g is the gravitational action in Einstein frame; $F_{\mu\nu}$ are the components of the electromagnetic field tensor; \mathcal{S}_m is the action of other matters; \mathcal{L}_ϕ is the Lagrangian of the scalar field ϕ . Noting that B_F takes the place of e^{-2} in Eq. (13) actually [43,48], one can easily see that the effective fine structure “constant” $\alpha = e^2/\hbar c$ is given by [31,42]

$$\alpha = \frac{\alpha_0}{B_F(\phi(\mathbf{x}, t))}. \quad (14)$$

Thus, we find that

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = \frac{1}{B_F(\phi)} - 1. \quad (15)$$

It is worth noting that the present value (at redshift $z=0$) of the coupling B_F should be 1 by definition. If $B_F(z=0) = B_{F0} \neq 1$, we can normalize it through rescaling the electromagnetic field, namely

$$F_{\mu\nu} \rightarrow \sqrt{B_{F0}} F_{\mu\nu}, \quad \text{while} \quad B_F \rightarrow \frac{B_F}{B_{F0}}. \quad (16)$$

Thus, we have

$$\frac{\Delta\alpha}{\alpha} = \frac{B_{F0}}{B_F} - 1. \quad (17)$$

3.2. Varying alpha in $f(R)$ theories

Following Bisabr's idea [27], here we briefly show why the fine structure "constant" should be varying in $f(R)$ theories. As is well known, in Jordan frame the action of $f(R)$ theories with matters (including electromagnetic fields here) reads (see e.g. [2–4])

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + \int d^4x \mathcal{L}_M(g_{\mu\nu}, \Psi_M), \quad (18)$$

where R is the Ricci scalar, and \mathcal{L}_M is the matter Lagrangian depending on $g_{\mu\nu}$ and matter fields Ψ_M (including electromagnetic fields here). It is well known that $f(R)$ theory can be equivalent to scalar-tensor theory [2–4]. Applying the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = \mathcal{F} \equiv f_R = \frac{\partial f}{\partial R}, \quad (19)$$

and introducing a new scalar field ϕ defined by

$$\kappa\phi \equiv \sqrt{3/2} \ln \mathcal{F}, \quad (20)$$

one can rewrite the action (18) to the one in Einstein frame [2–4], namely

$$\begin{aligned} \tilde{S} = & \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \\ & + \int d^4x \mathcal{L}_M(\mathcal{F}^{-1}(\phi) \tilde{g}_{\mu\nu}, \Psi_M), \end{aligned} \quad (21)$$

where a tilde denotes quantities in Einstein frame, and the potential of scalar field is given by

$$V(\phi) = \frac{\mathcal{F}R - f}{2\kappa^2 \mathcal{F}^2}. \quad (22)$$

From Eq. (21), it is easy to see that the matter fields (including electromagnetic fields here) are *inevitably* coupled with the scalar field ϕ in Einstein frame. So, α should be varying. Noting that $\sqrt{-\tilde{g}} = \Omega^{-4} \sqrt{-g}$, one can easily find that the coupling in Eq. (13) is given by [27]

$$B_F = \mathcal{F}^{-2} = f_R^{-2}. \quad (23)$$

From Eq. (15), the variation of the fine structure "constant" can be described by [27]

$$\frac{\Delta\alpha}{\alpha} = f_R^2 - 1. \quad (24)$$

As mentioned above, if $f_R(z=0) = f_{R0} \neq 1$, we can normalize it through Eq. (16), and then

$$\frac{\Delta\alpha}{\alpha} = \left(\frac{f_R}{f_{R0}} \right)^2 - 1. \quad (25)$$

In [27], Bisabr discussed the observational constraints on $f(R)$ theories with the temporal variation of the fine structure "constant", and found that the corresponding constraints are fairly tight. We refer to the original paper [27] for details.

4. $f(T)$ theories and varying alpha

As mentioned in Section 1, $f(T)$ theories are proposed in analogy to $f(R)$ theories. So, we extend Bisabr's idea [27] to constrain $f(T)$ theories also with the temporal variation of the fine structure "constant".

4.1. Varying alpha in a general $f(T)$ theory

As mentioned in [8,9], $f(T)$ theory can also be equivalent to scalar-tensor (torsion) theory. In Jordan frame, the relevant action with matters (including electromagnetic fields here) reads

$$S = \frac{1}{2\kappa^2} \int d^4x |e| f(T) + \int d^4x \mathcal{L}_M(e^i_\mu, \Psi_M). \quad (26)$$

Similarly, applying the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \leftrightarrow \tilde{e}^i_\mu = \Omega e^i_\mu, \quad \Omega^2 = \mathcal{F} \equiv f_T, \quad (27)$$

one can also rewrite the action (26) to the one in Einstein frame with a new scalar field ϕ [8,9]. Similar to the case of $f(R)$ theories, the matter action in Einstein frame is given by [9]

$$\tilde{S}_M = \int d^4x \mathcal{L}_M(\mathcal{F}^{-1/2}(\phi) \tilde{e}^i_\mu, \Psi_M). \quad (28)$$

Again, the matter fields (including electromagnetic fields here) are *inevitably* coupled with the scalar field ϕ in Einstein frame. So, α should be varying. Noting that $|e| = \Omega^{-4} |\tilde{e}|$, one can similarly find that

$$B_F = \mathcal{F}^{-2} = f_T^{-2}. \quad (29)$$

From Eq. (15), the variation of the fine structure "constant" can be described by

$$\frac{\Delta\alpha}{\alpha} = f_T^2 - 1. \quad (30)$$

As mentioned above, if $f_T(z=0) = f_{T0} \neq 1$, we can normalize it through Eq. (16), and then

$$\frac{\Delta\alpha}{\alpha} = \left(\frac{f_T}{f_{T0}} \right)^2 - 1. \quad (31)$$

In the followings, we will consider two concrete $f(T)$ theories, namely, $f(T) = T + \mu(-T)^n$ and $f(T) = T - \mu T(1 - e^{\beta T_0/T})$, which are the most popular $f(T)$ theories discussed extensively in the literature (see e.g. [7,8,15,16]).

4.2. $f(T) = T + \mu(-T)^n$

At first, we consider the case of $f(T) = T + \mu(-T)^n$, where μ and n are both constants. This is the simplest model, and has been considered in most papers on $f(T)$ theory. Obviously, if $n=0$, it reduces to Λ CDM model. Substituting it into the modified Friedmann equation (9), one can easily find that μ is not an independent model parameter, namely [8,15]

$$\mu = \frac{1 - \Omega_{m0}}{2n - 1} (6H_0^2)^{1-n} = \frac{1 - \Omega_{m0}}{2n - 1} (-T_0)^{1-n}, \quad (32)$$

where $\Omega_{m0} \equiv 8\pi G \rho_{m0}/(3H_0^2)$ is the present fractional energy density of dust matter. So, we have

$$f(T) = T + \frac{1 - \Omega_{m0}}{2n - 1} (-T_0) \left(\frac{T}{T_0} \right)^n, \quad (33)$$

and then

$$f_T = 1 + \frac{n(1 - \Omega_{m0})}{1 - 2n} E^{2(n-1)}, \quad f_{T0} = 1 + \frac{n(1 - \Omega_{m0})}{1 - 2n}, \quad (34)$$

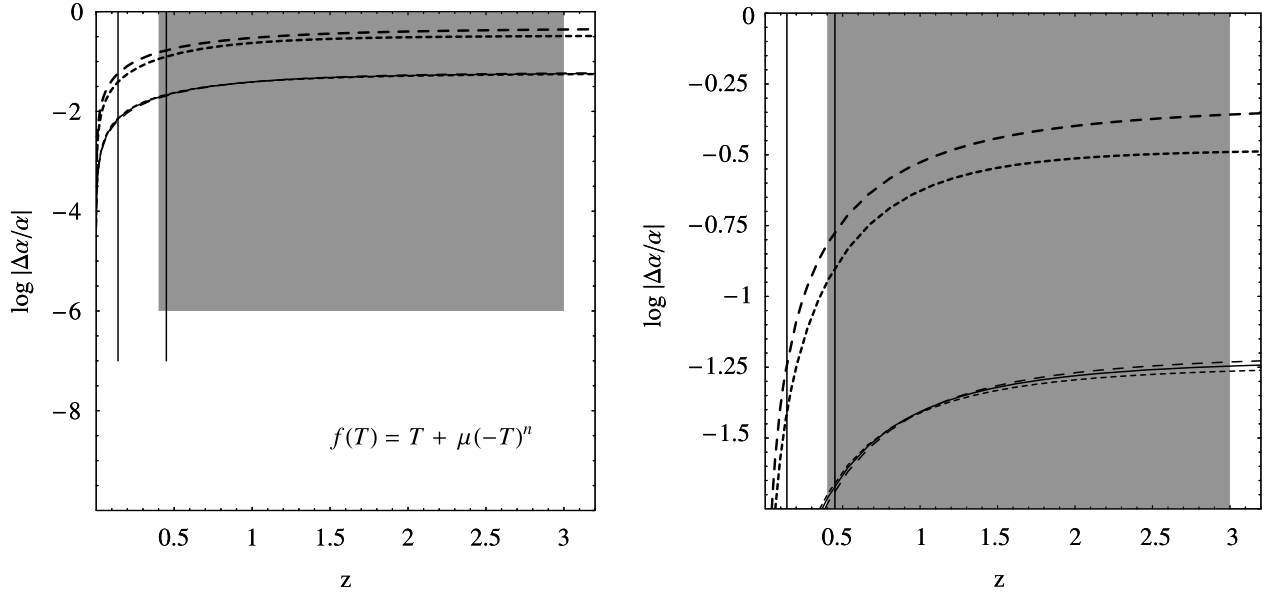


Fig. 1. $\log|\Delta\alpha/\alpha|$ as a function of redshift z for $f(T) = T + \mu(-T)^n$ with $\Omega_{m0} = 0.272$ and $n = 0.04$ (solid curve), $\Omega_{m0} = 0.272$ and $n = 0.26$ (thick long-dashed curve), $\Omega_{m0} = 0.272$ and $n = -0.29$ (thick short-dashed curve), $\Omega_{m0} = 0.240$ and $n = 0.04$ (thin long-dashed curve), $\Omega_{m0} = 0.308$ and $n = 0.04$ (thin short-dashed curve). Right panel is an enlarged part of left panel. Only the curves not overlapping the gray areas are phenomenologically viable. See text for details.

Table 1
The observational constraints on $\Delta\alpha/\alpha$.

$ \Delta\alpha/\alpha $	Redshift	Observation	Ref.
$\lesssim 10^{-2}$	$10^{10}-10^8$	BBN	[32,33]
$\lesssim 10^{-2}$	10^3	CMB	[33]
$\lesssim 10^{-6}$	3–0.4	quasars	[29,30,34,35]
$\lesssim 10^{-7}$	0.45	meteorite	[36]
$\lesssim 10^{-7}$	0.14	Oklo	[37]

where $E^2 = T/T_0 = H^2/H_0^2$. Substituting Eq. (34) into Eq. (31), one can finally obtain the explicit expression of $\Delta\alpha/\alpha$. In order to compare it with the observational data, we need to know $E(z)$ as a function of redshift z . Substituting $f(T) = T + \mu(-T)^n$ and Eq. (32) into the modified Friedmann equation (9), we find that

$$E^2 = \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})E^{2n}. \quad (35)$$

Obviously, if $n = 0$, it reduces to the one of Λ CDM model. If Ω_{m0} and n are given, we can numerically solve Eq. (35) and obtain $E^2(z)$ as a function of redshift z . Thus, $\Delta\alpha/\alpha$ is on hand.

Next, we compare $\Delta\alpha/\alpha$ with the observational data. In fact, as shown in e.g. [31,47], the constraints from the first two rows (at very high redshift) in Table 1 are very weak. Therefore, we only consider the last three rows (at low redshift) in Table 1 (and hence the radiation can be safely ignored). Note that in [15], this $f(T) = T + \mu(-T)^n$ model has been constrained by using the latest cosmological data, i.e., 557 Union2 SNIa dataset, BAO, and shift parameter from WMAP7. The corresponding 2σ results are given by [15]

$$\Omega_{m0} = 0.272^{+0.036}_{-0.032}, \quad n = 0.04^{+0.22}_{-0.33}. \quad (36)$$

At first, we try to see whether $\Delta\alpha/\alpha$ with the best-fit parameters of [15] and the corresponding 2σ edge can simultaneously satisfy the observational constraints in Table 1. In Fig. 1, we plot $\log|\Delta\alpha/\alpha|$ as a function of redshift z for $f(T) = T + \mu(-T)^n$ with $\Omega_{m0} = 0.272$ and $n = 0.04$ (solid curve), $\Omega_{m0} = 0.272$ and $n = 0.26$ (thick long-dashed curve), $\Omega_{m0} = 0.272$ and $n = -0.29$ (thick short-dashed curve), $\Omega_{m0} = 0.240$ and $n = 0.04$ (thin long-dashed curve), $\Omega_{m0} = 0.308$ and $n = 0.04$ (thin short-dashed curve), where \log indicates the logarithm to base 10. Obviously, one can see

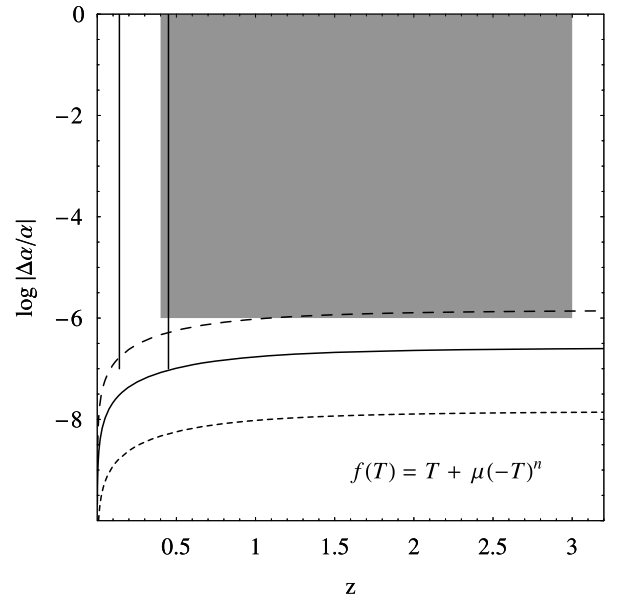


Fig. 2. $\log|\Delta\alpha/\alpha|$ as a function of redshift z for $f(T) = T + \mu(-T)^n$ with a fixed $\Omega_{m0} = 0.272$, and $n = \pm 1.8 \times 10^{-7}$ (solid curve), $n = \pm 10^{-6}$ (long-dashed curve), $n = \pm 10^{-8}$ (short-dashed curve). Only the curves not overlapping the gray areas are phenomenologically viable. See text for details.

that $\Delta\alpha/\alpha$ with the best-fit parameters of [15] and the corresponding 2σ edge cannot satisfy the observational constraints in Table 1. In addition, from Fig. 1, we find that the influence from n to $\Delta\alpha/\alpha$ is significantly larger than the one from Ω_{m0} . Thus, fixing $\Omega_{m0} = 0.272$, we try various n to find in which cases all the observational constraints in Table 1 could be simultaneously satisfied. From Fig. 2, it is easy to see that they can be all respected only for

$$|n| \leq 1.8 \times 10^{-7}. \quad (37)$$

This is the constraint on $f(T) = T + \mu(-T)^n$ theory from the observational $\Delta\alpha/\alpha$ data. It is a very severe constraint in fact. Noting

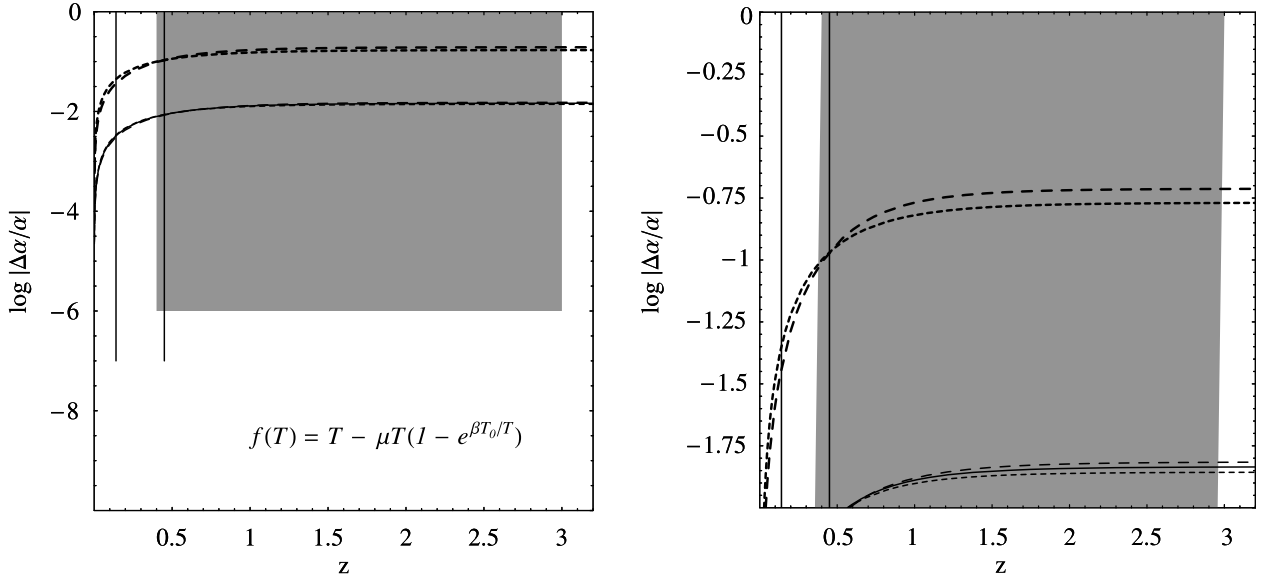


Fig. 3. $\log|\Delta\alpha/\alpha|$ as a function of redshift z for $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ with $\Omega_{m0} = 0.272$ and $\beta = -0.02$ (solid curve), $\Omega_{m0} = 0.272$ and $\beta = 0.29$ (thick long-dashed curve), $\Omega_{m0} = 0.272$ and $\beta = -0.22$ (thick short-dashed curve), $\Omega_{m0} = 0.238$ and $\beta = -0.02$ (thin long-dashed curve), $\Omega_{m0} = 0.308$ and $\beta = -0.02$ (thin short-dashed curve). Right panel is an enlarged part of left panel. Only the curves not overlapping the gray areas are phenomenologically viable. See text for details.

that $f(T) = T + \mu(-T)^n \rightarrow T + \text{const}$ when $n \rightarrow 0$, this $f(T)$ theory becomes almost indistinguishable from Λ CDM model.

4.3. $f(T) = T - \mu T(1 - e^{\beta T_0/T})$

Here, we consider the case of $f(T) = T - \mu T(1 - e^{\beta T_0/T})$, where μ and β are both constants. Obviously, $f(T) \rightarrow T + \mu\beta T_0 = T + \text{const}$ when $\beta \rightarrow 0$, it reduces to Λ CDM model. Substituting it into the modified Friedmann equation (9), one can easily find that μ is not an independent model parameter [8,15], i.e.,

$$\mu = \frac{1 - \Omega_{m0}}{1 - (1 - 2\beta)e^\beta}. \quad (38)$$

On the other hand, it is easy to obtain

$$\begin{aligned} f_T &= 1 - \mu + \mu(1 - \beta/E^2)e^{\beta/E^2}, \\ f_{T0} &= 1 - \mu + \mu(1 - \beta)e^\beta, \end{aligned} \quad (39)$$

where $E^2 = T/T_0 = H^2/H_0^2$. Substituting Eq. (39) into Eq. (31), one can finally obtain the explicit expression of $\Delta\alpha/\alpha$. In order to compare it with the observational data, we need to know $E(z)$ as a function of redshift z . Substituting $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ into the modified Friedmann equation (9), we find that

$$E^2 = \Omega_{m0}(1+z)^3 + \mu E^2 \left[1 - e^{\beta/E^2} + 2 \left(\frac{\beta}{E^2} \right) e^{\beta/E^2} \right]. \quad (40)$$

If $\beta \rightarrow 0$, we have $\mu\beta \rightarrow 1 - \Omega_{m0}$ from Eq. (38), and hence Eq. (40) reduces to the one of Λ CDM model. If Ω_{m0} and β are given, we can numerically solve Eq. (40) and obtain $E^2(z)$ as a function of redshift z . Thus, $\Delta\alpha/\alpha$ is on hand.

In [15], this $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ model has also been constrained by using the latest cosmological data, i.e., 557 Union2 SNIa dataset, BAO, and shift parameter from WMAP7. The corresponding 2σ results are given by [15]

$$\Omega_{m0} = 0.272^{+0.036}_{-0.034}, \quad \beta = -0.02^{+0.31}_{-0.20}. \quad (41)$$

Again, we try to see whether $\Delta\alpha/\alpha$ with the best-fit parameters of [15] and the corresponding 2σ edge can simultaneously satisfy the observational constraints in Table 1. In Fig. 3, we plot $\log|\Delta\alpha/\alpha|$

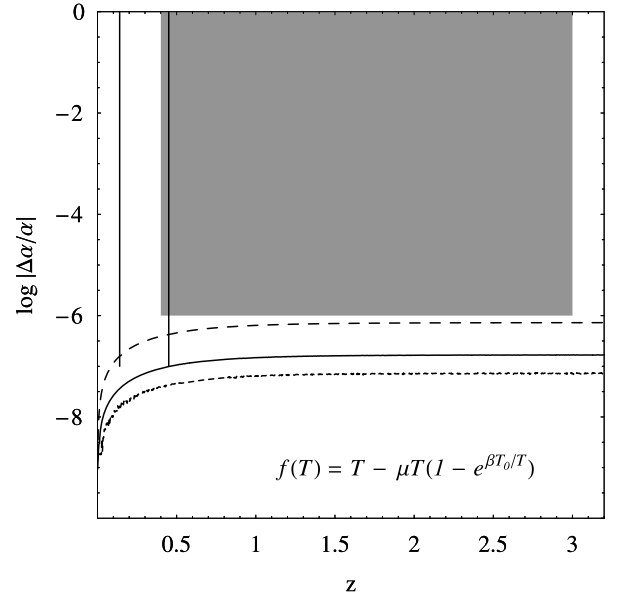


Fig. 4. $\log|\Delta\alpha/\alpha|$ as a function of redshift z for $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ with a fixed $\Omega_{m0} = 0.272$, and $\beta = \pm 2.3 \times 10^{-7}$ (solid curve), $\beta = \pm 10^{-6}$ (long-dashed curve), $\beta = \pm 10^{-7}$ (short-dashed curve). Only the curves not overlapping the gray areas are phenomenologically viable. See text for details.

as a function of redshift z for $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ with $\Omega_{m0} = 0.272$ and $\beta = -0.02$ (solid curve), $\Omega_{m0} = 0.272$ and $\beta = 0.29$ (thick long-dashed curve), $\Omega_{m0} = 0.272$ and $\beta = -0.22$ (thick short-dashed curve), $\Omega_{m0} = 0.238$ and $\beta = -0.02$ (thin long-dashed curve), $\Omega_{m0} = 0.308$ and $\beta = -0.02$ (thin short-dashed curve). Obviously, one can see that $\Delta\alpha/\alpha$ with the best-fit parameters of [15] and the corresponding 2σ edge cannot satisfy the observational constraints in Table 1. In addition, from Fig. 3, we find that the influence from β to $\Delta\alpha/\alpha$ is significantly larger than the one from Ω_{m0} . Thus, fixing $\Omega_{m0} = 0.272$, we try various β to find in which cases all the observational constraints in Table 1 could be simultaneously satisfied. From Fig. 4, it is easy to see that

they can be all respected only for

$$|\beta| \leq 2.3 \times 10^{-7}. \quad (42)$$

This is the constraint on $f(T) = T - \mu T(1 - e^{\beta T_0/T})$ theory from the observational $\Delta\alpha/\alpha$ data. It is a very severe constraint in fact. Noting that $f(T) = T - \mu T(1 - e^{\beta T_0/T}) \rightarrow T + \text{const}$ when $\beta \rightarrow 0$, this $f(T)$ theory becomes almost indistinguishable from Λ CDM model.

5. Concluding remarks

In analogy to $f(R)$ theory, recently $f(T)$ theory has been proposed to drive the current accelerated expansion without invoking dark energy. In the literature, the observational constraints on $f(T)$ theories were obtained mainly by using the cosmological data, such as type Ia supernovae (SNIa), baryon acoustic oscillation (BAO), and cosmic microwave background (CMB). In the present work, by extending Bisabr's idea [27], we instead try to constrain $f(T)$ theories with the varying fine structure "constant", $\alpha \equiv e^2/\hbar c$. We found that the constraints on $f(T)$ theories from the observational $\Delta\alpha/\alpha$ data are very severe. In fact, they make $f(T)$ theories almost indistinguishable from Λ CDM model.

Some remarks are in order. Firstly, in this work we only considered a spatially flat FRW universe. This is mainly motivated by the well-known inflation scenario and the very tight observational constraint on the spatial curvature term from WMAP7 data [50], namely, $\Omega_k = -0.0057^{+0.0067}_{-0.0068}$. Obviously, when a non-vanishing spatial curvature term is allowed, the observational constraints on $f(T)$ theories from the varying fine structure "constant" should be relaxed, since the number of free model parameters is increased. However, we can expect that the situation of $f(T)$ theories are still not improved, due to the very narrow range of the constrained Ω_k . For instance, even the constraints on n or β could be greatly relaxed from $\mathcal{O}(10^{-7})$ to $\mathcal{O}(10^{-4})$ (say), the corresponding $f(T)$ theories are still indistinguishable from Λ CDM model. Secondly, in fact the fine structure "constant" might be not only time-dependent but also space-dependent (see e.g. [51]). Of course, the space-dependent fine structure "constant" is still in controversy. On the other hand, the space-dependent fine structure "constant" might invoke an inhomogeneous scalar field ϕ . As is shown in Sections 3.2 and 4.1, the $f(R)$ and $f(T)$ theories in a homogeneous and isotropic FRW universe could not lead to a space-dependent fine structure "constant", and hence they are not constrained by the possibly spatial variation of the fine structure "constant". So, in this work we only considered the time-dependent fine structure "constant" for simplicity. Thirdly, it is worth noting that $f(T)$ gravity does not generally preserve the local Lorentz invariance and any theory of $f(T)$ gravity is always built on a local frame which is chosen on a specific spacetime point. As consequences, it is not always applicable for the conformal transformation in $f(R)$ gravity to be used in $f(T)$ gravity, and it is quite unclear how to explicitly define an Einstein or a Jordan frame in $f(T)$ gravity. Fortunately, we could use these conceptions in the homogenous and isotropic FRW background which is quite a special case (we thank the referee for pointing out this issue). Finally, we note that the two concrete $f(T)$ theories considered in this work contain only a single free parameter, which are very simple cases. In fact, it is reminiscent of the case of $f(R)$ theory. In the beginning, the forms of $f(R)$ are also very simple, such as the types of $1/R$ or R^n [1–4]. Later, these simple $f(R)$ forms have been easily ruled out by the cosmological observations and the local gravity tests. After several years, the viable $f(R)$ forms which can satisfy all the cosmological observations and the local gravity tests, e.g. Hu–Sawicki [52], Starobinsky [53] and Tsujikawa [54], have been toughly earned [1–4]. These three viable $f(R)$ forms are all complicated and delicate,

and they contain two or more free parameters. Similarly, we expect that the viable $f(T)$ forms, which satisfy all the cosmological observations, the local gravity tests and the constraints from the varying fine structure "constant", could be constructed with tough efforts in the future. Of course, it is easy to anticipate that they are also complicated and delicate, and contain many free parameters. In addition, at that time it is also interesting to see what kind of dark energy models could be mimicked by the viable $f(T)$ theories.

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