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Visualization of Conics through Augmented Reality

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Abstract

In this short paper we want to share some advances of our broader project dealing with the integration of augmented reality technology for the learning of mathematics. On this occasion we present the conic sections, a subject taught in high school. We stress the importance of a visual and gestural perception in the process of mathematics learning, possible now through the didactic design with this technology. We aim to empower with augmented reality the work of the mathematician G. P. Dandelin named as Dandelin Spheres, to connect the original idea of conic section with the algebraic representation commonly taught.

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1. Introduction

The learning of Mathematics can be enriched through the integration of the Augmented Reality (AR) technology because of its power to represent virtual 3D mathematical objects. This is a great opportunity to make mathematical content “tangible”; we cannot deny the motivational aspects that this feature could bring to students. Besides that, as mathematicians, and researchers in Mathematics Education, we believe that digital technology transforms the learning process of Mathematics. We have experienced that our own mathematics knowledge has been submitted to new challenges when we visualize the enhancement of our perception through the use of emergent digital technologies.

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The use of specialized technology in mathematics gave us also an opportunity to question the adequacy of a mathematical curriculum that has been established before the accelerated evolution of the available software. Our work involves the reconstruction of mathematical content to support a curriculum where mathematics makes sense to students. In this paper we want to share the work carried out by the authors motivated by the effort to enrich the study of Conics in Analytic Geometry.

We want to discuss about a didactic conflict situated in the traditional teaching of these curves which remains mainly in an algebraic fashion. Our aim is to give an alternative for the learning of conics through AR, focusing on the spatial visualization for understanding the particular features that this curves satisfy as conics, which eventually, will give place to their algebraic representation. This is a work in progress, and it is part of a wide project looking for the integration of the AR technology into the mathematics learning.

2. What is meant by the conics?

The conics are curves obtained by the intersection of a plane with a right circular conical surface. There we find the curves familiar to us by the names of circle, parabola, ellipse and hyperbola. Today we have access to eloquent images of these curves and the cone, as soon as we have an Internet connection. It is common to find there also this simple geometric description of the conics.

At Wolfram Web Resources, Weisstein[1] relates this description with the study of the ancient Greek, long before their application to inverse square law orbits. Apollonius of Perga (262 BC - 190 BC) Greek geometer and astronomer wrote the ancient book entitled "On Conics", where he alludes to his predecessor's work, like the one in Euclides´ four Books on Conics. It is well recognized the legacy of the Greek and their influence in later scholars, like Kepler (1571 - 1630), who noticed that planetary orbits were ellipses, and Newton (1643 – 1727) that, under the assumption that gravitational force goes as the inverse square of distance, was then able to derive the shape of the orbits mathematically using calculus.

Our claim with this work binds to that expressed by others: when the conics are studied in high school, the fact that they are derived from the cone is something mentioned in passing; they are quickly defined in the plane by their equations and tied to their focal properties[2].

Thus, on one hand, the attribute of being conic is illustrated with figures, which show the intersection of a plane with a cone. But on the other hand, these curves are defined as the locus of points which have certain conditions. As Weisstein[1] states, a more formally definition of a conic section sets the locus of a point P that moves in the plane of a fixed point F (focus) and a fixed line d (directrix, not containing F) such that the ratio of the distance of P from F to its distance from d is a constant e called eccentricity.

Different values for eccentricity determine the different conics, and once the definition is established, the common procedure is to use it to get algebraically the general equation for each of these curves. We found a conflict in the absence of a didactic explanations between this "being conic" and fulfilling the conditions of the formal definition. Where are the focus and directrix that seem imperative in order to establish its equations?

3. An active interaction through Augmented Reality

In this section we want to emphasize the impact of the active interaction of intersecting a plane with the cone. It is possible through AR technology. In figure 1 we show a mark displaying the virtual cone over a real surface in real time. Our student is interacting with the cone through the use of an additional mark that displays a yellow plane pane. In order to obtain the different conics, the student interacts freely seeking to find the largest number of different curves generated by his actions.

In our work discussions about the design of a didactical environment, we focus on an active visual perception promoted through the own physical actions performed by the students. This is a way in which we understand the term co-action, as stated by Moreno-Armella and Hegedus[3] where the digital artifact offers the opportunity to establish a cognitive relation, fostering an intentional interaction.

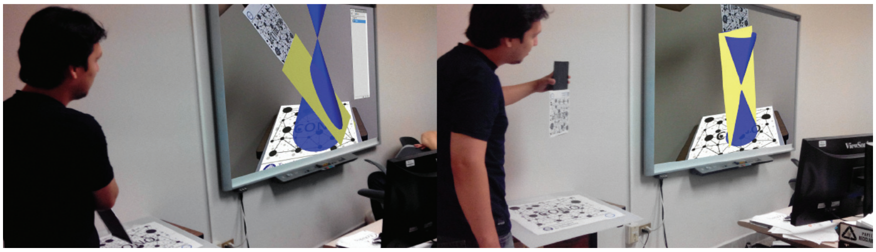

Fig. 1 A student interacting with AR using a mark displaying a yellow plane pane.

This way we are betting to offer students a tangible interaction with Mathematics, now enhanced through AR technology. In Figure 2 we present our student using a different mark, around his fist, which displays the cone in both directions. The gestural interaction is intended to reproduce the curves obtained before (when the cone was standing) now that the plane pane is fixed on the horizontal surface of the table.

Fig. 2 A student interacting with marks displaying the cone.

4. But, where are the focus and directrix?

As seen in last section, we seek to give the student a tangible perception to get the different conic curves through an Augmented Reality interaction. But this is not the only thing we propose. We want to illustrate what we think could be an important contribution from AR technology to identify the focus and directrix for each conic section. This is possible if we consider the "Dandelin Spheres", the masterpiece of the mathematician, soldier and professor of engineering Germinal Pierre Dandelin[4].

In Alsina[5] we find key information for our aim, to make a connection between the conic section method of the Greek, and the focus-directrix method used commonly in our curriculum of Analytic Geometry. It is about a Theorem preceded by a Lemma whose proof is due to Adolphe Quetelet (1796-1894) and Germinal-Pierre Dandelin (1794-1847). The fruit of this work is explaining how to derive the focal-directrix properties of the conic sections from their definition as sections of the cone.

We are not thinking on giving this proof to students, which elegance we could appreciate as mathematicians. Instead we are planning to produce an AR simulation introducing the Dandelin Spheres necessary for each conic section. We want to allow the visualization of the existence of those fixed points and straight lines, known as foci and directrices. In other words, after the student interaction producing a particular conic section, let say a parabola, there is a curve and a plane and the cone fixed. There, the AR simulation begins with the entry of a small sphere that will increase its size in order to become tangent to the cone and the plane. In Figure 3 we show some pictures of the simulation, now in the computer screen.

Once the AR simulation sets the focus and directrix corresponding the conic (the parabola in last figure), the truth that the Lemma and Theorem prove, require from the viewer an active visual perception.

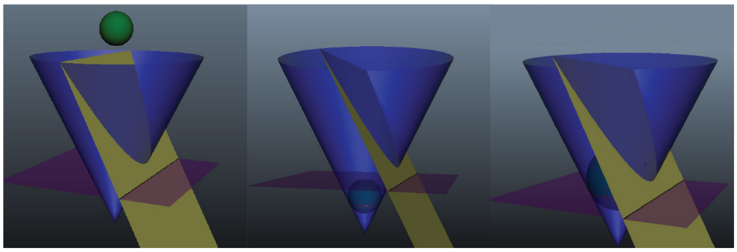

Fig. 3 Scenes in the simulation of the Dandelin Sphere for the parabola.

We are confident that the AR simulation allows the viewing from different angles, and the student could move around the AR scene, which is displaying the different elements in the mathematical proof. Our goal is that the student will be in good conditions to identify the focus and directrix for that particular parabola. This way, he will be able to recognize that property given in Analytic Geometry. In the case of the parabola, he will verify the equal length of the segments from any point P on the parabola, to the focus and to the directrix.

5. Conclusion

It is interesting to find information in Internet as the one shared by Bertrand[2]. As he says, he experienced an effective revelation finding the “ice cream cone proof” in Calculus, Volume 1 from Tom Apostol. There, using the Dandelin Spheres in the case of the ellipse, he produced what could be recognized as a proof by picture that a cone cut obliquely by a plane results in an ellipse as defined by its focal property.

The work in progress we share in this conference, seeks to bring students the opportunity of effective revelations through the interaction with AR technology. We believe that nowadays, augmented reality reveals a new level of visual perception, where the active interaction promotes an active understanding. The particular simulation with the contribution of Dandelin Spheres to determine the focus and directix of a parabola is our first product showing an AR Simulation. Next steps include the simulation corresponding to other conics. We are planning to pilot these products with students of different ages because these curves could become part of a mathematical culture developed through the use of AR technology.

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