



Self-accelerating solutions in the cascading DGP braneworld

Masato Minamitsuji

Center for Quantum Spacetime, Sogang University, Shinsu-dong 1, Mapo-gu, Seoul 121-742, South Korea

ARTICLE INFO

Article history:

Received 25 November 2009
 Received in revised form 8 January 2010
 Accepted 8 January 2010
 Available online 15 January 2010
 Editor: M. Trodden

Keywords:

Modified gravity
 Extra dimensions

ABSTRACT

The self-accelerating branch of the Dvali–Gabadadze–Porrati (DGP) five-dimensional braneworld has provided a compelling model for the current cosmic acceleration. Recent observations, however, have not favored it so much. We discuss the solutions which contain a de Sitter 3-brane in the cascading DGP braneworld model, which is a kind of higher-dimensional generalizations of the DGP model, where a p -dimensional brane is placed on a $(p + 1)$ -dimensional one and the p -brane action contains the $(p + 1)$ -dimensional induced scalar curvature term. In the simplest six-dimensional model, we derive the solutions. Our solutions can be classified into two branches, which reduce to the self-accelerating and normal solutions in the limit of the original five-dimensional DGP model. In the presence of the six-dimensional bulk gravity, the ‘normal’ branch provides a new self-accelerating solution. The expansion rate of this new branch is generically lower than that of the original one, which may alleviate the fine-tuning problem.

© 2010 Elsevier B.V. Open access under CC BY license.

Recent observational data with high precision suggest that our Universe is currently in an accelerating phase [1,2]. They are consistent with the presence of a nonzero cosmological constant or quantum vacuum energy, but its value must be extremely tiny. In the context of the braneworld, the Dvali–Gabadadze–Porrati (DGP) five-dimensional model has been a compelling model for the cosmic acceleration [3–5]. The DGP model contains a mechanism to modify the gravitational law just on cosmological scales by the effects of the four-dimensional Einstein–Hilbert term put into the action of our 3-brane Universe. Such an intrinsic curvature term would be induced due to quantum loops of the matter fields which are localized on the 3-brane. The effect of the four-dimensional intrinsic curvature term on the 3-brane recovers the Einstein gravity on small scales but on large distance scales gravitational law becomes five-dimensional. The DGP model realizes the so-called self-accelerating Universe that features a four-dimensional de Sitter phase even though our 3-brane Universe is completely empty. Recent studies, however, have indicated that the observational data have not favored the self-accelerating branch of DGP [6]. The self-accelerating solutions have also faced the disastrous issue of ghost excitations [7]. The energy is not bounded from below and therefore the theory is already pathological even at the classical level.

There are possibilities that the realistic cosmological model may be obtained by generalizing the five-dimensional DGP model to a higher-dimensional spacetime. An extension of the DGP model, where two 4-branes with the induced gravity terms are inter-

secting in the six-dimensional spacetime, has been investigated in Ref. [8]. Another interesting model is so-called the cascading DGP model [9]. The model is constructed by a set of branes of the different dimensionality, where a p -brane is placed on a $(p + 1)$ -dimensional brane and the p -brane action contains the $(p + 1)$ -dimensional induced scalar curvature term. For instance, in the simplest six-dimensional model, a 3-brane Universe whose action contains an induced four-dimensional scalar curvature term is placed on a 4-brane whose action contains an induced five-dimensional scalar curvature term, embedded into a (possibly infinitely extended) six-dimensional spacetime. An extension to the case of an arbitrary number of spacetime dimensions is straightforward in principle. It is expected that in such a kind of model, in the infrared region the gravitational force falls off sufficiently fast to exhibit ‘degravitation’ [9]. The linearized analysis has confirmed this idea in part at the level of the linearized theory [9].

One of crucial questions is the viability of the cascading DGP model. To answer to this question, of course, one should go beyond the linearized analysis and in particular investigate the cosmology. Non-linearities may detect effects which may not appear in the linearized treatment. In addition, cosmology can help to have a better understanding of the model and of the idea of gravity localized through intrinsic curvature terms on the 3- and 4-branes. As the first step to this direction, we will look for the solutions which contain a de Sitter 3-brane. They may give rise to the self-accelerating cosmological solutions in the simplest six-dimensional cascading DGP model.

The system of our interest is that our 3-brane Universe Σ_4 is placed on a 4-brane Σ_5 , embedded into the six-dimensional bulk

E-mail address: minamituji@sogang.ac.kr.

\mathcal{M}_6 . For simplicity, we suppress the matter terms in the bulk and on the branes. The total action is given by

$$S = \frac{M_6^4}{2} \int_{\mathcal{M}_6} d^6 X \sqrt{-G} {}^{(6)}R + \frac{M_5^3}{2} \int_{\Sigma_5} d^5 y \sqrt{-q} {}^{(5)}R + \frac{M_4^2}{2} \int_{\Sigma_4} d^4 x \sqrt{-g} {}^{(4)}R, \quad (1)$$

where G_{AB} , q_{ab} and $g_{\mu\nu}$ represent metrics in \mathcal{M}_6 , on Σ_5 and Σ_4 , respectively. ${}^{(i)}R$ ($i = 6, 5, 4$) are Ricci scalar curvature terms associated with respect to G_{AB} , q_{ab} and $g_{\mu\nu}$. For the later discussion, it is useful to introduce the crossover mass scales $m_5 := M_5^3/M_4^2$ and $m_6 := M_6^4/M_5^3$, which determine the energy scale where the five-dimensional and six-dimensional physics appear, respectively. We assume that $m_5 > m_6$. Then, it is natural to expect that the effective gravitational theory becomes four-dimensional for $H > m_5$, five-dimensional for $m_5 > H > m_6$, and finally six-dimensional for $H < m_6$, where H is the cosmic expansion rate.

We consider the six-dimensional Minkowski spacetime, which is covered by the following choice of the coordinates

$$ds_6^2 = G_{AB} dX^A dX^B = dr^2 + d\theta^2 + H^2 r^2 \gamma_{\mu\nu} dx^\mu dx^\nu, \quad (2)$$

where $\gamma_{\mu\nu}$ is the metric of the four-dimensional de Sitter spacetime with the expansion rate H . The r and θ coordinates represent two extra dimensions and x^μ ($\mu = 0, 1, 2, 3$) do the ordinary four-dimensional spacetime. The surface of $r = 0$ corresponds to a (Rindler-like) horizon and only the region of $r \geq 0$ is considered. Note that the boundary surface of $r = 0$ does not cause any pathological effect because it is not a singularity. We consider a 4-brane located along the trajectory $(r(|\xi|), \theta(|\xi|))$, where the affine parameter ξ gives the proper coordinate along the 4-brane. The 3-brane is placed at $\xi = 0$, and for decreasing value of $|\xi|$ one approaches the 3-brane. We assume the Z_2 -symmetry across the 4-brane and hence an identical copy is glued to the opposite side. Along the trajectory of the 4-brane $\dot{r}^2 + \dot{\theta}^2 = 1$, where the dot represents the derivative with respect to ξ . The induced metric on the 4-brane is given by

$$ds_5^2 = q_{ab} dy^a dy^b = d\xi^2 + H^2 r(|\xi|)^2 \gamma_{\mu\nu} dx^\mu dx^\nu. \quad (3)$$

The point where $r(\xi) = 0$ on the 4-brane corresponds to a horizon and the 4-brane is not extended beyond it. The 3-brane geometry is exactly de Sitter spacetime with the normalization condition $Hr(0) = 1$,

$$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu = \gamma_{\mu\nu} dx^\mu dx^\nu. \quad (4)$$

The nonvanishing components of the tangential and normal vectors to the 4-brane are given by

$$u^r = \dot{r}, \quad u^\theta = \dot{\theta}, \quad n^r = \epsilon \dot{\theta}, \quad n^\theta = -\epsilon \dot{r}. \quad (5)$$

We restrict that the region to be considered is to be $r > 0$ and the 3-brane is sitting on r -axis ($\theta = 0$). The 4-brane trajectory is Z_2 -symmetric across the 3-brane. $\dot{\theta} > 0$ for increasing ξ . In the case of $\epsilon = 1$, the bulk space is in the side of increasing r , while in the case of $\epsilon = -1$, the bulk space is the side of decreasing r .

The components of the extrinsic curvature tensor defined by $K_{ab} := \nabla_a n_b$ are given by

$$K_{\xi\xi} - K = -\frac{4\epsilon}{r} (1 - \dot{r}^2)^{1/2},$$

$$K_{\mu\nu} - q_{\mu\nu} K = \epsilon \left(-\frac{3}{r} (1 - \dot{r}^2)^{1/2} + \frac{\ddot{r}}{(1 - \dot{r}^2)^{1/2}} \right) q_{\mu\nu}. \quad (6)$$

The junction condition is given by

$$M_6^4 [K_{ab} - q_{ab} K] = (M_5^3 {}^{(5)}G_{ab} + M_4^2 {}^{(4)}G_{\mu\nu} \delta_a^\mu \delta_b^\nu \delta(\xi)), \quad (7)$$

where the square bracket denotes the jump of a bulk quantity across the 4-brane and the components of the Einstein tensor on the 4- and 3-branes are given by

$${}^{(5)}G_{\xi\xi} = -6 \frac{1 - \dot{r}^2}{r^2},$$

$${}^{(5)}G_{\mu\nu} = 3 \frac{\dot{r}^2 + r\ddot{r} - 1}{r^2} q_{\mu\nu}, \quad (8)$$

and ${}^{(4)}G_{\mu\nu} = -3H^2 \gamma_{\mu\nu}$. By taking the Z_2 -symmetry across the 4-brane into consideration, the matching condition becomes

$$-M_6^4 \epsilon \frac{4}{r} (1 - \dot{r}^2)^{1/2} = -3M_5^3 \frac{1 - \dot{r}^2}{r^2},$$

$$M_6^4 \epsilon \left(-\frac{3}{r} (1 - \dot{r}^2)^{1/2} + \frac{\ddot{r}}{(1 - \dot{r}^2)^{1/2}} \right) = \frac{3M_5^3}{2} \frac{\dot{r}^2 + r\ddot{r} - 1}{r^2}. \quad (9)$$

The way to construct the solution is essentially the same as the case of a tensional 3-brane on a tensional 4-brane (see Appendix A).

In our case, it is suitable to take $\epsilon = +1$ branch. Then, the junction condition tells that the trajectory of the 4-brane is given by $r(\xi) = a^{-1} \cos(a|\xi| - a\xi_0)$ with

$$a = \frac{4m_6}{3}, \quad (10)$$

where we assume $0 < a\xi_0 < \pi/2$. $r(\xi)$ vanishes at $|\xi| = |\xi_{\max}| = \pi/(2a) + \xi_0$. Note that, as mentioned before, the surface of $r = 0$ corresponds to a horizon and on the 4-brane there are horizons at $|\xi| = |\xi_{\max}|$, namely at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them [10]. Now an identical copy is attached across the 4-brane. The normalization of the overall factor of the metric function at the 3-brane place requires $\cos(a\xi_0) = a/H \leq 1$. Note that

$$H \geq \frac{4m_6}{3}. \quad (11)$$

The \ddot{r} term gives rise to the contribution proportional to $\delta(\xi)$. Here, by noting that

$$\frac{d}{d\xi} \arctan\left(\frac{\dot{r}}{\sqrt{1 - \dot{r}^2}}\right) = \frac{\ddot{r}}{\sqrt{1 - \dot{r}^2}} \quad (12)$$

and integrating the (μ, ν) -component of the junction equation (9) across $\xi = 0$, one finds

$$M_6^4 (4a\xi_0) = 6M_5^3 a \tan(a\xi_0) - 3H^2 M_4^2, \quad (13)$$

which with Eq. (10) leads to

$$\frac{H}{2m_5} - \left(\sqrt{1 - \frac{16m_6^2}{9H^2}} - \frac{2m_6}{3H} \arctan\left(\sqrt{\frac{9H^2}{16m_6^2} - 1}\right) \right) = 0. \quad (14)$$

The solution of Eq. (14) determines the value of the expansion rate H . The 3-brane induces the deficit angle $4a\xi_0$ in the bulk. The configuration of the bulk space is shown in Fig. 1. The bulk space is outside the curve of the 4-brane and has an infinite volume. As mentioned before, the surface of $r = 0$ corresponds to a horizon and, in particular, on the 4-brane there are horizons at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them. Note that this surface does not cause any pathological effect.

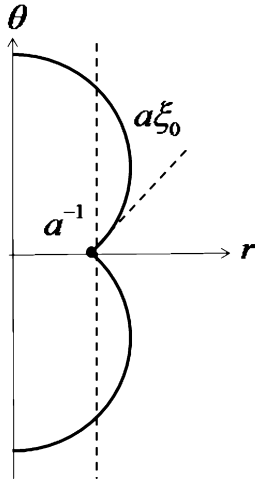


Fig. 1. The configuration of the bulk space is shown. The circle point and solid curve represent the 3- and 4-branes, respectively. According to the direction of the normal vector, the bulk space is outside the curve of the 4-brane. In this picture, each point represents the four-dimensional de Sitter spacetime. Because of the Z_2 -symmetry, an identical copy of this picture is glued across the 4-brane. The bulk space is outside the curve of the 4-brane and has an infinite volume. The surface of $r = 0$ corresponds to a horizon and, in particular, on the 4-brane there are horizons at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them.

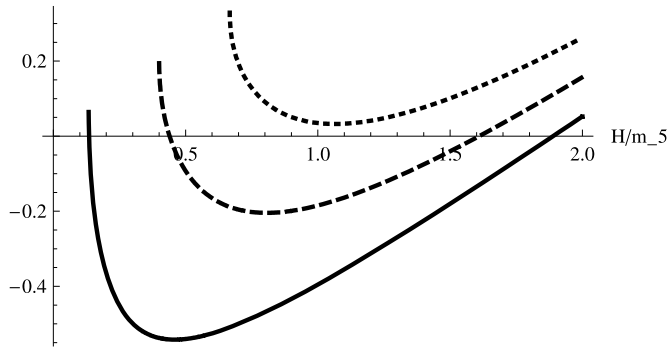


Fig. 2. The left-hand side of Eq. (14) is shown as a function of H/m_5 for fixed ratio m_6/m_5 . The solid, dashed and dotted curves correspond to the cases of $m_6/m_5 = 0.1, 0.3, 0.5$, respectively. In the last case, in which m_6/m_5 is above the critical value $(m_6/m_5)_{\text{crit}} = 0.46978$, there is no solution.

For generic values of m_6 , in Fig. 2, the left-hand side of Eq. (14) is shown as a function of H/m_5 for each fixed ratio m_6/m_5 . It is found that below the critical ratio $m_6/m_5 < (m_6/m_5)_{\text{crit}} \approx 0.46978$, there are two branches of solutions, which are here denoted by $H_+ > H_-$. On the other hand, for $m_6/m_5 > (m_6/m_5)_{\text{crit}}$, there is no solution of Eq. (14). In the marginal case of $m_6/m_5 = (m_6/m_5)_{\text{crit}}$, there is the degenerate solution given by $H \approx m_5$. For generic values of m_6/m_5 , in Figs. 3 and 4, the solutions H_+ and H_- are shown as functions of $m_6/m_5 (< (m_6/m_5)_{\text{crit}})$, respectively. In the limit of $m_6 \ll m_5$, another solution is given approximately given by

$$H_+ \approx 2m_5, \quad H_- \approx \frac{4m_6}{3}. \quad (15)$$

In the absence of the bulk gravity, $m_6 \rightarrow 0$, the (+)- and (-)-branches coincide with the ‘self-accelerating’ and ‘normal’ solutions in the DGP model, with $H_+ = 2m_5$ and $H_- = 0$, respectively. By taking the presence of the six-dimensional bulk into consideration, the self-accelerating branch essentially remains the same. But the normal branch solution provides a new self-accelerating solution if H_- , which could be much smaller than H_+ for $m_6 \ll m_5$.

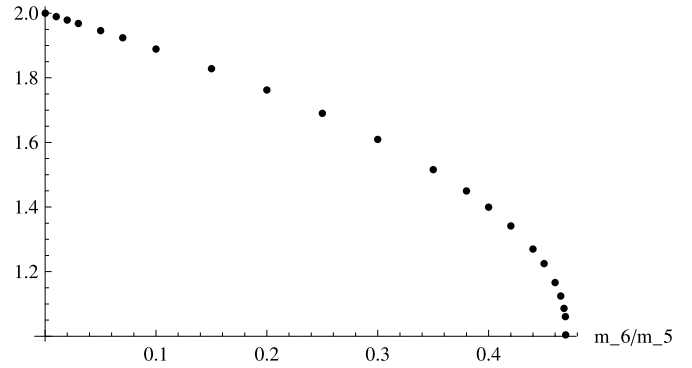


Fig. 3. The larger solution H_+ is shown as a function of m_6/m_5 , in the unit of m_5 .

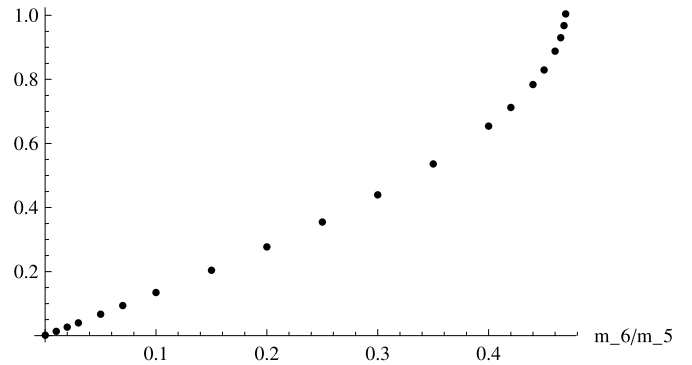


Fig. 4. The smaller solution H_- is shown as a function of m_6/m_5 , in the unit of m_5 .

Note that the existence of both of these new solutions relies on the presence of the 4-brane, since in the limit of $M_5 \rightarrow 0$ none of these solutions can exist.

As we mentioned, the self-accelerating branch of the original DGP model is not favored by recent observations and also suffers a ghost instability. What we found is that in the six-dimensional cascading DGP model, one of two branches, which corresponds to the ‘normal’ branch in the original DGP model, provides a new self-accelerating solution whose expansion rate could be much smaller than that in the other branch, which corresponds to the original ‘self-accelerating’ branch. Thus, the fine-tuning would be relaxed in some degrees. In the self-accelerating solution of the DGP model, the bulk spacetime is infinitely extended and a mode which satisfies the background solution is not normalizable. Thus, the scalar mode is hence different from the zero mode, which already implies the potential pathology about the ghost instability. In our new solutions the 4-brane where the 3-brane resides can never reach the infinity (see Fig. 1) and has a finite volume. Therefore, in analogy with the case of the standard DGP, it implies that the bending mode of the 3-brane would be normalized and hence solutions could be healthy, although the detailed investigations about the stability are left for a future work.

The idea of the cascading gravity may be extendable to the case of an arbitrary number of spacetime dimensions. In the case of n -dimensional spacetime, there would be 3-, 4-, ..., $(n-2)$ -branes, where a p -brane ($p = 3, 4, \dots, n-3$) is placed on a $(p+1)$ -dimensional brane and in the p -brane action the $(p+1)$ -dimensional scalar curvature term is induced with the coupling constant $1/M_{p+1}^{p-1}$. Assuming the hierarchal relation among the crossover scales $m_n \ll m_{n-1} \ll m_{n-2} \ll \dots \ll m_6 \ll m_5$, where $m_q = M_q^{q-2}/M_{q-1}^{q-3}$ ($q = 5, 6, \dots, n$), the solution with the smallest expansion rate would be given by $H \simeq m_n$. Thus, the resultant ex-

pansion rate becomes tiny and the presence of enough number of branes may resolve the fine-tuning problem.

Acknowledgement

The author wishes to thank the anonymous reviewers for his/her comments. This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 20090063070).

Appendix A. The case of pure tension branes

In the system composed of tensional 3- and 4-branes in the six-dimensional bulk, the action is given by

$$S = \frac{M_6^4}{2} \int_{\mathcal{M}_6} d^6 X \sqrt{-G^{(6)}} R + \int_{\Sigma_5} d^5 y \sqrt{-q} (-\sigma_4) + \int_{\Sigma_4} d^4 x \sqrt{-g} (-\sigma_3), \quad (\text{A.1})$$

where σ_3 and σ_4 are tensions of branes. We assume that both the brane tensions are positive. Then, we look for the de Sitter 3-brane solution.

The ansatz of the spacetime metric is assumed to be the same as the case in the text, discussed in Eqs. (2)–(4). Then, the junction condition becomes

$$M_6^4 [K_{ab} - q_{ab} K] = \sigma_4 q_{ab} + \sigma_3 g_{\mu\nu} \delta_a^\mu \delta_b^\nu \delta(\xi). \quad (\text{A.2})$$

The matching conditions on the 4-brane are given by

$$-M_6^4 \epsilon \frac{4}{r} (1 - \dot{r}^2)^{1/2} = \frac{\sigma_4}{2}, \quad (\text{A.3})$$

$$M_6^4 \epsilon \left(-\frac{3}{r} (1 - \dot{r}^2)^{1/2} + \frac{\ddot{r}}{(1 - \dot{r}^2)^{1/2}} \right) = \frac{\sigma_4}{2}.$$

To obtain a 4-brane with a positive tension, it is suitable to choose $\epsilon = -1$ branch. The trajectory is given by $r(\xi) = a^{-1} \cos(a|\xi| + a\xi_1)$ with

$$\sigma_4 = 8M_6^4 a, \quad (\text{A.4})$$

where we assume $0 < a\xi_1 < \pi/2$. $r(\xi)$ vanishes at $|\xi| = |\xi_{\max}| = \pi/(2a) - \xi_1$. Note that the surface of $r=0$ corresponds to a horizon and on the 4-brane there are horizons at $|\xi| = |\xi_{\max}|$, namely at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them [10]. On the other hand, the 3-brane induces the deficit angle given by $4a\xi_1$, which is determined through the 3-brane junction condition as

$$\sigma_3 = M_6^4 (4a\xi_1), \quad (\text{A.5})$$

which is the standard tension-deficit relation for a conical singularity. The normalization condition of 3-brane metric provides $\cos(a\xi_1) = a/H$. It gives the expansion rate H in terms of the 3- and 4-branes tension as

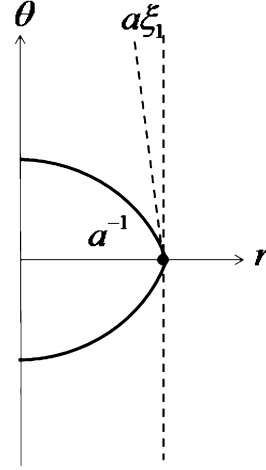


Fig. 5. The configuration of the bulk space is shown. The circle point and solid curve represent the 3- and 4-branes, respectively. According to the direction of the normal vector, the bulk space is inside the curve of the 4-brane. In this picture, each point represents the four-dimensional de Sitter spacetime. Because of the Z_2 -symmetry, an identical copy of this picture is glued across the 4-brane. The bulk space is inside the curve of the 4-brane and has a finite volume. The surface of $r=0$ corresponds to a horizon and, in particular, on the 4-brane there are horizons at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them.

$$H = \frac{\sigma_4}{8M_6^4 \cos\left(\frac{\sigma_3}{4M_6^4}\right)}, \quad (\text{A.6})$$

which is the six-dimensional generalization of the case without the bulk cosmological constant, discussed in Ref. [10]. The configuration of the bulk space is shown in Fig. 5. The bulk space is inside the curve of the 4-brane and has a finite volume. As mentioned before, the surface of $r=0$ corresponds to a horizon and, in particular, on the 4-brane there are horizons at a finite proper distance from the 3-brane. The 4-brane is not extended beyond them. Note that this surface does not cause any pathological effect.

References

- [1] S. Perlmutter, et al., Nature 391 (1998) 51; A. Riess, et al., Astron. J. 116 (1998) 1009.
- [2] D.N. Spergel, et al., Astrophys. J. Suppl. 148 (2003) 175.
- [3] G.R. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485 (2000) 208.
- [4] C. Deffayet, Phys. Lett. B 502 (2001) 199.
- [5] C. Deffayet, G.R. Dvali, G. Gabadadze, Phys. Rev. D 65 (2002) 044023.
- [6] Z.K. Guo, Z.H. Zhu, J.S. Alcaniz, Y.Z. Zhang, Astrophys. J. 646 (2006) 1; R. Maartens, E. Majerotto, Phys. Rev. D 74 (2006) 023004; K. Yamamoto, B.A. Bassett, R.C. Nichol, Y. Suto, Phys. Rev. D 74 (2006) 063525; S. Rydbeck, M. Fairbairn, A. Goobar, JCAP 0705 (2007) 003; L. Amendola, M. Kunz, D. Sapone, JCAP 0804 (2008) 013; W. Fang, S. Wang, W. Hu, Z. Haiman, L. Hui, M. May, Phys. Rev. D 78 (2008) 103509; J.Q. Xia, Phys. Rev. D 79 (2009) 103527.
- [7] K. Koyama, Phys. Rev. D 72 (2005) 123511; D. Gorbunov, K. Koyama, S. Sibiryakov, Phys. Rev. D 73 (2006) 044016; C. Charmousis, R. Gregory, N. Kaloper, A. Padilla, JHEP 0610 (2006) 066; C. Deffayet, G. Gabadadze, A. Iglesias, JCAP 0608 (2006) 012.
- [8] O. Corradini, K. Koyama, G. Tasinato, Phys. Rev. D 77 (2008) 084006.
- [9] G. Dvali, S. Hofmann, J. Khoury, Phys. Rev. D 76 (2007) 084006; C. de Rham, S. Hofmann, J. Khoury, A.J. Tolley, JCAP 0802 (2008) 011.
- [10] L. Grisa, O. Pujolas, JHEP 0806 (2008) 059.