Material characteristics at 3D-mixed-mode-loadings

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Abstract

Cracks in structures often grow due to local multiaxial loading situation. Saving structure’s reliability requires fracture mechanical criteria, which were validated by experimental investigations. This article presents 3D-mixed-mode criteria for cyclic loadings, which will be compared with experimental determined threshold values under 3D-mixed-mode-loadings for different materials as well as crack kinking and twisting angles. Using specially developed specimens and loading devices the experiments are performed for pure mode I-loading, pure mode II-loading, pure mode III-loading and 2D- as well as 3D-mixed-mode-loading combinations. The comparison of mixed-mode threshold values, resulting from fatigue experiments, with 3D-criterion by Richard reveals a widely validity and a generally conservative behaviour of the criterion by Richard.

Keywords: 3D-mixed-mode criteria; CTSR-specimen; fatigue crack growth; threshold values

1. Introduction

Cracks often are subjected to spatial mixed-mode-loadings. Therefor different reasons exist. While manufacturing process of structures and parts cracks, which in general are orientated arbitrarily to the loading direction, can already exist. Changing the loading direction during the operation can equally cause a mixed-mode-loading situation at the crack front, Richard et al. (2003 a, b).

It is imperatively required to predict the crack growth at 3D-mixed-mode-loading as effective as possible. Very important for that is the determination of characteristic fracture mechanical values, which allow to make a prediction for stable as well as unstable crack growth and, of course, the effect of mixed-mode-loading on the crack growth rate.

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In addition, the calculation of the crack growth direction respectively orientation is important too for characterising the crack growth behaviour under combined loading completely. For the prediction of those characteristic values some theoretical concepts were developed over decades (Richard et al. (2003 a, b), Richard et al. (2005), Schöllmann et al. (2001), Schöllmann et al. (2002)). In contrast to in-plane mixed-mode loading conditions experimental investigations and results for 3D-mixed-mode-loadings to validate the hypotheses only a few exist or are still missing.

With the development of the CTSR-specimen with corresponding loading device since 2009 (Schirmeisen and Richard (2009)) the possibility was found to perform experimental investigations under nearly any three-dimensional mixed-mode-conditions. This contribution shows and discusses threshold values for mixed-mode-loading and criteria’s validity.

### 2. Fatigue crack growth under 3D-mixed-mode-loading

The $K$-concept for mode I-loading means that fatigue crack growth occurs as soon as the cyclic stress intensity factor for mode I $\Delta K_I$ reaches the threshold value $\Delta K_{I,th}$. At cyclic 3D-mixed-mode-loading the stress field near the crack front is not only defined by $\Delta K_I$ but also by the stress intensity factor for mode II $\Delta K_{II}$ as well as for mode III $\Delta K_{III}$. Out of this three cyclic stress intensity factors a cyclic comparative stress intensity factor $\Delta K_V$ can be formulated as (Richard et al. (2014)):

$$\Delta K_V = \frac{\Delta K_I}{2} + \frac{1}{2} \sqrt{\Delta K_{II}^2 + 5.336 \cdot \Delta K_{II}^2 + 4 \cdot \Delta K_{III}^2}$$  \hspace{1cm} (1)

A crack under spatial mixed-mode-loading is able to propagate, if the cyclic comparative stress intensity factor $\Delta K_V$ on the one hand is greater than the threshold value $\Delta K_{I,th}$ and on the other hand lower than a critical stress intensity factor $\Delta K_{IC}$ as Equation 2 shows:

$$\Delta K_{I,th} < \Delta K_V < \Delta K_{IC}$$  \hspace{1cm} (2)

Consequently, 3D-limit surfaces for fatigue crack growth under spatial mixed-mode-loading arise from this correlation, which can be displayed in a $K_I$-$K_{II}$-$K_{III}$-diagram as in Figure 1 illustrated.

![Fig. 1. $K_I$-$K_{II}$-$K_{III}$-diagram for 3D-mixed-mode-loading with fracture limit surface and threshold limit surface](image)

Unstable crack growth will occur, if the local loading condition along the crack front reaches a point on the 3D fracture limit surface (Figure 1). Is the local loading condition at the crack front, i.e. the cyclic comparative stress...
intensity factor $\Delta K_V$, below the 3D threshold limit surface (Figure 1) then no crack growth will occur. Furthermore, the crack growth direction for 3D-mixed-mode-loading condition can easily be predict using the Equation 3 for the crack kinking angle $\varphi_0$ and Equation 4 for the crack twisting angle $\psi_0$ (Richard et al. (2014)):

$$\varphi_0 = \pm \left[ 140^\circ \cdot \frac{|K_{II}|}{|K_I| + |K_{II}| + |K_{III}|} - 70^\circ \cdot \left( \frac{|K_{II}|}{|K_I| + |K_{II}| + |K_{III}|} \right)^2 \right]$$

$$\psi_0 = \pm \left[ 78^\circ \cdot \frac{|K_{II}|}{|K_I| + |K_{II}| + |K_{III}|} - 33^\circ \cdot \left( \frac{|K_{II}|}{|K_I| + |K_{II}| + |K_{III}|} \right)^2 \right]$$

This concept described above, is the 3D-criterion by RICHARD. Based on the stress hypotheses it is helpful for practical application and was developed due to the fact that engineers often use the classical stress hypotheses. Other existing criteria using a comparative stress intensity factor predicting the crack growth under spatial mixed-mode-loading conditions are e.g. the 3D-criterion by POOK (Pook (1980), Pook (2000)) and the $\sigma_1'$ -criterion by SCHÖLLMANN et al. (Schöllmann et al. (2001), Schöllmann et al. (2002)), which are validated by the crack deflection angles in Section 4.

3. CTSR-specimen and loading device

The threshold experiments were performed using the CTSR-specimen (Compact-Tension-Shear-Rotation-specimen) with the corresponding loading device developed by Schirmeisen (2012) and can be also found in Eberlein (2016). Figure 2 illustrates the adjustment of the fracture modes (mode I, mode II and mode III) by the loading angles $\alpha$ and $\beta$ on the loading device.

$$\begin{align*}
\alpha = 0^\circ; \beta = 0^\circ: & \text{ mode I} \\
\alpha = 90^\circ; \beta = 0^\circ: & \text{ mode II} \\
\alpha = 0^\circ; \beta = 90^\circ: & \text{ mode III}
\end{align*}$$

Fig. 2. CTSR-concept: Adjustment of loading angles $\alpha$ and $\beta$ on loading device

The corresponding loading device basically consists of two sickles and two inboard so-called turrets, where the specimen is fixed. By varying the loading angle $\alpha$ in the range of 0° till 90° by 15°-steps the mode I-ratio to mode II respectively mode III is regulated. A mounting position of the loading device of $\alpha = 0^\circ$ corresponds with a pure mode I-loading at the crack front of the CTSR-specimen. Mounting the loading device with the specimen in a position of $\alpha = 90^\circ$ and varying the loading angle $\beta$ by rotating the turrets in the range of 0° till 90° by 15°-steps the loading situation at the crack front of the specimen can be adjusted from pure mode II-loading to pure mode III-loading. Are
both loading angles in a range between 15° and 75° this concept enables investigating the crack growth under 3D-mixed-mode-loading conditions.

4. Experimental results in comparison with fracture criteria

Within this contribution mixed-mode threshold values for Al 7075-T651 were determined using the load rising amplitude test (Campbell and Ritchie (2000), Nalla et al. (2002), Tabernig and Pippan (2002)). Before the fatigue test the specimen were pre-cracked under cyclic compression. The advantage of pre-cracking the specimen in cyclic compression are, however, the left residual tensile stresses, which may cause cyclic plastic deformation and crack initiation (Tabernig and Pippan (2002)). The threshold tests were performed at a constant load ratio of \( R = 0.1 \) by increasing the load amplitude in steps until the threshold value is reached. A more detailed information to the experimental procedure and chosen parameters can be found in Eberlein (2016). Furthermore, a comparison of the determined mixed-mode threshold values for the mentioned aluminium alloy as well as of threshold values of other materials given in literature with the criterion by RICHARD is shown too. In addition, threshold values of different materials and specimen types for mode I-, mode III- as well as for mixed-mode I + III-loading are compared. At the end of this section the validity of the criteria mentioned above is discussed on the measured crack deflection angles.

4.1. Measured threshold values and its comparison with fracture criteria

Fig. 3. Determined mixed-mode threshold values compared to the 3D-criterion by RICHARD
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The mixed-mode threshold values determined by the procedure described above are illustrated in Figure 3. The threshold values of Al7075-T651 measured by Schirmeisen (2012) in comparison to the threshold value surface of the criterion by RICHARD show a good congruence for pure mode I-loading, 2D-mixed-mode-loading and for mixed-mode II + III-loading with small mode III-ratio. A significant variation, with around a factor of 2.2 above the hypothesis (Richard et al. (2012)), depict the threshold values for pure mode III-loading. The threshold values measured within the experiments by Eberlein (2016) in total are closer to the threshold value surface of the hypothesis. The variation in average is around factor 1.8 above the hypothesis. In addition, a comparison of other materials investigated by Vojtek and Pokluda (2012) as well as by Vojtek et al. (2013) show partially similar threshold value ratios $\Delta K_{II,th}/\Delta K_{I,th}$ and $\Delta K_{III,th}/\Delta K_{I,th}$ (see threshold values for austenitic steel in Fig. 3). Ferritic steel exhibits completely other threshold value ratios. Nevertheless, the determined mixed-mode threshold values for different materials in comparison with the criterion by RICHARD point out that this criterion possesses a widely validity and a generally conservative behaviour.

4.2. Threshold values for different materials and specimen types

In the literature some experimental determined threshold values for the loading combination of mode I and mode III are given by e.g. (Pook (1985), Pook and Crawford (1991), Yates and Miller (1989), Yoshioka et al. (1984)). The comparison of the characteristic values with values determined by the CTSR-specimen, shown in Figure 4, reveals that the spread of the values given in literature is considerable, which is connected to various applied specimen types. Threshold values determined by the CTSR-specimen and the corresponding loading device are within the spread of the threshold values in Figure 4 and thereby demonstrate its solid performance.

![Fig. 4. Mode I-, mode III- and mixed-mode I + III-threshold values for different materials and specimen types in comparison](image)

4.3. Criteria’s validity by measured crack kinking and crack twisting angles

Based on the measured crack kinking angle $\phi_0$ and crack twisting angle $\psi_0$ the criteria’s validity predicting the crack growth orientation is examined. Concerning this, Figure 5 shows the measured crack deflection angles for each criterion mentioned herein. The blue dots are measured by Schirmeisen (2012) and are also published by Eberlein (2016). The green dots are values measured within this investigation. The comparison of the crack kinking angle $\phi_0$
exhibits the least deviations for the $\sigma'_I$-criterion by SCHÖLLMANN et al. as well as the 3D-criterion by RICHARD. These criteria predict the crack kinking angle very good.

Fig. 5. Comparison of measured crack deflection angles $\varphi_0$ and $\psi_0$ with the criteria by RICHARD (a), POOK (b) and SCHÖLLMANN et al. (c)
Moreover, it should be noted that the greatest deviations of the 3D-criterion by Richard occur in a range of low mode II-parts between \(0.1 \leq K_{II} \leq 0.2\). Here always smaller crack kinking angles are calculated. However, the greatest measuring differences at \(\sigma'_1\)-criterion by Schöllmann et al. for loading combinations of mode II and mode III are in an area of higher \(K_{II}^o\)-parts with \(K_{II}^o > 0.6\). All in all, the average deviation of the crack kinking angle, measured by Schirmeisen (2012), is around ± 7°. Within this experiment no significant crack kinking angle at pure mode III-loading was measured. By comparison hereto, Schirmeisen (2012) measured for pure mode III-loading a crack kinking angle \(\phi_0\) of ca. 11°. Generally, for pure mode II-loading slightly smaller crack kinking angles, with a maximum deviation of ca. 7° and an average deviation of ca. 4° from the predicted value by criteria, were measured. The criterion by Pook compared to the others exhibits the greatest discrepancies to the resulting values by the prediction of the crack kinking angle as well as crack twisting angle.

A very good accordance of criteria by Richard and by Schöllmann et al. arises by the forecast of the crack twisting angle \(\psi_0\). The average deviation to the measured crack deflection angles for both criteria is ca. 4° (Schirmeisen (2012)). Within this research the crack twisting angles for pure mode III-loading are around 3° smaller as the criteria predict. A comparison of the crack deflection angles between the \(\sigma'_1\)-criterion by Schöllmann et al. and FE-simulations is i. a. published by Kullmer et al. (2013).

The measurement resp. determination of the crack twisting angle generally is very difficult. The crack twisting angle should be measured at the local crack initiation point as close as possible. Fatigue crack under spatial mixed-mode-loading with high mode III-part propagates by segmenting in several daughter-cracks along the crack front. Therefore many crack initiation points exist, where the cracks propagate at first faceted and then each daughter-crack after further crack growth unify to one continuous crack front. Each daughter-crack or so-called facet is a new crack front with a corresponding crack twisting angle. In addition, it can be supposed that plastic deformations during rupturing the specimen at each facet occur. This plastic deformations distort, of course, the real crack twisting angles.

Furthermore, some typical fractured surfaces for pure mode I-, pure mode II as well as pure mode III-loading developed within this investigations are pictured in Figure 6. Under pure mode I-loading the fatigue crack grows perpendicular to the loading direction. A pure mode II-loading leads the crack to a characteristic kinking of ca. 70°, while a crack under pure mode III-loading conditions twists by nearly 45° and propagates in all directions radially even back to the starter notch.

![Fig. 6. Typical fractured surfaces under pure mode I-, pure mode II- as well as pure mode III-loading](image-url)

**References**


