

for  $t \geq 0$  and  $(y, z) \in Y \times Z$ . It turns out that in all cases studied so far  $Y$  and  $Z$  are commutative semigroups and  $H$  is a homomorphism from  $Y \times Z$  into  $\mathbb{R}$  or  $\mathbb{C}$  with multiplication:

$$H(y_1 + y_2, z) = H(y_1, z)H(y_2, z),$$

$$H(y, z_1 + z_2) = H(y, z_1)H(y, z_2);$$

$H$  is said to be a duality between the semigroups  $Y$  and  $Z$ .

Examples of semigroups in duality will be discussed. A simple one is:  $Y = [0, 1]$  with  $+$  = supremum;  $Z = [0, 1]$  with  $+$  = infimum;  $H(y, z) = 1$  if  $y \leq z$ , 0 elsewhere.

A general class of Markov processes in duality will be presented in this context. Both  $\eta$  and  $\zeta$  are random walks with state-dependent clock, stopping at the zeros of the semigroups  $Y$  and  $Z$ , so that these zeros are absorbing states. The clock speed of one random walk is the logarithm of the characteristic function of the steps of the other random walk. The special case of the groups  $Y = Z^d$ ,  $Z = (\mathbb{R}/\mathbb{Z})^d$  with  $+$  = addition and  $H(y, z) = e^{2\pi i y z}$  has been considered by Holley and Stroock (1979).

## References

- [1] R. Holley and D. Stroock, Dual processes and their application to infinite interacting systems, *Advances in Mathematics* 32 (1979) 149–174.
- [2] T.M. Liggett, *Interacting Particle Systems* (Springer-Verlag, Berlin, 1985).

## Quick Simulations of Networks of GI/GI/1 Queues

Jean Walrand, *University of California, Berkeley, CA, USA*

A heuristic to speed up simulations of large backlogs in networks of GI/GI/1 queues is proposed. We use an importance sampling technique where the change of measure is based on large deviation estimates.

The change of measure is performed so that the modified system is again a network of GI/GI/1 queues. The service times and interarrival times (or the independent renewal arrival processes) are modified by an exponential change of measure. The routing probabilities are also modified. The calculations required for finding the new measures depend only on the number of queues, not on the backlog level to be investigated. Also, the necessary likelihood ratio calculations are recursive and inexpensive.

We present simulation results (in the case of Jackson networks, for simplicity) that demonstrate significant speed-ups.