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Wess–Zumino supersymmetric phase and superconductivity in graphene

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ABSTRACT

Supersymmetry is expected to exist in nature at high energies, but must be spontaneously broken at ordinary energy scales. The required energy scale in elementary particle physics is currently inaccessible, but condensed matter could furnish low energy realizations of supersymmetry. In graphene, electrons behave as 'relativistic' massless fermions in $1+2$ dimensions. Here we propose phenomenologically, assuming that some microscopic parameters can be fine-tuned in graphene, the existence of a supersymmetric Wess–Zumino phase. The supersymmetry breaking leads to a superconductor phase, described by a relativistic Ginzburg–Landau phenomenology.

Modern quantum field theories are firmly grounded on the concept of symmetry [1]. A fundamental symmetry known as supersymmetry, yet unobserved, connects particle fields obeying opposite statistics, *i.e.*, transforms bosons into fermions and vice-versa. It also unifies space–time with gauge symmetries, and is expected to unify gravity with the other fundamental forces of nature when properly gauged [1–5]. Supposedly, supersymmetry exists at very high energies and is spontaneously broken at the scales currently accessible in the study of elementary particle physics. Fortunately, condensed matter physics provides a fertile ground for the search of supersymmetric models that emulate the world of elementary particles at a much lower energy scale. In recent years, the most conspicuous example of a solid state system allowing the study of fundamental aspects of elementary particles is graphene, a genuine two-dimensional material. Graphene has a peculiar band structure, in which electrons emulate the behavior of massless Dirac fermions in a $(1+2)$-D relativistic space–time near the so-called Dirac points of the Brillouin zone of a honeycomb lattice [6,7], providing an interesting bridge between condensed matter and relativistic high-energy physics [8].

Concerning supersymmetry, the simplest theory is provided by the Wess–Zumino (WZ) model [1–3,5,9], yielding a graded Lie algebra of the Poincaré group that allows the unified description of a spin-$1/2$ fermion field and a spin-0 boson field related to each other by a supersymmetric transformation. Since supersymmetry implies that both fields, also known as superpartners, must possess the same mass m, it is difficult to encounter an example of this case in nature at low energy scales. We then speculate that such an example has to be sought in systems of massless particles. Recently, an optical lattice realization of the WZ model was proposed, in which a cold atom–molecule mixture provides the emergence of supersymmetry by fine-tuning the atomic and molecular interactions [10]. Here we raise the question whether graphene could furnish another condensed matter prototype of the WZ model. The breaking of supersymmetry would imply superconductivity, as it was conjectured long ago to take into account the behavior of high-T_c superconductors [11,12]. Earlier studies show that one of the requirements for the emergence of supersymmetry is the presence of Dirac points in the Brillouin zone [13], favoring the formation of Cooper pairs which would act as Klein–Gordon bosons under special circumstances. Graphene certainly accomplishes that goal, at least near its Fermi energy. The Dirac points are located at the corners of the hexagonal Brillouin zone, with two inequivalent points, which in the literature are called $\mathbf{K}$ and $\mathbf{K}'=-\mathbf{K}$ [8]. The emergence of supersymmetry from a tight-binding model in the honeycomb lattice at a critical point was demonstrated by Lee for spinless fermions [14] which, possess fewer degrees of freedom than electrons in graphene. Concerning superconductivity, graphene properties as a superconductor are still under intense debate [15–18]. There is experimental evidence for intrinsic superconductivity in doped samples of graphite, which consists of stacked graphene layers [19]. The precise microscopic pairing mechanisms providing the superconducting instabilities are not well understood but a number of possibilities were proposed, such as phonon- and plasmon-mediated interactions [15] and resonant valence bonds and density waves in lattice models [20–22]. A detailed analysis of conventional electron–phonon

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mediated superconductivity in doped graphene showed that the critical temperature is in the range of $T_c \sim 10$ K [23]. Theories dealing with unconventional pairing mechanisms, involving spin triplet pairing, have also been proposed [24,17,25,26]. Recent experiments suggest that proximity effects induce a supercurrent in monolayer graphene contacted by superconducting electrodes [27] and a theoretical calculation showed that the supercurrent can be tuned with high efficiency at the charge neutrality point through mechanical strain [28].

Here we will adopt a more phenomenological route to describe a supersymmetric Wess–Zumino phase in monolayer graphene, assuming it exists, regardless of the microscopic mechanisms leading to the emergence of supersymmetry. The breaking of supersymmetry imply the existence of a superconducting phase, in which electrons are paired. Since the electrons near a Dirac point behave as massless Dirac fermions, we postulate that the Cooper pairs formed by massless electrons in a singlet state will be described by relativistic Klein–Gordon scalar bosons in $(1+2)$-D space–time, where the Fermi velocity v_F plays the role of the speed of light c. Our starting point is the Wess–Zumino Lagrangian density in the form below:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \partial_\mu\phi^*\partial^\mu\phi - i\frac{g'}{2}\left(\psi^T\Gamma\psi\phi^* + \bar{\psi}\Gamma\bar{\psi}^T\phi\right) - \frac{g^2}{8}|\phi|^4, \tag{1}$$

where ϕ is a boson field and $\psi = (\xi_\uparrow, \xi_\downarrow, \zeta_\uparrow, \zeta_\downarrow)^T$ is an 8-component Dirac spinor, $\bar{\psi} = \psi^\dagger\gamma^0$ is the adjoint fermion field, $\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2)$ are the Dirac matrices, $\partial_\mu = \partial/\partial x^\mu$ is the derivative operator, $x^\mu = (x^0 = v_F t, x^1, x^2)$ are the space–time coordinates in $1+2$ dimensions, the Fermi velocity v_F plays the role of the speed of light c and the index μ runs from 0 to 2. The two-component spinor $\xi_s(\zeta_s)$ describes an electron at the $\mathbf{K}(\mathbf{K}')$ Dirac point with genuine spin $s = (\uparrow, \downarrow)$. Usually one defines an index known as the valley pseudospin α, corresponding to the Dirac points $\mathbf{K}(\alpha = +1)$ and $\mathbf{K}' = -\mathbf{K}(\alpha = -1)$, *i.e.*, the spinors ξ and ζ correspond to valley pseudospin $+1$ and -1, respectively. Besides the genuine spin s and the valley pseudospin α there is an additional sublattice pseudospin, associated with the Pauli matrices $\vec{\tau} = (\tau_x, \tau_y, \tau_z)$, such that for an electron at $\mathbf{K}$ we have $\tau_z\xi_s = \pm 1\xi_s$, where the eigenvalue $+1(-1)$ represents an electron located at the sublattice A (B). This way, the Dirac matrices satisfying the anti-commuting relation, $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, where $g^{\mu\nu} = \mathrm{diag}(1, -1, -1)$ is the Minkowski metric tensor, can be explicitly represented by $\gamma^0 = \mathrm{diag}(\tau_z, \tau_z, \tau_z, \tau_z)$, $\gamma^1 = \mathrm{diag}(i\tau_x, i\tau_x, -i\tau_x, -i\tau_x)$ and $\gamma^2 = \mathrm{diag}(i\tau_y, i\tau_y, i\tau_y, i\tau_y)$. Finally, the parameters g' and g are effective couplings.

Taking into account the pseudospin indices (α and τ) and the genuine spin (s) the totally anti-symmetric wave function of an electron pair in graphene can be expressed as a tensor product $\Psi_{pair} = \psi_{K,K'} \otimes \psi_{A,B} \otimes \psi_{s,s'}$. Conventional pairing imply a spin-singlet state, allowing only the triplet-triplet or singlet–singlet product of sublattice and valley isospin functions. Restricting our attention to the singlet–singlet–singlet pairing, which corresponds to the scalar boson field of our tight-binding model, we chose the coupling matrix Γ to be:

$$\Gamma = \begin{pmatrix} 0 & 0 & 0 & \tau_y \\ 0 & 0 & \tau_y & 0 \\ 0 & -\tau_y & 0 & 0 \\ -\tau_y & 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

We suppose that the parameters of the above model can be fine-tuned applying external electric and magnetic fields or by doping [15]. The quantum critical point $g' = g$ leads to the emergence of the Wess–Zumino supersymmetric model [5]:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \partial_\mu\phi^*\partial^\mu\phi - i\frac{g}{2}\left(\psi^T\Gamma\psi\phi^* + \bar{\psi}\Gamma\bar{\psi}^T\phi\right) - \frac{g^2}{8}|\phi|^4, \tag{3}$$

where $g = -\upsilon/\lambda$ is the effective coupling constant in the supersymmetric phase. The Lagrangian density (3) is invariant (apart from a total divergence) under a supersymmetric transformation of the form:

$$\psi' = \psi - (\gamma^\mu\partial_\mu\phi)\eta + \frac{g}{4}\Gamma\bar{\eta}^T|\phi|^2, \tag{4}$$

$$\phi' = \phi + i\bar{\eta}\psi, \tag{5}$$

where η is an 8-component constant Majorana spinor and $\bar{\eta}$ its adjoint. Eqs. (4) and (5) are responsible for the mixing of bosons and fermions. The supersymmetric phase is highly unstable under de-tuning of the coupling parameters.

Notice that the third and fourth terms in Eq. (3), *i.e.* $\psi^T\Gamma\psi\phi^*$ and $\bar{\psi}\Gamma\bar{\psi}^T\phi$, represent the coupling between the Dirac massless electron field ψ and the boson field ϕ, corresponding to the annihilation (creation) of two electrons and creation (annihilation) of an excitation of the boson field. Since the fermionic field ψ is associated with the electrons, it must be charged under the electromagnetic $U(1)$ gauge group. Consistency between the fermion–boson interactions and charge conservation imply that if the field ψ possess electric charge $q = -e$, the excitation of the field ϕ must carry charge $Q = -2e$. Thus, the field ϕ is interpreted as the order parameter describing the macroscopic wave function of the condensate of Cooper pairs. The gauge invariance under the $U(1)$ symmetry group, for which the fields are transformed according to $\psi' = e^{ie\Lambda}\psi$ and $\phi' = e^{2ie\Lambda}\phi$, being Λ an arbitrary space–time function, is achieved replacing ordinary derivatives ∂_μ by their covariant versions, $D_\mu\psi = (\partial_\mu - ieA_\mu)\psi$ and $D_\mu\phi = (\partial_\mu - 2ieA_\mu)\phi$, where A_μ is the electromagnetic potential describing the in-plane components E_x, E_y of the electric field and the perpendicular component B_z of the magnetic field, subjected to the gauge transformations $A'_\mu = A_\mu + \partial_\mu\Lambda$. The $U(1)$ gauge symmetry is responsible for the breaking of WZ supersymmetry in the massless case, because the fields ψ and ϕ carry distinct values of electric charge. The equations of motion for the field ϕ, written explicitly below:

$$D_\mu D^\mu\phi + \frac{g^2}{4}|\phi|^2\phi = -i\frac{g}{2}\psi^T\Gamma\psi, \tag{6}$$

is a relativistic version of the Gross–Pitaevskii equation with a source term $-i(g/2)\psi^T\Gamma\psi$. Within the mean field approximation, we make the replacement $\langle -i\psi^T\Gamma\psi\rangle = \alpha\phi/g$. This procedure decouples ϕ from the fermion field and leads to the following Lagrangian density for ϕ and A_μ:

$$\mathcal{L} = D_\mu\phi D^\mu\phi^* + \frac{\alpha}{2}|\phi|^2 - \frac{g^2}{8}|\phi|^4 + \mathcal{L}_{em}. \tag{7}$$

At this point, we must emphasize the main difference of the above model of graphene superconductivity from the usual 2D superconductors. While the former is described by a relativistic Lagrangian density, meaning that the Cooper pairs behave as relativistic particles, the later is usually described by non-relativistic Cooper pairs. The Lagrangian density $\mathcal{L}_{em}$ related to the electromagnetic field cannot be given by the usual term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor, because the photon field lives in a $(3+1)$-D space–time and interacts with fermions confined on a plane. To get the correct description of the photon field the starting point is usual action for A_μ in $3+1$ space–time and the matter current confined to the plane. Integrating out the z-coordinate perpendicular to the plane one obtains the Lagrangian density of the electromagnetic field, describing B_z, E_x and E_y [29,30]:

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} \frac{1}{\sqrt{\partial^2}} F^{\mu\nu}, \tag{8}$$

where $\partial^2 = \partial_\mu \partial^\mu$ is the d'Alembertian operator in $2+1$ space–time.

Expression (7) is reminiscent of the Ginzburg–Landau theory and the sign of the parameter α determines whether the vacuum spontaneously breaks a gauge symmetry or not. Directly from Noether's theorem we get an expression for the conserved current:

$$J_\mu = -2ie\big[\phi^*(\partial_\mu \phi) - (\partial_\mu \phi^*)\phi\big] - 8e^2|\phi|^2 A_\mu. \tag{9}$$

The absence of an explicit mass parameter in the definition of the current J_μ is a consequence of the fact that Cooper pairs described by the field ϕ behave as "relativistic" particles with speed v_F. In the superconducting phase we assume that α and g^2 appearing in (7) are both positive definite, such that the potential energy $V(\phi) = -\alpha|\phi|^2/2 + g^2|\phi|^4/8$ has a minimum at $|\phi| \neq 0$, meaning that the macroscopic density of condensed Cooper pairs is non-vanishing. The vacuum expectation value of the field ϕ, which minimizes the potential $V(\phi)$, is given by $|\phi_0| = \sqrt{2\alpha/g^2}$, allowing us to expand the field about the minimum through new field variables φ and θ in the form $\phi(x^\mu) = [|\phi_0| + \varphi(x^\mu)]e^{i\theta(x^\mu)}$. It can be straightforwardly shown that the field φ picks up a mass $m_\varphi^2 = \alpha$, which is directly related to the superconductor energy gap $\Delta = \sqrt{\alpha}$. In the superconducting state $\phi = |\phi_0|$, the supercurrent turns out to be $J_\mu = -8e^2|\phi_0|^2 A_\mu$, which results in the breaking of the $U(1)$ gauge symmetry, since $\partial_\mu J^\mu = 0 \to \partial_\mu A^\mu = 0$ the photon field A_μ acquires mass, $m^2 = 8e^2|\phi_0|^2$. From the London equations describing the electrodynamics of a superconductor, we obtain a relation between the photon mass and the London penetration depth:

$$\lambda_L = \frac{1}{m} = \frac{1}{\sqrt{8}e|\phi_0|}. \tag{10}$$

A measurement of the London penetration depth λ_L and independently the superconductor gap Δ, provides knowledge of parameters α, g and $|\phi_0|$:

$$\alpha = \Delta^2, \tag{11}$$

$$g^2 = 16e^2\lambda_L^2\Delta^2, \tag{12}$$

$$|\phi_0| = \frac{1}{\sqrt{8}e\lambda_L}. \tag{13}$$

For pure graphene the chemical potential μ is located exactly at the Dirac points in the Brillouin zone. Therefore, in pure and unbiased graphene the value of $|\phi|^2$ should be zero, since the electronic density of states vanishes at those points. To be precise, the density of states of massless relativistic particles of energy ε in two space dimensions, including spin degeneracy, is given by $\rho(\varepsilon) = |\varepsilon|/(\pi v_F^2)$, where $\varepsilon = 0$ denotes the Dirac point. However, we can finely tune the chemical potential by doping graphene, in order to locate the Fermi level above (electron-like) or below (hole-like) the Dirac points. A critical issue in doing so, is not disturbing the dispersion relation of Dirac electrons. If, for instance, the chemical potential can be moved slightly above the Dirac point and assuming that all the occupied electron states condense, the macroscopic density of Cooper pairs will be given approximately by $|\phi_0|^2 \propto \mu^2/(2\pi v_F^2)$.

In summary, we discussed from a phenomenological standpoint the existence of a supersymmetric phase in monolayer graphene at a critical point, described by the Wess–Zumino model for massless Dirac fermions in $1+2$ space–time dimensions. In graphene, it is supposed that the coexistence of nearly free fermions and Cooper pairs acting as bosons are allowed. The usual spin-singlet bound pair was assumed for simplicity. Finally, we interpret the possible existence of graphene superconductivity, described by a 'relativistic' Ginzburg–Landau phenomenological model, as a result of supersymmetry breaking.

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