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Extended general relativity: Large-scale antigravity and short-scale gravity with $\omega = -1$ from five-dimensional vacuum

José Edgar Madriz Aguilar^{a,b}, Mauricio Bellini^{b,c,*}

^a Instituto de Física de la Universidad de Guanajuato, C.P. 37150, León Guanajuato, Mexico

^b Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, C.P. 7600, Mar del Plata, Argentina

^c Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina

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ABSTRACT

Considering a five-dimensional (5D) Riemannian spacetime with a particular stationary Ricci-flat metric, we obtain in the framework of the induced matter theory an effective 4D static and spherically symmetric metric which give us ordinary gravitational solutions on small (planetary and astrophysical) scales, but repulsive (anti gravitational) forces on very large (cosmological) scales with $\omega = -1$. Our approach is an unified manner to describe dark energy, dark matter and ordinary matter. We illustrate the theory with two examples, the solar system and the great attractor. From the geometrical point of view, these results follow from the assumption that exists a confining force that make possible that test particles move on a given 4D hypersurface.

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1. Introduction

In much the term antigravity has come to present all those physical phenomena in which the usual gravitational potential is modified to accommodate repulsive gravitational forces. This is a fascinating subject which has many implications from the possible check of antigravity against experiments to the several theoretical issues that are involved, the 4D principle of equivalence and energy conservation among others [1]. The idea of antigravity has been subject of different approaches through the last decades. Scherk [2] considered this phenomenon in the framework of supergravity related to fermionic generators. In essence, antigravity can be treated as one where the gravitational and other forces between certain objects in a field theory can mutually cancel. Antigravity has been studied from five-dimensional Kaluza–Klein (KK) theory, where the extra dimension is compact [3]. The original version of the KK theory assures, as a postulate, that the fifth

dimension is compact. A few years ago, a non-compactified approach to KK gravity known as Induced Matter (IM) theory was proposed by Wesson and collaborators [4]. In this theory all classical physical quantities, such as matter density and pressure, are susceptible of a geometrical interpretation. Wesson's proposal also assumes that the fundamental 5D space in which our usual spacetime is embedded, should be a solution of the classical 5D vacuum Einstein equations: $R_{AB} = 0$. The mathematical basis of it is the Campbell–Magaard theorem [5], which ensures an embedding of 4D general relativity with sources in a 5D theory whose field equations are apparently empty. That is, the Einstein equations $G_{\alpha\beta} = (8\pi G/c^4)T_{\alpha\beta}$ are embedded perfectly in the Ricci-flat equations $R_{AB} = 0$. In simple terms, Wesson uses the fifth dimension to model matter. More recently, has been suggested that antigravity can be originated as the repulsion effect of matter and antimatter [6]. The relationship between antigravity and antimatter has been studied also in [7]. In the framework of brane world models, has been suggested that antigravity effects could be important for very large scales when gravitons are metastable [8]. The possibility of obtaining an infrared modification to gravity on cosmological scales from extra dimensions has been considered in [9]. Strong antigravity has been obtained by compactification on a manifold with flat directions from a higher-dimensional model [10]. Very

* Corresponding author at: Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, C.P. 7600, Mar del Plata, Argentina.

E-mail addresses: madriz@mdp.edu.ar (J.E. Madriz Aguilar), mbellini@mdp.edu.ar (M. Bellini).

recently was proposed an experiment to measure antigravity with an antihydrogen beam [11].

On the other hand, when applied to cosmic structure on galactic and larger scales, standard 4D general relativity and its Newtonian weak-field limit fail at describing the observed phenomenology. To reconcile the theory with observations we need to assume that $\sim 85\%$ of the mass is seen only through its observational effect and that $\sim 74\%$ of the energy content of the universe is due to either to an arbitrary cosmological constant or to a not well defined dark energy fluid. The cosmological constant problem appears to be so serious as the dark matter problem. The Einstein equations admit the presence of an arbitrary constant Λ . The Friedmann solutions with a positive Λ fit very satisfactorily the observational evidence of an accelerating universe. The problem arises when one wishes to attach a physical interpretation to Λ . Since observations indicate $\Lambda > 0$, the dark energy fluid has negative pressure. Current observations suggest $\omega = -1$ at all probed epochs [12], so models more sophisticated than a simple Λ could seem in principle unnecessary. However, in the context of quantum field theory, the Λ problem translates into an extreme fine-tuning problem, because $\rho_v(t_p)/(\sum \Delta\rho_v) = (1 + 10^{-108})$ is extremely close to 1, but not exactly 1. This problem would disappear if Λ were exactly zero [15]. An alternative conclusion we can draw from this failure is that standard 4D General Relativity must be modified on these cosmic scales, or, in other words that the equation of state for matter is not $\omega_m = 0$, but could be $\omega_m = -1$. In this Letter we explore this idea from a 5D vacuum state using some ideas of the STM theory.

2. The field equations on 4D hypersurfaces

We start by considering a 5D spacetime with a Ricci-flat metric g_{ab} determined by the line element [13]

$$dS^2 = \left(\frac{\psi}{\psi_0}\right)^2 \left[c^2 f(r) dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] - d\psi^2, \quad (1)$$

where $f(r) = 1 - (2G\zeta\psi_0/(rc^2))[1 + c^2r^3/(2G\zeta\psi_0^3)]$ is a dimensionless function, $\{t, r, \theta, \phi\}$ are the usual local spacetime spherical coordinates employed in general relativity and ψ is the space-like extra dimension that following the approach of the induced matter theory, will be considered as non-compact. In this line element ψ and r have length units, θ and ϕ are angular coordinates, t is a time-like coordinate, c denotes the speed of light, ψ_0 is an arbitrary constant with length units and the constant parameter ζ has units of $(\text{mass})(\text{length})^{-1}$.

Now let us to assume that the 5D spacetime can be foliated by the family of hypersurfaces $\{\Sigma_0: \psi = \psi_0\}$. On every generic hypersurface Σ_0 the induced metric is given by the 4D line element

$$dS_{\text{ind}}^2 = c^2 f(r) dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

Given the symmetry properties of the 5D spacetime, it seems natural to assume that the induced matter on Σ_0 can be globally described by a 4D energy momentum tensor of a perfect fluid $T_{\alpha\beta} = (\rho c^2 + P)U_\alpha U_\beta - P g_{\alpha\beta}$ where $\rho(t, r)$ and $P(t, r)$ are respectively the energy density and pressure of the induced matter. From the relativistic point of view, observers that are on Σ_0 move with $U^\psi = 0$ [see Section 3]. The Einstein field equations on the hypersurface Σ_0 for the metric in (2), read

$$r \frac{df}{dr} - 1 + f = -\frac{8\pi G}{c^2} r^2 \rho, \quad (3)$$

$$r \frac{df}{dr} - 1 + f = \frac{8\pi G}{c^4} r^2 P. \quad (4)$$

The resulting equation of state is

$$P = -\rho c^2 = -\frac{3c^4}{8\pi G} \frac{1}{\psi_0^2}, \quad (5)$$

which technically corresponds to a vacuum equation of state. On the other hand, regarding that the metric in (2) has spherical symmetry, we can associate the energy density of induced matter ρ to a mass density of a sphere of physical mass $m \equiv \zeta \psi_0$ and radius r_0 . If we do that, it follows that M and the radius r_0 of such a sphere are related by the expression $\zeta = r_0^3/(2G\psi_0^3)$. An immediate consequence of this expression is that in principle given an specific value of r_0 , depending of the value of ψ_0 we could induce a large massive object or a mini-massive object. This way we can say that is possible in this case to treat the induced matter as a massive compact object embedded in a 5D vacuum. Some information that we can obtain by simple inspection of the metric (1) is that when $G\zeta = \sqrt{3}/9$ there is a single Schwarzschild radius. In this case the Schwarzschild radius is $r_{\text{Sch}} = \psi_0/\sqrt{3} \geq r_0$. When it is greater than the radius of the sphere of parameter ζ , the compact object has properties very close to those of a black hole on distances $1 \gg r/\psi_0 > r_{\text{Sch}}/\psi_0$, this condition holds when $G\zeta \leq 1/(2\sqrt{27}) \simeq 0.096225$. For $G\zeta > \sqrt{3}/9$ one obtains that $f(r) < 0$ and there is not Schwarzschild radius. When $G\zeta \leq \sqrt{3}/9$ there are two Schwarzschild radius, an interior r_{S_i} and an exterior one r_{S_e} , such that by definition $f(r_{S_i}) = f(r_{S_e}) = 0$. This last case has very interesting properties and we will focus on the study of that properties in some scenarios at astrophysical and cosmological scales. When we assume that the present universe we live in can be modeled on the 4D hypersurface $\Sigma_{H_0}: \psi_0 = cH_0^{-1}$, H_0 being $H_0 = 73 \frac{\text{km}}{\text{s}} \text{Mpc}^{-1}$ the present Hubble constant, we found that the exterior Schwarzschild radius r_{S_e} becomes of the order of size of the Hubble radius which is the size of the present observable universe. On the other hand as the interior Schwarzschild radius r_{S_i} depends strongly of the value of $G\zeta$ then when $G\zeta \ll 1$, the interior Schwarzschild radius r_{S_i} approximates to zero. What makes interesting this case is that an observer located between these two Schwarzschild radius could be able to see a compact object with a horizon event determined by r_{S_i} immersed in our observable universe whose size is determined by the Hubble horizon given by r_{S_e} . This particular case is our interest and in the preceding sections we will study in detail more properties of it.

3. Particle trajectories

In order to study with more detail properties of the metric (2), we shall describe the geodesic trajectories of non-massive and massive test particles. All of these are described by the equation

$$\frac{dU^a}{dS} + \Gamma^a_{bc} U^b U^c = 0, \quad (6)$$

where $U^a = \frac{dx^a}{dS}$ and the five-dimensional velocity conditions are fulfilled respectively

$$g_{ab} U^a U^b = 0, \quad (7)$$

$$g_{ab} U^a U^b = c^2, \quad (8)$$

for non-massive and massive test particles that moves on the 5D Ricci-flat metric (1).

3.1. Non-massive particles

For non-massive particles on the metric (1) the condition of 5D null-geodesics: $g_{ab} U^a U^b = 0$, can be written as

$$\frac{p_t^2}{c^2 f(r)} \left(\frac{\psi_0}{\psi} \right)^2 - \left(\frac{\psi}{\psi_0} \right)^2 \frac{(U^r)^2}{f(r)} - \frac{p_\phi^2}{r^2} \left(\frac{\psi_0}{\psi} \right)^2 - (U^\psi)^2 = 0. \quad (9)$$

For a photon moving radially and with no motion along the fifth coordinate Eq. (9) gives

$$\frac{dr}{dt} = \pm c f(r). \quad (10)$$

For a class of observers located at Σ_{H_0} the coordinate function $f(r)$ only remains positive when $r_{S_i} < r < r_{S_e}$, thus the metric (1) maintains its signature only in the region given by $r_{S_i} < r < r_{S_e}$. Hence the metric (1) is valid on such interval.

Expressing Eq. (9) in terms of the angular coordinate ϕ and of the variable $u(\phi)$ it yields

$$\left(\frac{du}{d\phi} \right)^2 + p_\phi^{-2} \left(\frac{\psi}{\psi_0} \right)^2 (U^\psi)^2 - \frac{1}{c^2} p_t^2 p_\phi^{-2} + u^2 - \frac{2G\zeta\psi_0}{c^2} u^3 - 1 + p_\phi^{-2} \left(\frac{\psi}{\psi_0} \right)^2 (U^\psi)^2 \left[\frac{2G\zeta}{c^2} - (\psi_0 u)^{-3} \right] (\psi_0 u) = 0. \quad (11)$$

If the observer moves with velocity $U^\psi = 0$ on the hypersurface Σ_0 , Eq. (11) reduces to

$$\left(\frac{du}{d\phi} \right)^2 - \frac{p_t^2 p_\phi^{-2}}{c^2} + u^2 - \left(\frac{2Gm}{c^2} \right) u^3 - 1 = 0. \quad (12)$$

From the geodesical point of view, Eq. (6) implies that

$$\frac{dU^\alpha}{dS} + \Gamma^\alpha_{\beta\gamma} U^\beta U^\gamma + F^\alpha = 0, \quad (13)$$

$$\frac{dU^\psi}{dS} + F^\psi = 0, \quad (14)$$

where

$$F^\alpha = 0, \quad (15)$$

$$F^\psi = \Gamma^\psi_{\beta\gamma} U^\beta U^\gamma = - \left(\frac{\psi}{\psi_0} \right)^2 U^\mu U_\mu \Big|_{\psi_0}. \quad (16)$$

Notice that $F^\psi \neq 0$, so that this force acts as a confining force that make possible that non-massive particles move on the hypersurface Σ_0 [14], with $U^\psi = 0$. One is allowed to do it because of the warped product structure of the 5D metric. However, this force is perpendicular to all possible particle trajectories, so that the system remains conservative on Σ_0 .

Differentiating with respect to ϕ we get simply

$$\frac{d^2 u}{d\phi^2} + u = \left(\frac{3Gm}{c^2} \right) u^2. \quad (17)$$

This is the orbit equation for non-massive particles for instead photons. Clearly the expression (17) remains the same as the deflection equation for light obtained in standard 4D general relativity and therefore we can say that in this formalism the photons deflection usually obtained in general relativity can be integrally recovered on Σ_0 .

3.2. Massive particles

For a massive test particle outside of a spherically symmetric compact object in 5D with exterior metric given by (1) the 5D Lagrangian can be written as

$$\begin{aligned} {}^{(5)}L &= \frac{1}{2} g_{ab} U^a U^b \\ &= \frac{1}{2} \left(\frac{\psi}{\psi_0} \right)^2 \left[c^2 f(r) (U^t)^2 - \frac{(U^r)^2}{f(r)} \right. \\ &\quad \left. - r^2 (U^\theta)^2 - r^2 \sin^2 \theta (U^\phi)^2 \right] - \frac{1}{2} (U^\psi)^2. \end{aligned} \quad (18)$$

As it is usually done in literature we can, without loss of generality, confine the test particle to orbits with $\theta = \pi/2$. Thus, from the Lagrangian (18), it can be easily seen that only t and ϕ are cyclic coordinates, so their associated constants of motion p_t and p_ϕ , are

$$p_t \equiv \frac{\partial {}^{(5)}L}{\partial U^t} = c^2 \left(\frac{\psi}{\psi_0} \right)^2 f(r) U^t, \quad (19)$$

$$p_\phi \equiv \frac{\partial {}^{(5)}L}{\partial U^\phi} = - \left(\frac{\psi}{\psi_0} \right)^2 r^2 U^\phi. \quad (20)$$

The five-velocity condition gives

$$\begin{aligned} c^2 \left(\frac{\psi}{\psi_0} \right)^2 f(r) (U^t)^2 - \left(\frac{\psi}{\psi_0} \right)^2 \frac{(U^r)^2}{f(r)} \\ - \left(\frac{\psi}{\psi_0} \right)^2 r^2 (U^\phi)^2 - (U^\psi)^2 = c^2. \end{aligned} \quad (21)$$

By expressing Eq. (21) in terms of the constants of motion given by (19) and (20) we obtain

$$\left(\frac{\psi_0}{\psi} \right)^2 \frac{p_t^2}{c^2 f(r)} - \left(\frac{\psi}{\psi_0} \right)^2 \frac{(U^r)^2}{f(r)} - \frac{p_\phi^2}{r^2} \left(\frac{\psi}{\psi_0} \right)^2 - (U^\psi)^2 = c^2. \quad (22)$$

After rearranging some terms and using the expression for $f(r)$, Eq. (22) can be written as

$$\frac{1}{2} (U^r)^2 + \frac{1}{2} \left(\frac{\psi_0}{\psi} \right)^2 (U^\psi)^2 + V_{\text{eff}}(r) = E, \quad (23)$$

where the effective potential $V_{\text{eff}}(r)$ and the total energy E are given by the expressions

$$\begin{aligned} V_{\text{eff}}(r) &= - \left(\frac{\psi_0}{\psi} \right)^2 \frac{G\zeta\psi_0}{r} + \left(\frac{\psi_0}{\psi} \right)^4 \left[\frac{p_\phi^2}{2r^2} - \frac{G\zeta\psi_0 p_\phi^2}{c^2 r^3} \right] \\ &\quad - \frac{1}{2} \left(\frac{\psi_0}{\psi} \right)^2 \left[(U^\psi)^2 \left(\frac{2G\zeta\psi_0}{c^2 r} - \frac{r^2}{\psi_0^2} \right) - \left(\frac{rc}{\psi_0} \right)^2 \right], \end{aligned} \quad (24)$$

$$E = \frac{1}{2} \left(\frac{\psi_0}{\psi} \right)^4 (p_t^2 c^{-2} + p_\phi^2 \psi_0^{-2}) - \frac{c^2}{2} \left(\frac{\psi_0}{\psi} \right)^2. \quad (25)$$

When one takes $U^\psi = 0$, the induced potential $V_{\text{ind}}(r)$ on the hypersurface Σ_0 is given by

$$V_{\text{ind}}(r) = - \frac{Gm}{r} + \frac{p_\phi^2}{2r^2} - \frac{Gm}{c^2} \frac{p_\phi^2}{r^3} - \frac{c^2}{2} \left(\frac{r}{\psi_0} \right)^2, \quad (26)$$

where $m = \zeta\psi_0$ is the effective 4D physical mass. As in the example studied in Section 3.1, the confining force is given by Eq. (15), and the system is conservative on Σ_0 , because F^ψ is perpendicular to the penta-velocities U^b on all the hypersurface Σ_0 . The first two terms in the right-hand side of (26) correspond to the classical potential, the third term is the usual relativistic contribution and the last term is a new contribution coming from the 5D metric solution (1). The Newtonian acceleration associated to the induced potential (26) reads

$$a = -\frac{Gm}{r^2} + \frac{p_\phi^2}{r^3} - \frac{3Gm}{c^2} \frac{p_\phi^2}{r^4} + \frac{rc^2}{\psi_0^2}. \quad (27)$$

In order to describe purely gravitational effects, let us to consider the case when $p_\phi = 0$. In this case, Eq. (27) reduces to

$$a = -\frac{Gm}{r^2} + \frac{rc^2}{\psi_0^2}. \quad (28)$$

This acceleration becomes null when

$$r_{ga} = \left(\frac{Gm}{c^2} \psi_0^2 \right)^{1/3}. \quad (29)$$

By simple inspection of Eq. (28) it can be easily seen that for $0 < r < (Gm\psi_0^2/c^2)^{1/3}$ the acceleration a is negative, which means that the force acting on the test particle is attractive. For distances $r > (Gm\psi_0^2/c^2)^{1/3}$ the acceleration experimented by the particle is positive and so it is the force, meaning this that on this range the test particle is experimenting a repulsive force. This fact can be interpreted as before $r = (Gm\psi_0^2/c^2)^{1/3}$ the force is gravitational in nature and after this value the force becomes anti-gravitational. We define the radius on which this force is null as the gravitational-antigravitational radius, and will denote it by r_{ga} . A particular limit, which is very interesting because one recover the Schwarzschild case, is obtained when we take the limit $\psi_0 \rightarrow \infty$. In this case

$$\lim_{\psi_0 \rightarrow \infty} r_{ga} \rightarrow \infty,$$

and the effective 4D force on massive particles is purely attractive. Furthermore, as in standard general relativity one obtains the equation of state for matter: $P|_{\psi_0 \rightarrow \infty} = -\rho c^2|_{\psi_0 \rightarrow \infty} = 0$.

The condition for circular motion of the test particle ($dV_{ind}/dr = 0$) acquires the form

$$r^5 - \frac{Gm}{c^2} \psi_0^2 r^2 + \frac{p_\phi^2 \psi_0^2}{c^2} r - \frac{3Gm}{c^4} p_\phi^2 \psi_0^2 = 0. \quad (30)$$

By expressing Eq. (23) as a function of the angular coordinate ϕ (indeed assuming $r = r(\phi)$), we obtain

$$\begin{aligned} & \left(\frac{du}{d\phi} \right)^2 + \left(1 - \frac{2G\zeta\psi_0}{c^2} u \right) (u^2 + (\psi/\psi_0)^2 p_\phi^{-2}) + \frac{(U\psi)^2}{p_\phi^2} \left(\frac{\psi}{\psi_0} \right)^2 \\ & + \frac{(U\psi)^2}{p_\phi^2} \left(\frac{\psi}{\psi_0} \right)^2 \frac{2G\zeta\psi_0}{c^2} \left(1 - \frac{c^2(\psi_0 u)^{-3}}{2G\zeta} \right) u \\ & - c^2 (\psi/\psi_0)^2 p_\phi^{-2} (u\psi_0)^{-2} = c^{-2} p_t^2 p_\phi^{-2} + \psi_0^{-2}, \end{aligned} \quad (31)$$

where we have introduced the new variable $u = 1/r$. This expression is the equation of motion for a massive test particle in the metric (1). When we go down from 5D to 4D the expression (31) yields

$$\begin{aligned} & \left(\frac{du}{d\phi} \right)^2 + \left(1 - \frac{2Gm}{c^2} u \right) (p_\phi^{-2} + u^2) - c^2 p_\phi^{-2} (u\psi_0)^{-2} \\ & = c^{-2} p_t^2 p_\phi^{-2} + \psi_0^{-2}. \end{aligned} \quad (32)$$

In order to simplify the algebraic structure of (32) we derive it with respect ϕ , obtaining

$$\frac{d^2 u}{d\phi^2} + u + c^2 p_\phi^{-2} \psi_0^{-2} u^{-3} = \left(\frac{Gm}{c^2} \right) p_\phi^{-2} + \left(\frac{3Gm}{c^2} \right) u^2. \quad (33)$$

This equation is almost the same that the one usually obtained in the 4D general theory of relativity for an exterior Schwarzschild metric with the exception of the third term on the left-hand side. This new term could be interpreted as a new contribution coming in this case from the extra coordinate.

4. Some applications

To illustrate the approach here presented, we shall study two examples which are of astrophysical interest. As a first application we consider a star of one solar mass and in second place we will consider an object of 6×10^{15} solar masses, which is the mass estimated for the core of the great attractor. In both cases we regard $\psi_0 = cH_0^{-1}$ which means that in this case the size of the fifth dimension corresponds to the present Hubble radius. For simplicity, we shall consider the case $p_\phi = 0$.

4.1. Solar system

The core of our solar system is the sun, which has a mass $m = M_\odot$. Its mass is approximately the 98% of all the mass of the solar system. On the other hand, the Oort cloud is a spherically symmetric cloud of comets which surround the core of the solar system and has a maximum size of approximately 1×10^5 AU $\simeq 1.5 \times 10^{16}$ mts [16]. It defines the outer gravitational limit of the solar system. In order to test our theory we shall calculate the radius r_{ga} , which should be bigger than this gravitational limit.

The expression (29) and the metric function $f(r)$ become

$$r_{ga} = \left(\frac{Gm}{c^3} H_0 \right)^{1/3} \frac{c}{H_0}, \quad (34)$$

$$f(r) = 1 - \frac{2Gm}{rc^2} - \frac{H_0^2 r^2}{c^2}. \quad (35)$$

Employing Eqs. (34) and (35) and using the observables $H_0 = 73$ (km/s)/Mpc $^{-1}$, $c = 299792458$ (mts/s) and $G = 6.6743 \times 10^{-11}$ mts 3 /(kg s 2) we obtain that for a star of mass $m = 1M_\odot$ the gravitational and antigravitational radius is given approximately by $r_{ga} = 2.8771 \times 10^{18}$ mts which in parsecs is $r_{ga} = 93.24$ pc. The interior Schwarzschild radius for this case is $r_{s_i} = 2953.2359$ mts which coincides exactly with the usual Schwarzschild radius obtained in 4D general relativity. The exterior Schwarzschild radius is $r_{s_e} = 1.27 \times 10^{26}$ mts which coincides with the size of the observable universe i.e. it coincides with the present Hubble horizon. With these data the picture we can get from our solar system is that for length scales greater than 93.24 pc the gravity of the sun will present a different face in this case antigravity will be present. This result is compatible with observable structures trapped by the gravitational field of the sun (the Oort cloud which is located approximately to 1×10^5 AU), which are approximately two orders of magnitude below r_{ga} . Notice that this is only an estimation, because we are considering $p_\phi = 0$.

4.2. Great attractor

The Great Attractor is the largest and most important cluster concentration of galaxies in the local Universe. Although the motion of galaxies towards the GA is significant, there is no evidence for any backside infall onto the GA. This suggests that the galaxy flow in this region is just a part of a larger flow, caused by some more massive attractive center. Its mass can be estimated in $m = 6 \times 10^{15} M_\odot$ [17], the radius r_{ga} take the value $r_{ga} = 5.22808 \times 10^{23}$ mts $\simeq 16.94305$ Mpc. The exterior Schwarzschild radius is $r_{s_e} = 1.26699 \times 10^{26}$ mts which is the present day Hubble horizon. Our results for the gravitational influence of the core of the SC are in very good agreement with observation. The r_{ga} estimated in our model for the great attractor is of the same order of magnitude than the observed one. For distances greater than this radius antigravitational effects will appear according to our model. These antigravitational effects could explain why at until astrophysical scales 4D general relativity works predicting purely

gravitational interactions and why at cosmological scales the effects of an accelerated expansion can be affecting matter distributions.

5. Conclusions

In this Letter we have studied on the framework of an extended version of general relativity a 5D vacuum state solution given by the Ricci-flat metric (1). In our approach, matter is considered as a $\omega = p_m/\rho_m = -1$ 4D vacuum state, such that the pressure on the effective 4D manifold is $P = -3c^4/(8\pi G\psi_0^2)$, being $\psi_0 = c/H_0$ the Hubble radius. The effective 4D metric (2), is static, exterior and describes spherically symmetric matter (ordinary matter, dark matter and dark energy) on scales $r_0 < r_{\text{Sch}} < c/H_0$ for black holes or $r_{\text{Sch}} < r < c/H_0$ for ordinary stars with r_0 being the radius of the star. The metric (2) describes both, gravity (for $r < r_{ga}$) and antigravity (for $r > r_{ga}$), r_{ga} being the radius for which the effective Newtonian acceleration (28) becomes zero. Notice that in that calculation we have considered null angular momentums $p_\phi = 0$, fact that allow us to interpret the repulsive force acting on a test particle as a genuine antigravitatory effect. The radius r_{ga} is very important because delimitates distances for which dark energy and ordinary matter (dark matter and ordinary matter) are dominant: $r > r_{ga}$ ($r < r_{ga}$). In resume we conclude that all of these, ordinary matter, dark matter and dark energy can be considered as matter subject to a generalized gravitational field which is attractive on scales $r < r_{ga}$ and repulsive on scales $r > r_{ga}$. In simple words gravity from a 5D vacuum state can have two facets, an attractive nature known as gravity and a repulsive one that we interpret as antigravitational, every one acting on different length scales: astrophysical and cosmological respectively.

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