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# Leading low-energy effective action in the 6D hypermultiplet theory on a vector/tensor background

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ABSTRACT

We consider a six-dimensional $(1,0)$ hypermultiplet model coupled to an external field of vector/tensor system and study the structure of the low-energy effective action of this model. Manifestly a $(1,0)$ supersymmetric procedure of computing the effective action is developed in the framework of the superfield proper-time technique. The leading low-energy contribution to the effective action is calculated.



## 1. Introduction

In our recent paper [1] we have developed the harmonic superfield formulation of the 6D vector/tensor system and constructed its coupling to 6D hypermultiplet. One of the important and interesting applications of such a coupling is a problem of the effective action induced by the hypermultiplet interaction with the vector/tensor background. In the paper [1] we introduced the corresponding effective action, which is a harmonic superfield functional of the vector/tensor system, and computed the structure of its divergences. The present paper is devoted to continuation of the research originated in [1]. Our basic purpose here is to calculate the finite first leading low-energy contribution to the effective action. The main motivation to studying the low-energy effective action in the theory under consideration is related to a description of the low-energy dynamics of M5-branes in terms of field theory.

As it is known, the M2- and M5-branes arise as states of the strong coupling phase of M-theory (see e.g. [2] for a review and references). The low-energy dynamics of a single M5-brane is described by the Abelian $\mathcal{N}=(2,0)$ tensor multiplet [3]. The field content of this multiplet is determined as follows. There are five scalars which arise as the Goldstone bosons from spontaneous breaking of the eleven-dimensional translational invariance by a brane. The M5-brane is a 1/2-BPS object and therefore there are eight fermionic degrees of freedom. The three additional bosonic degrees of freedom are provided by an Abelian 2-form gauge field $B_{ab}$ which has a self-dual field strength $H_{abc}$. This 2-form originates from breaking the gauge symmetry of the 3-form potential which exists in M-theory. However a Lagrangian description of such a system faces a problem: the kinetic term for the 2-form gauge field is identically zero because of the self-duality condition. In the non-Abelian case, there is an additional problem since an appropriate generalization of the tensor gauge symmetry is still unknown [4].[1] In addition, there are the inevitable problems of quantization of such models and whether the conformal symmetry is preserved at the quantum level.

The low energy theory of multiple M5-branes is an interacting six-dimensional conformal field model with $(2,0)$ supersymmetry (see e.g. [6] for a review and references). The existence of such field theories, as well as all of their known properties, has originated from string theory, where they occur in various related contexts: the IR limit of the M5 or IIA NS-5 brane world-volume theory, IIB string theory on an ALE singularity [7], M theory on $AdS_7 \times S^4$ [8], etc. The IR-limit of these theories are $(2,0)$ superconformal field models which obey an ADE-classification: $SU(N)$, $SO(2N)$, or $E_{6,7,8}$ [9], but have no other parameters. It is worth pointing out that all that is known about an interacting 6D, $\mathcal{N}=(2,0)$ field theory has been obtained from string theory. In par-

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[1] Various proposals for dealing with this problem have been suggested (see e.g. [5] for a review and references).

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ticular, the non-trivial $SO(5)_R$ 't Hooft anomaly was found in [10] in the context of 11d M-theory, which gave the anomaly for the case $G = SU(N)$, realized as N parallel M5 branes. The corresponding anomaly coefficient for the $SU(N)$ case was found with help of M theory on $AdS_7 \times S^4$ in [11] to be $c_{SU(N)} = N^3 - N$.

In a series of works [12] it was considered the possibility of constructing the $(2,0)$ theory of multiple M5-branes using $(1,0)$ supersymmetry in the framework so-called the non-Abelian hierarchy of $p$-form fields [13]. In this case the following supermultiplets are used: tensor multiplet, hypermultiplet and super Yang–Mills multiplet. In the framework of these models the SYM multiplet should be auxiliary analogous to non-propagating gauge fields in the BLG or ABJM theory for multiple M2-branes. Such models are parameterized by a set of dimensionless constant tensors, which are constrained to satisfy a number of algebraic identities. A concrete model is defined by the explicit choice of the gauge group and representations and the above associated invariant tensors. All these theories can be treated as belonging to the same universality class of theories which are dual to $AdS_7 \times S^4$ and possibly describe multiple M5-branes. Several explicit examples which satisfy all algebraic consistency conditions have been discussed in the literature (see e.g. [12]).

Superfield formulation of the tensor hierarchy has been studied in the paper [14] where a set of constraints on the super-$(p+1)$-form field strengths of non-Abelian super-$p$-form potentials in the $(1,0)$ 6D superspace has been proposed. In [1] we considered six-dimensional hypermultiplet, vector and tensor multiplet models in $(1,0)$ harmonic superspace and discussed the corresponding superfield actions (see also [15–17]). The superfield actions for a free $(2,0)$ tensor multiplet and for an interacting vector/tensor multiplet system in terms of $(1,0)$ superfields have been constructed for the first time in [1]. To construct $(2,0)$ theory, one adds $n_T$ $(1,0)$ superconformal hypermultiplets to the above $(1,0)$ vector/tensor system. It is worth mentioning that there is no direct interaction between hypermultiplets and tensor multiplets, a coupling between these multiplets is provided by a vector multiplet (see e.g. [12]). Such a coupling comes through the auxiliary fields, which are described by the algebraic field equation $d_{Irs}(Y^s_{ij}\phi^I - 2\bar\lambda^s_{(i}\chi^I_{j)}) - \ldots = 0$. In general, this equation implies constraints on the elementary fields [12] but inclusion of Abelian factors or tensor multiplet singlets, allows us to bypass constrains on the elementary fields and, in particular, leads to the interaction terms of the form $\mathcal{L}_{\phi^0\mathcal{F}^2}$. In that case there is a unique solution for the auxiliary fields $Y_{ij}$. The resulting scalars can take any values and then the vev of the tensor multiplet scalar acts as an inverse Yang–Mills coupling constant in the conformal broken phase. This effect is similar in many aspects of the "M2 to D2" scenario [18] proposed for the BLG theory which teaches us that the M2-brane field theory is the strongly-coupled limit of the D2-brane theory where the type IIA string theory transforms into M-theory. Such a circumstance allows us to consider the Coulomb brunch of the theory and study of the perturbative properties of the models on this branch.

The next natural question is, what are the higher-order corrections to the M5-brane action where the fields of the vector multiplet become dynamic degrees of freedom. One of the direct ways to answer this question is to derive the effective action by calculating the open string scattering amplitudes. This program for the Abelian case yielded the full higher-derivative purely bosonic terms in the Dirac–Born–Infeld approximation [19]. In addition, there exists a remarkable connection between (i) partial supersymmetry breaking, (ii) nonlinear realizations of extended supersymmetry, (iii) BPS solitons, and (iv) nonlinear Born–Infeld–Nambu type actions [20–22].[2] On the other hand, the systems of D5-branes have complementary descriptions in terms of gauge theory (see e.g. [23]). As one of the consequences, the leading-order interaction potential between separated branes admits representation as a leading term in the quantum gauge theory effective action. The agreement between the supergravity and the gauge theory expressions for the potential is possible because of the existence of certain non-renormalization theorems on the gauge theory side (see e.g. [24]). Since the hypermultiplet has a universal coupling to the vector multiplet, one can expect that, in the context of field theory, it will be possible to derive directly the leading higher order 6D supersymmetric correction to the classical action. Precisely this problem is considered in the present paper.

We begin with harmonic superfield 6D hypermultiplet coupled to an external field of vector/tensor system and compute the one-loop effective action depending on the superfields of the vector/tensor system. To develop the method of calculating of the effective action and study of its possibilities we consider the simplest case when all the fields are Abelian. As the result we find superfield action which corresponds to the 6D $(1,0)$ superconformal '$F^4$' term in the components.

## 2. Model of 6D hypermultiplet coupled to vector/tensor system

We consider the hypermultiplet model coupled to an external field of the vector/tensor system in the framework of the formalism of the $(1,0)$ harmonic superspace.[3] Our main aim is to compute the leading low-energy contribution to the superfield effective action depending on the superfields of the vector/tensor system.

Let us briefly discuss the structure of the vector/tensor system. The $(1,0)$ superconformal 6D field theory of the vector/tensor system describes a hierarchy of non-Abelian scalar, vector and tensor fields $\{\phi^I, A^r_a, Y^{ij\ r}, B^I_{ab}, C_{abc\ r}, C_{abcd\ A}\}$ and their supersymmetric partners which are labeled by the indices $r = 1,\ldots,n_V$ and $I = 1,\ldots,n_T$ (see the details e.g. in [12]). The non-Abelian field strengths of the vector and two-form gauge potentials are given as

$$\begin{aligned}\mathcal{F}^r_{ab} &= \partial[a A^r_{b]} - f_{st}{}^r A^s_a A^t_b + h^r_I B^I_{ab},\\ \mathcal{H}^I_{abc} &= \frac{1}{2}\mathcal{D}_{[a}B^I_{bc]} + d^I_{rs}A^r_{[a}\partial b A^s_{c]}\\ &\quad - \frac{1}{3}f_{pq}{}^s d^I_{rs}A^r_{[a}A^p_b A^q_{c]} + g^{Ir}C_{abc\ r}.\end{aligned} \tag{1}$$

Here $f_{[st]}{}^r$ are the structure constants, $d^I_{(rs)}$ are the $d$-symbols, defining the Chern–Simons couplings, and $h^r_I$, $g^{Ir}$ are the covariantly constant tensors, defining the general Stückelberg-type couplings among the forms of different degrees. The existence of the non-degenerate Lorentz-type metric $\eta_{IJ}$, such that $h^r_I = \eta_{IJ}g^{Jr}$, $b_{Irs} = 2\eta_{IJ}d^J_{rs}$, is also assumed. The covariant derivatives are defined as $\mathcal{D}_a = \partial a - A^r_a X_r$ with the gauge generators $X_r$ acting on the different fields as follows: $X_r \cdot \Lambda^s \equiv -(X_r)^s_t\Lambda^t$, $X_r \cdot \Lambda^I \equiv -(X_r)^I_J\Lambda^J$. The covariance of the field strengths (1) requires that the gauge group generators in the various representations should have the form

$$(X_r)^t_s = -f_{rs}{}^t + g^t_I d^I_{rs}, \quad (X_r)^J_I = 2d^J_{rs}g^s_I - g^{Js}d_{Isr},$$

in terms of the invariant tensors parameterizing the system (see the details in [12]). The field strengths (1) are defined in such a

[2] Due to the large number of relevant papers we have no possibility to cite a large number of papers on these aspects.

[3] We follow the harmonic superspace conventions of [25] to which we refer for definitions, notations and additional references. Its application to vector/tensor system is discussed in [1].

way that they transform covariantly under the set of non-Abelian gauge transformations

$$\delta A_a = \mathcal{D}_a \Lambda^r - h^r_I \Lambda^I_a,$$
$$\delta B^I_{ab} = \mathcal{D}_{[a}\Lambda^I_{b]} - 2d^I_{rs}(\Lambda^r \mathcal{F}^s_{ab} - \frac{1}{2}A^r_{[a}\delta A^s_{b]}) - g^{Ir}\Lambda_{ab\, r}. \quad (2)$$

The superspace realization of the tensor hierarchy was developed in the paper [14] in framework of the conventional 6D, $(1,0)$ superspace by means of study of the consistency conditions for the generalized Bianchi identities. In [1] we reformulated the 6D hypermultiplet, vector and tensor multiplet models in $(1,0)$ harmonic superspace and discussed the corresponding superfield actions. Further, we will use the results of the works [14,1]. It is convenient to introduce the generalized superfield strength

$$\mathcal{W}^{i\alpha\, r} = W^{i\alpha\, r} + g^r_I \mathcal{V}^{i\alpha\, I}, \quad (3)$$

where the $W^{i\alpha\, r}$ is the superfield strength of the super Yang–Mills theory (defined in [15,16]) and $\mathcal{V}^{i\alpha\, I}$ is the superpotential of the tensor multiplet (defined in [17]), and write the generalized Bianchi identities in its terms. Then one can see that the conventional strength $F_{ab}$ of the vector multiplet and the potential $B_{ab}$ of the tensor multiplet enter into $\mathcal{W}^{i\alpha\, r}$ in the gauge covariant form $\mathcal{F}^r_{ab} = F^r_{ab} + g^r_I B^I_{ab}$. The other superfield strengths of the vector/tensor multiplet are defined as

$$\mathcal{Y}^{++r} = \frac{1}{4}\mathcal{D}^+_\alpha \mathcal{W}^{+\alpha r}, \quad g^r_I \Phi^I = \frac{1}{4}(\mathcal{D}^-_\alpha \mathcal{W}^{+\alpha r} - \mathcal{D}^+_\alpha \mathcal{W}^{-\alpha r}),$$
$$\Psi^{\pm I}_\alpha = -\frac{i}{2}\mathcal{D}^\pm_\alpha \Phi^I, \quad g^r_I \mathcal{H}^I_{abc} = \mathcal{D}_{[a}\mathcal{F}^r_{bc]}. \quad (4)$$

The algebra of the covariant derivatives $\mathcal{D}^\pm_\alpha$, $\mathcal{D}^{\pm\pm}$, $\mathcal{D}_a$ is described in [1]. By applying a harmonic-dependent gauge transformation, one can choose a $\lambda$-frame where $\mathcal{D}^+_\alpha \to D^+_\alpha$, $\mathcal{D}^{++} = D^{++} + V^{++}$, $\mathcal{D}^{--} = D^{--} + \mathcal{V}^{--}$, with $V^{++}$ the analytic prepotential for the off-shell vector multiplet, and the other harmonic connection $\mathcal{V}^{--}$ is the linear combination of the non-analytic potential $V^{--}$ for vector multiplet and the potential $\mathcal{V}^{(-2)}$ for on-shell tensor multiplet (see [16,17,1] for more details). By using these superfields one can define the superfield action in harmonic superspace as follows

$$S = \frac{1}{8}\int d\zeta^{(-4)} du\, g_{Ir}\{\Phi^I \mathcal{D}^{++}\mathcal{Y}^{++\, r} + D^+_\alpha \Phi^I \mathcal{D}^{++}\mathcal{W}^{+\alpha r}\}, \quad (5)$$

where $d\zeta^{(-4)}$ denotes the analytic subspace integration measure. The action (5) depends both on superfields $V^{++}$, $W^{\alpha i}$ of the vector multiplet and on superfields $\Phi$, $\mathcal{V}^{\alpha i}$ responsible for the tensor multiplet. If a vev of $\Phi$ is a constant $1/f^2$, this action takes the form of SYM action [15,16]

$$S \sim \frac{1}{f^2}\int d^6x d^8\theta du V^{++}V^{--},$$

as discussed above. The equation of motion for this action is $Y^{++} = (D^+)^4 V^{--} = 0$.

As a further step towards to a $(2,0)$ theory it was proposed in the papers [12] to complement the non-Abelian vector/tensor by superconformal hypermultiplets and construct the corresponding coupling. The Lagrangian for these theories consists of two pars. One part involves vector and tensor multiplets, and the second part contains hypermultiplets coupled to the vector/tensor system. These two parts are independently $(1,0)$ supersymmetric.

A conformally invariant hypermultiplet model can be formulated in six-dimensional $(1,0)$ harmonic superspace [16]. The corresponding superfield action in general case is written as follows

$$S = -\frac{1}{2}\int d\zeta^{(-4)} du (q^{+A}\mathcal{D}^{++}q^+_A + L^{(+4)}(q^+, u)). \quad (6)$$

The potential $L^{(+4)}(q^+, u)$ determines a hypermultiplet self-interaction [26], it is irrelevant for our purposes and will be omitted further. We want to emphasize that the superfield $V^{++}$ here is related to the superfield $\mathcal{V}^{--}$ through zero curvature equation (see [25]). The superfield strengths $\mathcal{W}^{+\alpha} = -\frac{1}{4}(D^+)^{3\alpha}\mathcal{V}^{--}$, involving the superfield $\mathcal{V}^{--\ r} = V^{--\ r} + g^r_I \mathcal{V}^{--\ I}_T$, obey the Bianchi identities which contain the superfields $\Phi$, $\Psi^i_\alpha$, $\mathcal{H}_{abc}$ related to tensor multiplet (see [1] for the details). As a result the action (6) describes the interaction of a hypermultiplet with a vector/tensor system.

## 3. Construction of effective action

We will discuss here the procedure of calculating the effective action corresponding to the hypermultiplet theory in an external field of a vector/tensor system (6). The effective action is defined by integrating out hypermultiplet and keeping the $U(1)$ vector/tensor system as a background.

A formal relation for the effective action follows from (6) in the form

$$\Gamma = i\,\mathrm{Tr}\ln \mathcal{D}^{++} = -i\,\mathrm{Tr}\ln G^{(1,1)}, \quad (7)$$

where the $G^{(1,1)}(\zeta_1, \zeta_2)$ is the hypermultiplet Green function, satisfying the equation:

$$\mathcal{D}^{++}_1 G^{(1,1)}(\zeta_1, \zeta_2) = \delta^{(3,1)}_A(\zeta_1, \zeta_2),$$
$$G^{(1,1)}(1|2) = -\frac{1}{4\,\widehat{\Box}_1}(\mathcal{D}^+_1)^4(\mathcal{D}^+_2)^4 \delta^{14}(z_1 - z_2)\frac{1}{(u^+_1 u^+_2)^3}. \quad (8)$$

Here $\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2)\delta^8(\theta_1 - \theta_2)$ is the delta-function in conventional superspace, $\delta^{(3,1)}_A(\zeta_1, \zeta_2)$ is the appropriate covariantly analytic delta-function $\delta^{(3,1)}_A(\zeta_1, \zeta_2) = (\mathcal{D}^+)^4\delta^{14}(z_1 - z_2)\delta^{(-1,1)}(u_1, u_2)$ and $(u^+_1 u^+_2)^{-3}$ a special harmonic distribution [25]. In Eq. (8), $\widehat{\Box}$ is the covariantly analytic d'Alembertian which arises when $(\mathcal{D}^+)^4(\mathcal{D}^{--})^2$ acts on the analytical superfield

$$\widehat{\Box} = -\frac{1}{8}(\mathcal{D}^+)^4(\mathcal{D}^{--})^2$$
$$= \mathcal{D}_a\mathcal{D}^a + \mathcal{W}^{+\alpha}\mathcal{D}^-_\alpha + \mathcal{Y}^{++}\mathcal{D}^{--} - \mathcal{Y}^{+-} - \Phi. \quad (9)$$

The operator $\widehat{\Box}$ (9) possesses the important properties

$$[\mathcal{D}^+_\alpha, \widehat{\Box}] = 0,$$
$$[\mathcal{D}^{++}, \widehat{\Box}]V^{(p)} = \mathcal{Y}^{++}(p-1)V^{(p)}, \quad (10)$$

where $V^{(p)}(\zeta, u)$ is an arbitrary analytic superfield of $U(1)$ charge $p$. To prove the above identities, one should make use of the following properties of the 6D, $(1,0)$ gauge covariant derivatives in harmonic superspace [15,16,1]

$$[\mathcal{D}^+_\alpha, \mathcal{D}^-_\beta] = 2i\mathcal{D}_{\alpha\beta}, \quad [\mathcal{D}^\pm_\gamma, \mathcal{D}_{\alpha\beta}] = -2i\varepsilon_{\alpha\beta\gamma\delta}\mathcal{W}^{\pm\gamma}. \quad (11)$$

The field strength $\mathcal{W}^{\pm\alpha}$ obeys the generalized vector/tensor Bianchi identities

$$\mathcal{D}^-_\alpha \mathcal{W}^{+\beta\, r} = \delta^\beta_\alpha(\mathcal{Y}^{+-\, r} + \frac{1}{2}\Phi^I g^r_I) + \frac{1}{2}\mathcal{F}^{\beta\, r}_\alpha,$$
$$\mathcal{D}^\pm_\alpha \mathcal{Y}^{+-\, r} = \pm i(\mathcal{D}_{\alpha\beta}\mathcal{W}^{\pm\beta\, r} + i\mathcal{D}^\pm_\alpha \Phi^I g^r_I). \quad (12)$$

These properties follow from the 6D $(1,0)$ vector/tensor multiplet formulation [14] in conventional superspace.

The definition (7) of the one-loop effective action is purely formal. The actual evaluation of the effective action can be done in various ways (see e.g. [27,28]). Further we mainly will follow [28] with some special differences and use the relation

$$\Gamma = \Gamma_{y=0} + \int_0^1 dy \partial y \Gamma(yV) = -i \,\mathrm{Tr} \int_0^1 dy (V^{++} G^{(1,1)}(y)), \quad (13)$$

where

$$\mathrm{Tr}(V^{++} G^{(1,1)}) = \int du_1 d\zeta_1^{(-4)} V^{++}(1) G^{(1,1)}(1|2)|_{1=2}. \quad (14)$$

Here $G^{(1,1)}(yV)$ means the Green function depending on the superfield $yV^{++}$. Now one substitutes the expression (8) for Green function $G^{(1,1)}(1|2)$ into (13) and uses a proper-time representation for the inverse operator $\frac{1}{\widehat{\square}}$. To avoid the divergences in the intermediate steps of calculation one considers the regularized inverse operator in the form ($\omega$ – regularization)

$$-\frac{1}{\widehat{\square}} = \int_0^\infty d(is)(is\mu^2)^\omega e^{is\widehat{\square} - \varepsilon s}. \quad (15)$$

The divergent part of the effective action has already been found in [1], it was shown that it defines a charge renormalization in the vector/tensor action (5), and a higher derivative SYM action, found in [16]. We calculate the effective action in the local approximation where the effective action is represented as a series in background fields and their derivatives and expressed in terms of the effective Lagrangian in the form

$$\Gamma = \int d\zeta^{(-4)} du \mathcal{L}^{(+4)}. \quad (16)$$

The further analysis is based on the following identity involving the product of $\mathcal{D}$-factors presenting in the Green function (see derivation of this identity for 4$D$ and 5$D$ cases in [28])

$$(\mathcal{D}_1^+)^4 (\mathcal{D}_2^+)^4 \frac{1}{(u_1^+ u_2^+)^3} = (\mathcal{D}_1^+)^4 \{(u_1^+ u_2^+)(\mathcal{D}_1^-)^4 - (u_1^- u_2^+)\Delta^{--} - 4\widehat{\square} \frac{(u_1^- u_2^+)^2}{(u_1^+ u_2^+)}\}. \quad (17)$$

Here

$$\Delta^{--} = i\mathcal{D}^{\alpha\beta}\mathcal{D}_\alpha^- \mathcal{D}_\beta^- + 4\mathcal{W}^{-\alpha}\mathcal{D}_\alpha^- - (D_\alpha^- \mathcal{W}^{-\alpha}). \quad (18)$$

Now we will discusses the restrictions on background. To find the leading low-energy contribution to effective action it is sufficient to consider a covariantly constant vector/tensor multiplet in the absence of the auxiliary fields (‘on-shell’ background)

$$\mathcal{D}_a \mathcal{W}^{\pm\alpha} = 0, \quad \mathcal{Y}^{ij} = 0. \quad (19)$$

For self-consistency of the relations (19) we should supplement the above relations by the following relations

$$D_\alpha^i \Phi = 0, \qquad D_\alpha^i \mathcal{F}_{ab} = 0. \quad (20)$$

In this case the operators $\widehat{\square}$ and $\Delta^{--}$ take a simple form and depend only on the background fields $\mathcal{W}^{+\alpha}$, $D_\alpha^- \mathcal{W}^{+\beta}$ and $\Phi$. Since the form of the effective Lagrangian is defined by the coefficients of these operators we can conclude that on the background under consideration, the effective Lagrangian should have the following general form

$$\mathcal{L}^{(+4)} = \mathcal{L}^{(+4)}(\mathcal{W}^{+\alpha}, D_\alpha^- \mathcal{W}^{+\beta}, \Phi). \quad (21)$$

Further we will see that in leading approximation the effective Lagrangian does not depend on $D_\alpha^- \mathcal{W}^{+\beta}$.

## 4. Leading low-energy contribution to effective action

We will consider now a computation of the leading low-energy quantum contribution to the effective action. First of all we substitute the expression for the Green function (8) into the expression for effective action (13) and use the identity (17). It leads to

$$\Gamma = \frac{i}{4} \int_0^1 dy \int d\zeta_1^{(-4)} du V_1^{++} \frac{1}{\widehat{\square}_1} (\mathcal{D}_1^+)^4 \{(u_1^+ u_2^+)(\mathcal{D}_1^-)^4 - (u_1^- u_2^+)\Delta_1^{--} - 4\widehat{\square} \frac{(u_1^- u_2^+)^2}{(u_1^+ u_2^+)}\} \delta^{14}(z_1 - z_2)|_{2=1}. \quad (22)$$

To get the leading low-energy contribution to the effective action one analyzes the terms in the expression (22) for the background under consideration. First of all we take into account that we should use eight $D_\alpha^\pm$-factors in this expression to eliminate the $\delta$-function of anticommuting variables via the identity

$$(D^+)^4 (D^-)^4 \delta^8(\theta - \theta')|_{\theta=\theta'} = 1. \quad (23)$$

Consider the last term in (22). We see that the operator $\widehat{\square}$ is cancelled and then there is no enough number of $D$-factors to eliminate the above $\delta$-function. Therefore this term is zero. Now consider the first term in (22). This term was analyzed in [1], it was shown that it is proportional to $\mathcal{Y}^{++}$, which is equal to zero on the background under consideration.[4] Now let us analyze the contributions of $\Delta^{--}$. The third term here is proportional to $\mathcal{Y}^{--}$ and hence, it vanishes on the background under consideration. Now we will use the proper-time representation (15) of the inverse operator $\widehat{\square}$ in (22) and expend $e^{is\widehat{\square}} = e^{is(\square - \Phi)} e^{is\mathcal{W}^+\mathcal{D}^-}$ in the power series in $\mathcal{W}^{+\alpha}\mathcal{D}_\alpha^-$. The leading contribution arises in the third order in this expansion. Consider the contribution of the second term in (18) after the above expansion. Schematically it has the form

$$\int d\zeta^{(-4)} du V^{++} \mathcal{W}^- (\mathcal{W}^+)^3 = \int d\zeta^{(-4)} du V^{++} \mathcal{D}^{--} (\mathcal{W}^+)^4 = \int d\zeta^{(-4)} du \mathcal{V}^{--} \mathcal{D}^{++} (\mathcal{W}^+)^4 = 0.$$

Thus, the leading low-energy contribution to effective action is given by the following expression

$$\Gamma = -\frac{i}{4} \int_0^1 dy \int_0^\infty d(is) \int d\zeta^{(-4)} du V^{++} i\mathcal{D}^{\alpha\beta}\mathcal{D}_\beta^- \mathcal{D}_\alpha^- \times \frac{1}{6}(is\mathcal{W}^{+\delta}\mathcal{D}_\delta^-)^3 e^{is(\square - \Phi)} (D^+)^4 \delta^{14}(z - z')|_{2=1}. \quad (24)$$

Then let us integrate by parts with respect of the operator $\mathcal{D}^{\alpha\beta}\mathcal{D}_\beta^-$. The following transformations of $\int d\zeta^{(-4)} du V^{++} i\mathcal{D}^{\alpha\beta}\mathcal{D}_\beta^- \mathcal{L}_\alpha^{(+3)}$ look schematically like

$$= \int d\zeta^{(-4)} du \mathcal{D}_\beta^- \frac{1}{4} \varepsilon^{\alpha\beta\gamma\delta} D_\gamma^+ \mathcal{D}_\delta^- V^{++} \mathcal{L}_\alpha^{(+3)}$$
$$= \int d\zeta^{(-4)} du \mathcal{D}_\beta^- \frac{1}{4} \varepsilon^{\alpha\beta\gamma\delta} D_\gamma^+ (\mathcal{D}^{--} D_\delta^+) V^{++} \mathcal{L}_\alpha^{(+3)}$$
$$= -\int d\zeta^{(-4)} du \mathcal{D}_\beta^- \frac{1}{4} \varepsilon^{\alpha\beta\gamma\delta} D_\gamma^+ D_\delta^+ (\mathcal{D}^{--} V^{++}) \mathcal{L}_\alpha^{(+3)}$$

[4] This term determines the divergences of the effective action [1]. In particular, it means that the effective action is finite on the background under consideration.

$$= -\int d\zeta^{(-4)} du \mathcal{D}^-_\beta \frac{1}{4}\varepsilon^{\alpha\beta\gamma\delta} D^+_\gamma D^+_\delta (\mathcal{D}^{++}\mathcal{V}^{--})\mathcal{L}^{(+3)}_\alpha$$
$$= \int d\zeta^{(-4)} du D^+_\beta \frac{1}{4}\varepsilon^{\alpha\beta\gamma\delta} D^+_\gamma D^+_\delta \mathcal{V}^{--}\mathcal{L}^{(+3)}_\alpha$$
$$= \int d\zeta^{(-4)} du d\zeta^{(-4)} du \mathcal{W}^{+\alpha}\mathcal{L}^{(+3)}_\alpha.$$

The expression $\mathcal{L}^{(+3)}_\alpha$ is seen from (24). Here we have used that $D^+_\alpha V^{++} = 0$, $D^{++}(W^+)^3 = 0$ and that for a non-zero result there should be eight $D$-factors acting on $\delta^8(\theta - \theta')$. As a result we obtain that the effective Lagrangian depends only on $\mathcal{W}^{+\alpha}$ and $\Phi$. After these transformations one gets the integrand in (24) in the form

$$\sim (is)^3 \mathcal{W}^{+\alpha}\mathcal{W}^{+\beta}\mathcal{W}^{+\gamma}\mathcal{W}^{+\delta}\mathcal{D}^-_\alpha\mathcal{D}^-_\beta\mathcal{D}^-_\gamma\mathcal{D}^-_\delta(\mathcal{D}^+)^4\delta^8(\theta-\theta')$$
$$\sim (is)^3(\mathcal{W}^+)^4.$$

By using the relation $e^{is\Box}\delta^6(x-x')|_{x=x'} = \frac{i}{(4\pi is)^3}$, we finally obtain

$$\Gamma[\mathcal{W}^+, \Phi] = \frac{1}{12(4\pi)^3}\int d\zeta^{(-4)} du \frac{(\mathcal{W}^+)^4}{\Phi}. \tag{25}$$

The effective action is given as an integral over the analytic subspace of harmonic superspace of the effective Lagrangian $\mathcal{L}^{(+4)}$. It is necessary to point out here that this effective Lagrangian satisfies the condition of analyticity only on the background under consideration where $\mathcal{D}^+_\alpha \mathcal{W}^{+\alpha} = 0$ and $\Phi = \text{const}$. For a generic background we should take into account the terms containing the superfields $\mathcal{Y}^{++}$ and the derivatives of the superfields $\mathcal{W}$, $\Phi$, but all this lies beyond the leading low-energy approximation.

Now we will consider the component structure of the effective Lagrangian in the bosonic sector. By integrating over the anticommuting coordinates $\int d^4\theta^+ = (\mathcal{D}^-)^4$, one gets

$$(\mathcal{D}^-)^4(\mathcal{W}^+)^4$$
$$= \frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\varepsilon^{\alpha'\beta'\gamma'\delta'}\mathcal{D}^-_{\alpha'}\mathcal{W}^{+\alpha}\mathcal{D}^-_{\beta'}\mathcal{W}^{+\beta}\mathcal{D}^-_{\gamma'}\mathcal{W}^{+\gamma}\mathcal{D}^-_{\delta'}\mathcal{W}^{+\delta}$$
$$\sim \frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\varepsilon^{\alpha'\beta'\gamma'\delta'}\mathcal{N}^\alpha_{\alpha'}\mathcal{N}^\beta_{\beta'}\mathcal{N}^\gamma_{\gamma'}\mathcal{N}^\delta_{\delta'} = \frac{1}{4!}\det\mathcal{N}, \tag{26}$$

where we have denoted $\mathcal{N}^\beta_\alpha \equiv \mathcal{D}^-_\alpha\mathcal{W}^{+\beta}|_{\theta=0}$ for (12). A direct calculation of the determinant gives

$$\det\mathcal{N} = (\mathcal{N})^4 - 6(\mathcal{N})^2\mathcal{N}^\beta_\alpha\mathcal{N}^\alpha_\beta + 8(\mathcal{N})\mathcal{N}^\beta_\alpha\mathcal{N}^\gamma_\beta\mathcal{N}^\alpha_\gamma$$
$$- 6\mathcal{N}^\beta_\alpha\mathcal{N}^\gamma_\beta\mathcal{N}^\delta_\gamma\mathcal{N}^\alpha_\delta + 3(\mathcal{N}^\beta_\alpha\mathcal{N}^\alpha_\beta)^2, \tag{27}$$

where $(\mathcal{N}) \equiv \mathcal{N}^\alpha_\alpha = 2\Phi$. It is also evident that $\mathrm{tr}\,\mathcal{N}^3 = 0$. This expression in the limiting case $\Phi = 0$ is in agreement with earlier perturbative calculations of the low-energy effective action of superstrings (see a review and references in [19] and restrictions implied by supersymmetry in 6D [22]). In the bosonic sector we have $\mathcal{N} = \frac{1}{2}(\phi + \mathcal{F})$ where

$$\mathcal{F}_\alpha^{\ \beta} = F_\alpha^{\ \beta} + B_\alpha^{\ \beta}.$$

It follows from the definition (1) in the Abelian case. Here $\phi$ is a scalar bosonic component of the superfield $\Phi$, $F_\alpha^{\ \beta} = (\gamma^{ab})_\alpha^\beta F_{ab}$ is the strength of Abelian vector field and $B_\alpha^{\ \beta} = (\gamma^{ab})_\alpha^\beta B_{ab}$ is the antisymmetric tensor field. Then it is evident that if we substitute relation (27) into expression (25) and consider the bosonic sector, we get the following terms $\phi^3$, $\phi\mathcal{F}^2$, $\frac{1}{\phi}\mathcal{F}^4$ as the quantum corrections induced by the one-loop effect of the hypermultiplet.

## 5. Conclusion

Let us briefly summarize the main results. We have considered a problem of the induced effective action in the 6D $(1,0)$ hypermultiplet theory coupled to an external field of vector/tensor system. The theory is formulated in six-dimensional $(1,0)$ harmonic superspace in terms of an unconstrained analytic hypermultiplet superfield in the external superfields corresponding to an Abelian vector/tensor system. The effective action is formulated in the framework of superfield proper-time technique which allows us to preserve a manifest $(1,0)$ supersymmetry. To calculate the low-energy effective action it is sufficient to consider a special background (19), (20). We have developed a generic procedure for calculating the effective action on such a background and found the leading low-energy contribution to the effective action (25). The divergences in this theory have been computed in our previous paper [1]. It is worth mentioning that the divergences are absent on the background (19), (20).

We expect that the obtained results can have a relation to the problem of the effective action of a single isolated D5-brane [23]. However, to calculate the complete effective action for such a D5-brane we should study a quantum vector/tensor + hypermultiplet system. Of course, such a problem requires a special consideration. Another aspect, which is essential for finding the effective action of a D5-brane, is a necessity to curry out the calculations on a conformally broken phase of the 6D non-Abelian supersymmetric gauge theory (see definition of this phase e.g. in [12]). Nevertheless, we hope that the methods, developed in this paper, can be used to analyze the general problem of the effective action of a D5-brane. The methods and results of the present work can be generalized in the following directions: (i) calculation of the low-energy effective action beyond the leading approximation, (ii) calculation of the effective action in a non-Abelian theory in the broken phase, (iii) calculation of the effective action of the quantum vector/tensor + hypermultiplet system.

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