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Tolerance Analysis of rotating Mechanism based on Skin Model Shapes in discrete Geometry

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Abstract

Geometric deviations are inevitably observable on every manufactured workpiece. These deviations affect the function and quality of mechanical products and have therefore to be controlled by geometric tolerances. Computer-aided tolerancing aims at supporting design, manufacturing, and inspection by determining and quantifying these effects of geometric deviations on the product quality and the functional behaviour. However, most established tolerance representation schemes imply abstractions of geometric deviations and are not conform with the standards for geometric dimensioning and tolerancing. These limitations led to the development of a Skin Model inspired framework for the tolerance analysis, which is based on a representation of non-ideal workpieces employing discrete geometry representation schemes, such as point clouds and surface meshes. In this contribution, this Skin Model inspired framework for computer aided tolerancing is extended to systems in motion and applied to the tolerance analysis of rotating mechanism with higher kinematic pairs. For this purpose, the generation of non-ideal part representatives, as well as their processing with algorithms for registration and computational geometry are highlighted. Finally, the results are visualized and interpreted. The procedure as well as the simulation model itself are shown in a case study of a disk cam mechanism.

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Keywords: Skin Model; Tolerance Analysis; Rotating Mechanism; Discrete Geometry; Computational Geometry.

Nomenclature

CAD	Computer Aided Design
CAT	Computer Aided Tolerancing
(F)KC	(Functional) Key Characteristic
GPS	Geometric Product Specification and Verification
ISO	International Organization for Standardization
ICP	Iterative Closest Point

1. Introduction

Geometric deviations are inevitably observable on every manufactured workpiece. These deviations affect the functional compliance and quality of mechanical products and have therefore to be controlled by geometric tolerances. Thus, tolerancing is a key activity in order to realize high quality mechanism manufactured at moderate costs. It is a responsible task, which requires a high level of expertise. Computer-aided tolerancing (CAT) aims at supporting design, manufacturing, and inspection by determining and quantifying the effects of geometric deviations on the product quality and the functional behaviour.

In the context of CAT, the representation of geometric deviations is still a key issue in tolerance simulation modelling, since most established tolerance representation schemes imply abstractions of geometric deviations. Many models for the representation of geometric *deviations*, which are subsumed as variational geometry approaches and used for the displacement accumulation, and for the representation of geometric *tolerances*, which are referred to as tolerance zone models and used for the tolerance accumulation, have been proposed [1,2]. However, most of these models only consider translational and rotational defects of part features [3,4]. Furthermore, many of the available tolerance simulation tools are not conform with the standards for geometric dimensioning and tolerancing [5].

These limitations led to the development of a Skin Model inspired framework for the tolerance analysis [6,7], which is based on a representation of non-ideal workpieces employing discrete geometry representation schemes, such as point clouds and surface meshes. These workpiece representatives are referred to as Skin Model Shapes, since they can be interpreted as outcomes of the Skin Model as a basic concept in the standards for geometric product specification and verification. In

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this regard, Skin Model Shapes are particular finite Skin Model representatives and each single Skin Model Shape is a specific outcome comprising deviations from manufacturing and assembly [6,8].

In this contribution, this Skin Model inspired framework is extended to systems in motion and applied to the tolerance analysis of rotating mechanism with higher kinematic pairs. In the following section, a brief state of the art with regard to tolerance analysis of mechanism is given. Thereafter, the framework for the skin model based tolerance analysis of mechanism is explained and the employed simulation models are highlighted. Finally, a conclusion and an outlook are given.

2. Tolerance Analysis of Mechanism and Systems in Motion

Tolerancing is a basic task in design and comprises the tolerance analysis, i. e. the prediction of the effects of geometric tolerances on the product function and quality [9]. In this regard, tolerance analysis methods can be classified as one-dimensional tolerance stack-up, two-dimensional, and three-dimensional tolerance analysis [10]. All of these tolerance analysis methods require the representation of geometric deviations by mathematical models. Some of these mathematical approaches for the representation of geometric tolerances are Vectorial Tolerancing [11], the model of Technologically and Topologically Related Surfaces [12], the Direct Linearization Method [13], the Deviation Domain [14] based on the Small Displacement Torsor [15], and Tolerance Maps [16].

Based on these tolerance and deviation representation schemes, various approaches for the tolerance analysis of mechanism have been proposed. For example, vectorial tolerancing has been employed for the tolerance analysis of mechanism with lower kinematic pairs considering different kinds of geometric deviations, such as manufacturing-inherent deviations, deviations caused by elastic deformations and thermal expansion, and clearance in linkages [17]. The approach has been extended with regard to the consideration of interactions between these deviations [18] and has also been used for the tolerance-cost optimization of systems in motion [19]. Furthermore, vectorial tolerancing has been employed for the tolerance analysis of mechanism with higher kinematic pairs (bevel gears) utilizing a numerical contact analysis approach [1]. In contrast to that, a parametric tolerance analysis approach for planar mechanism is proposed in [20] and the tolerance zone approach has been used for the computation of the envelope of rotating parts in [21]. The Direct Linearization Method has been employed for the tolerance analysis of mechanism considering position errors in kinematic linkages [22] and taking into account part flexibility in [23]. Apart from this, a rich survey on multi-body systems with imperfect kinematic joints can be found in [24]. Moreover, a discrete geometry approach for the tolerance analysis of gears has been proposed in [25].

However, since these approaches ground on mathematical models for the representation of geometric deviations and tolerances, which imply severe assumptions, most of them only respect translational and rotational defects of part features. Moreover, they involve complex mathematical models for the evaluation of the effects of geometric deviations and their embedding in an integrated CAT process comprising design, manufacturing, inspection, and product testing is difficult.

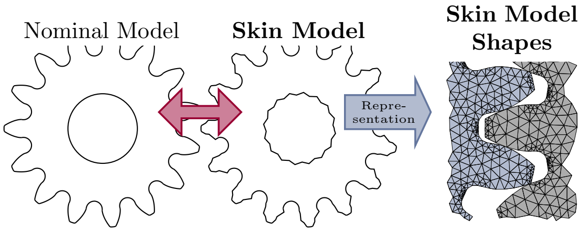


Fig. 1. The Nominal Model, the Skin Model, and the Concept of Skin Model Shapes

3. The Concept of Skin Model Shapes

In order to facilitate and to link the different activities of geometric variations management in design, manufacturing, and inspection, GeoSpelling and the Skin Model concept have been developed and adopted in the standards for GPS (ISO 17450-1) [26], where the Skin Model is a model of the physical interface between a workpiece and its environment. In contrast to the Nominal Model as the designers ideal product geometry proposal, the Skin Model comprises geometric deviations introduced by manufacturing and assembly. Since an infinite description is required to consider all different kinds of geometric deviations from a macro to a micro scale, there exists no possibility for identification and simulation of the Skin Model [27]. Due to this, the concept of Skin Model Shapes has been developed [6], which can be seen as an operationalization of the Skin Model. In this regard, Skin Model Shapes are specific finite outcomes of the Skin Model and serve as virtual part representatives considering geometric deviations. The concept of Skin Model Shapes is not linked to a certain geometry representation scheme, such as discrete (point cloud, surface mesh) or parametric ones (NURBS, Splines). However, a discrete geometry representation is employed for the implementation of Skin Model Shapes, since it is available and processable throughout design, manufacturing, and inspection. For example, a point cloud as well as a surface mesh representation of Skin Model Shapes can be obtained during design by tessellation, whereas part inspection routines by tactile or optical measurement systems lead to such representations during manufacturing and inspection. The difference between the Nominal Model, the Skin Model, and the concept of Skin Model Shapes can be seen from Fig. 1. Since a focus is set on a tolerance analysis approach for rotating mechanism, which requires a contact analysis based on a closed part surface description, a triangle mesh representation of Skin Model Shapes is employed in this contribution.

4. Approach for the Tolerance Analysis of rotating Mechanism based on Skin Model Shapes

In the following, the proposed approach for the tolerance analysis of rotating mechanism with higher kinematic pairs based on Skin Model Shapes is highlighted, where a focus is set on the simulation model for the part assembly and contact evaluation. For the sake of comprehension, it is applied to a study case of a disk cam mechanism as an irregular transmission, which can be seen from Fig. 2. The mechanism is to transmit a circular motion into a longitudinal motion, where the functional key characteristic (FKC) is the altitude of the bolt h_b.

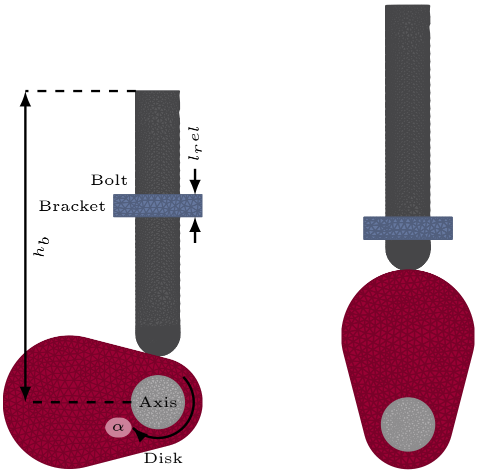


Fig. 2. Surface Mesh Representation of the Disk Cam Mechanism for a Cam Disk rotation of $\alpha = 0°$ (left) and $\alpha = 90°$ (right)

4.1. General Approach

The approach for the tolerance analysis of rotating mechanism based on Skin Model Shapes can be divided in a pre-processing, a processing, and a post-processing stage as can be seen from Fig. 3.

In the *Pre-Processing* phase, Skin Model Shapes are generated either based on the nominal model gathered from CAD (prediction stage) or by using observations from manufacturing process simulations or measurement data (observation stage) [8]. During the prediction stage, the Skin Model Shape generation comprises the tessellation of the nominal model as well as the modelling of systematic and random geometric deviations. For this purpose, several mathematical approaches have been proposed, such as second order shapes for the simulation of systematic deviations and the Gibbs method or Gaussian random fields for the sampling of random deviations [8,28]. In contrast to that, statistical shape analysis can be used at the observation stage in order to increase the number of Skin Model Shapes based on a limited set of observations [8].

The generated Skin Model Shapes are then processed in assembly and contact simulation models during the *Processing* stage. In this regard, firstly, the assembly and contact simulation model has to be established. For this purpose, all relevant assembly and contact features have to be extracted from the Skin Model Shapes with the help of GeoSpelling operations, such as partition and extraction [26]. Thereafter, the contact simulation is performed for each relevant time step t_i of the motion cycle. As a result, the part positions for each of the time steps t_i are obtained.

These part positions can then be used to determine the relevant functional key characteristics for each motion step t_i at the *Post-Processing* stage. The results of the simulation models and the development of the FKC over the motion cycle have to be visualized and interpreted. For this purpose, e. g. a plot of the FKC trajectories or a parallel coordinates plot can be used. Finally, a comparison for conformance has to be drawn in order to check if the geometric deviations lead to a violation of the requirements.

In the following, each stage of this approach is highlighted.

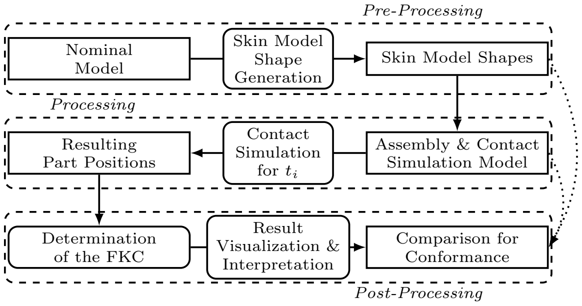


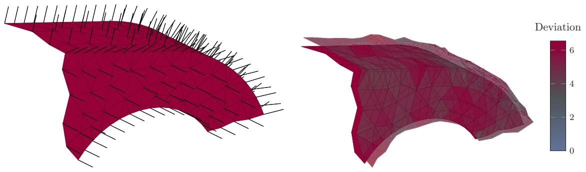
Fig. 3. General Approach for the Tolerance Analysis of rotating Mechanism based on Skin Model Shapes

4.2. Pre-Processing: Generation of Skin Model Shapes

As pointed out, the generation of Skin Model Shapes can be performed by modelling geometric deviations employing mathematical methods in early design stages (prediction stage) or by using observations from manufacturing process simulations or measurement data in later design stages (observation stage) [8]. In the following, the first procedure is pursued.

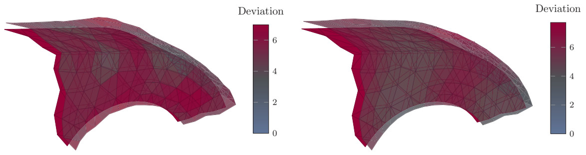
Several approaches for the modelling of systematic and random geometric deviations in discrete geometry have been proposed [8,28], such as second order shapes, the Multi-Gaussian method, and random fields. Any of these methods can be used for the generation of Skin Model Shapes, but for the sake of comprehensibility and generalization, a focus is set on random geometric deviation modelling by Gaussian random fields. The underlying idea of the approaches for the discrete shape modelling is to deviate each point of a discrete point set along a predefined direction. This direction can either be a global direction, as for example the feature normal, which can be determined by the Principal Component Analysis, or a local normal vector. For the Gaussian random field approach, the value of deviations along this direction is given by a collection of spatially correlated Gaussian random variables. The amount of correlation between these variables is defined by a correlation function $\rho(<>, l_\rho)$ with a characteristic parameter l_ρ, which is denoted by correlation length and affects the amount of correlation between the random variables. Furthermore, the random variables are described by their mean μ and standard deviation σ. In summary, samples ξ of these spatially correlated random variables are obtained by $\xi = \mu + \sqrt{C} \cdot \psi$, where C is the covariance matrix with the (i, j)th element of $C(i, j) = \rho(i, j, l_\rho)\ \sigma_i\ \sigma_j$ is the covariance between the deviation of the ith and the jth vertex and ψ is a vector of samples from a standard Gaussian distribution $(\mu = 0, \sigma = 1)$. In the following, the values for the modelling of random geometric deviations by Gaussian random fields are set as: $\mu_{D,B} = 0$, $\sigma_D = 5$, $\sigma_B = 2.5$, $l_{\rho,D} = 50$, $l_{\rho,B} = 10$, where the subscript D marks the disk and B the bolt, respectively. Specific outcomes of Skin Model Shapes for the disk with different correlation lengths are shown in Fig. 4.

Since several approaches for the generation of Skin Model Shapes exist, such as modelling of systematic and random deviations as well as employing results from manufacturing process simulations or real-life measurements of part prototypes, approaches for the digital measurement of these shapes are required. This is because the tolerance analysis accounts for a relationship between geometric deviations measured as toler-



(a) Detail of the Nominal Geometry of the Disk with local Vertex Normals

(b) Skin Model Shape with l_ρ = 5



(c) Skin Model Shape with l_ρ = 10

(d) Skin Model Shape with l_ρ = 25

Fig. 4. Outcomes of Skin Model Shapes for the Disk Surface with different Correlation Lengths

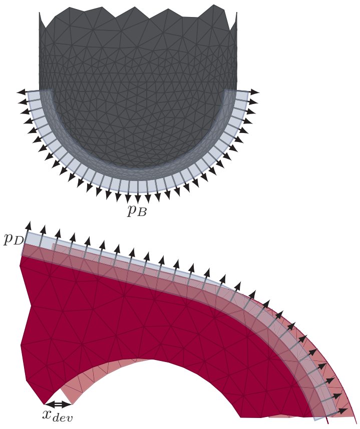
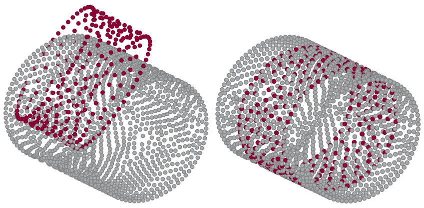
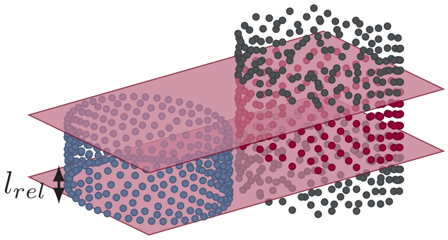


Fig. 5. Considered Geometric Deviations of the Disk Cam Mechanism



(a) ICP-based Assembly for Cylindrical Features



(b) Modification for the Bolt Assembly

Fig. 6. Assembly of the Cylinders by Registration

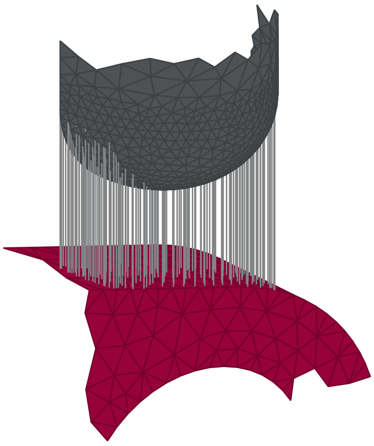


Fig. 7. Raytracing for the Contact Analysis

ances and the functional key characteristic. Nowadays, geometric deviations are evaluated by algorithms implemented in measurement machines, which process point clouds obtained from tactile or optical measurement systems. These algorithms can be used for determining the geometric deviations of Skin Model Shapes. In this contribution, geometric form deviations of the cam disk and the bolt as well as position deviations between the disk axis and the disk are evaluated. Since the projection of nominal points along their corresponding vertex normals onto the measured points is common practice in the context of topography evaluations, this procedure is applied for the form deviation determination of the bolt p_B and the disk p_D, where the respective tolerance zones can be seen from Fig. 5. Furthermore, the position deviation of the disk in x-direction is considered as x_{dev}.

4.3. *Processing: Assembly and Contact Simulation*

4.3.1. *Assembly and Contact Simulation for Skin Model Shapes*

In order to determine the relationship between the geometric part deviations and the functional key characteristic in the proposed approach for the tolerance analysis of mechanism, the Skin Model Shapes have to be assembled. For the accompanying example, this requires the assembly simulation of cylindrical fits between the disk and its axis as well as the bolt and its bracket, respectively. For this purpose, the well-known Iterative Closest Point algorithm (ICP) [29] is used to fit the cylinder of the disk and its axis as can be seen from Fig. 6 (a). This procedure is adapted for the fit between the bolt and its bracket, where the relevant points of the bolt for the registration by the ICP are selected by GeoSpelling extraction operations according to the nominal bolt altitude l_{rel} as illustrated in Fig. 6 (b). Since the ICP minimizes the sum of squared Euclidean distances between the selected vertices of the bolt and the bracket, the selection of the relevant bolt vertices is required in order to avoid adulterated results for the assembly fit.

The assembly simulation is then completed by a ray trace algorithm [30], which is employed to determine the height between the pre-assembled bolt and the disk as illustrated in Fig. 7. In this regard, the vertices of the bolt are traced along the direction given by the cylinder axis of the bracket. The shortest distance between any of these vertices and the surface of the disk can then be found as the contact constraint and is therefore used to adapt the altitude of the bolt. This can be seen from Fig. 8.

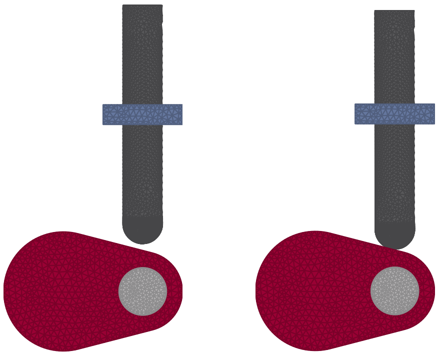


Fig. 8. Assembly Position before (left) and after (right) the Ray Tracing

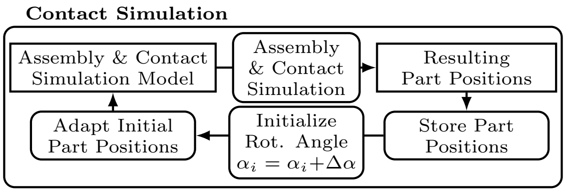


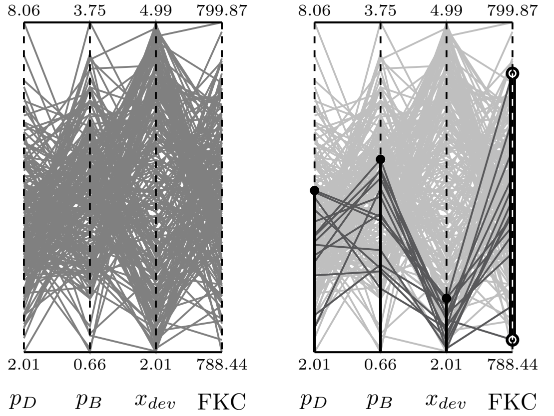
Fig. 9. Model for the System Behaviour

4.3.2. *Model for the System Behaviour*

The model for the assembly simulation of the mechanism can be used to evaluate the system behaviour for selected positions of interest, for example for a disk rotation angle of $\alpha = 0°$ or $\alpha = 90°$. However, in order to determine the behaviour of the mechanism in motion, a time-discretization is performed, i. e. the rotation of the disk is apportioned in discrete angle steps α_i with distance $\Delta\alpha$. For each of these motion steps, the system behaviour of the disk cam mechanism is analysed based on the assembly and contact simulation models. For this purpose, the initial assembly positions of the parts are adapted taking into account the disk rotation as well as the bolt revolution. Thus, the model for the system behaviour is a sequence of assembly steps, with varying initial part positions, as can be seen from Fig. 9. In this regard, the simulation of the different motion steps can be performed by parallel computing, which decreases the required computing time.

4.4. *Post-Processing: Result Visualization and Interpretation*

The interpretation of tolerance analysis results is an important step to finally derive proper tolerancing decisions. Especially for systems in motion considering not only dimensional but also form deviations, this task can become complex and requires user experience. Thus, in order to enable the result interpretation and to ease the decision making process, adequate visualization methods have to be employed. These methods are to visualize and to reveal relationships between the geometric part deviations and the functional key characteristics. For this purpose, the parallel coordinates plot [31] is a suitable approach for the visualization of tolerance analysis results for time invariant key characteristics, where all input and output parameters are shown in one plot and highlighted as a connecting line. Such a parallel coordinates plot for the accompanying example of the disk cam can be seen from Fig. 10 (a), where the functional key characteristic is the maximum bolt altitude h_b over the motion cycle. Based on the parallel coordinates plot, also the effects of geometric part specifications on the FKC can be analysed. In this regard, Fig. 10 (b) shows the effects of part tolerances of $p_D = 5$, $p_B = 2.5$, and $x_{dev} = [0; 2.5]$ on the FKC. It can be seen, that these requirements lead to values of the FKC from 788.86 to 798.10.



(a) Parallel Coordinates Plot without Tolerance Specifications

(b) Parallel Coordinates Plot with Tolerance Specifications

Fig. 10. Parallel Coordinates Plot of the Tolerance Specifications and the FKC

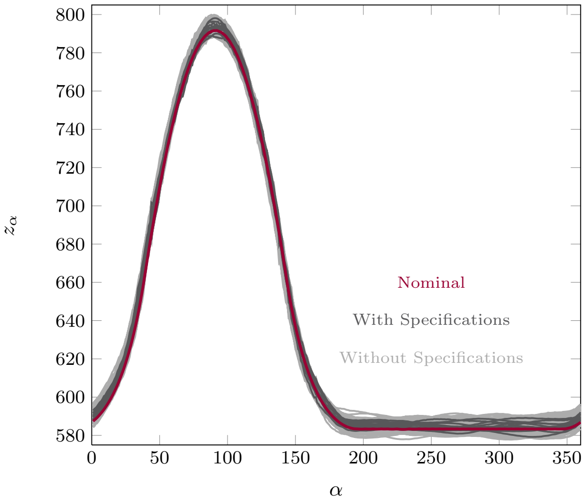


Fig. 11. Trajectories of the FKC with and without Tolerance Specifications

The parallel coordinates plot aims at revealing the relationship between the geometric deviations and one or more functional key characteristics. However, time variant key characteristics can hardly be visualized by this approach. In order to overcome this problem, a straightforward solution is to plot the FKC over the motion cycle. This can be seen from Fig. 11, where the bolt altitude is plotted against the motion steps t_i for a disk revolution angle α from 0° to 360°. The red line highlights the bolt altitude for nominal parts, whereas the dark lines highlight the results for Skin Model Shapes, which conform to the tolerance requirements as specified ($p_D = 5$, $p_B = 2.5$, $x_{dev} = [0; 2.5]$). It can be seen, that the specification of part tolerances results in a less volatile developing of the bolt altitude over the motion cycle, which can be traced back to the profile tolerances.

5. Conclusion and Outlook

The prediction of the effects of geometric part deviations on the functional compliance and quality of mechanism by adequate simulation models is an ongoing focus of research in the field of computer aided tolerancing. However, up to now, most presented approaches lack of form deviation considerations and do not ground on a complete and coherent language for the geometric product specification and verification. This hinders the consideration of manufacturing and inspection aspects in computer aided tolerancing during design. Therefore, an approach for the tolerance analysis of mechanism is proposed in this contribution, which grounds on a surface mesh representation of deviated workpiece representatives denoted to as Skin Model Shapes. It employs methods known from registration and computational geometry, such as the ICP and ray trace algorithms, to compute the assembly positions and the contact between non-ideal parts. Based on the visualization and interpretation of the obtained results, robust tolerancing decisions for mechanism can be derived. Future research will focus on the integration of results obtained from computer aided manufacturing tools and on the consideration of various physical phenomena, such as gravity and friction.

It can be concluded, that the discrete geometry representation of deviated workpieces, which are referred to as Skin Model Shapes, enables the consideration of various kinds of geometric deviations in the tolerance analysis. Therefore, a more holistic image of the product behaviour during use can be drawn in engineering design. Furthermore, it is a further step towards a coherent and complete computer aided tolerancing process considering design, manufacturing, and inspection aspects.

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